This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

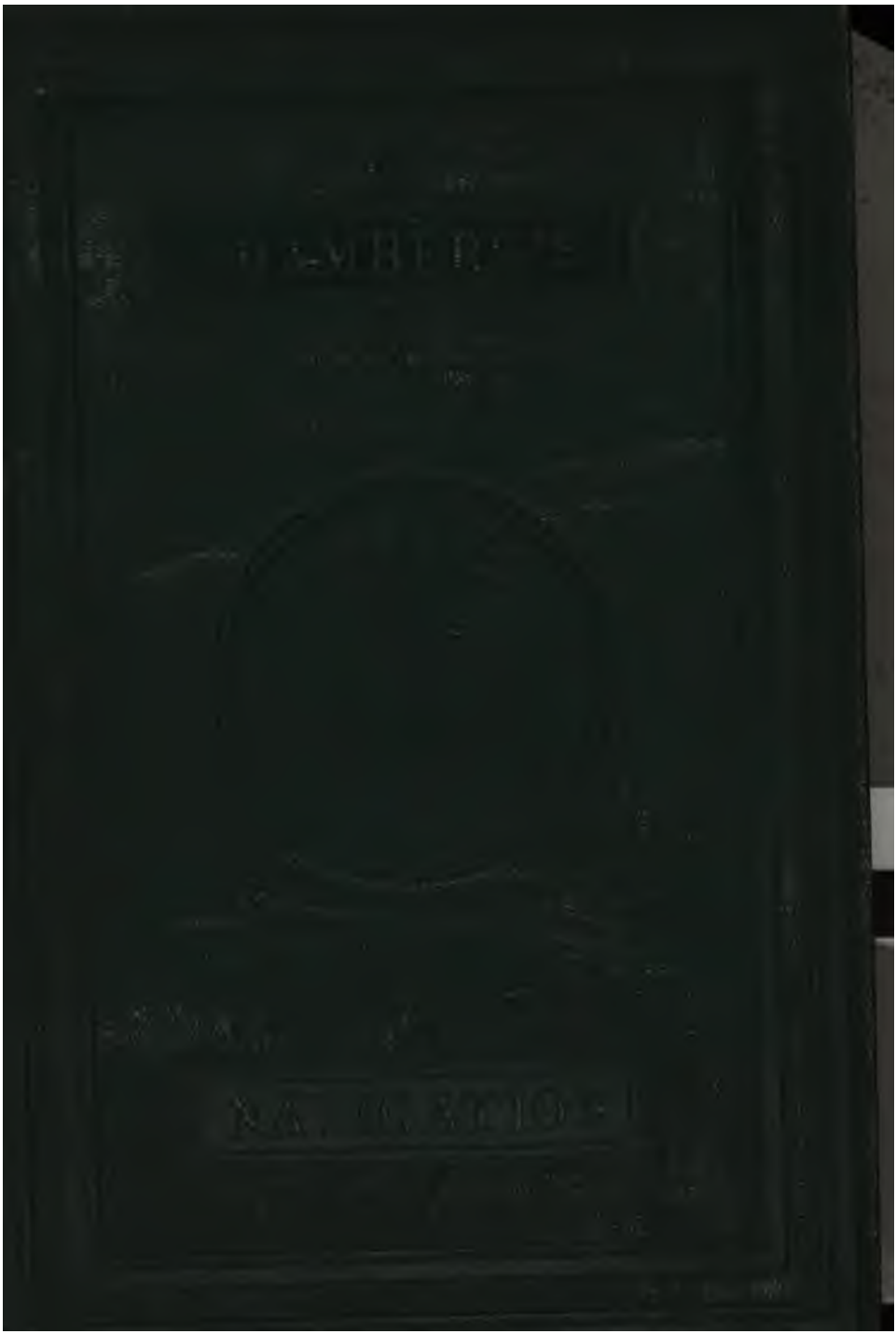
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`



CHAMBERS'S EDUCATIONAL COURSE.

Chambers's Narrative Series of Standard Reading Books.

MESSRS CHAMBERS can recommend with the utmost confidence this series of IMPROVED READING BOOKS, which, since their first editions, have undergone careful revision and improvement, and to which a KEY to the Arithmetical Exercises (*for Teachers only*) has been added. Their objects are:

1. To furnish the means of teaching to read rapidly.
2. To interest the pupil in reading.
3. To be easy and thoroughly consistent with progress.

INFANT SCHOOL READING SHEETS, Nos. 1 to 14, . each 1d.
INFANT SCHOOL PRIMER, 36 pages, cloth 2½d., or wrappers, . 1½d.

BOOK I.	.	80 pages	.	4d.	
BOOK II.	.	112	"	6d.	
BOOK III.	.	160	"	8d.	
BOOK IV.	.	208	"	1s. 0d.	
BOOK V.	.	272	"	1s. 3d.	

BOOK VI. . 320 pages 1s. 6d.
Key to Arithmetic. Price 6d.
SELECT POETRY for FIFTH STANDARD, 200 pages 1s. 0d.

READINGS IN ENGLISH PROSE, 236 pages, Price 2s.
READINGS IN ENGLISH POETRY, 240 Pages, Price 2s.
READINGS IN ENGLISH LITERATURE, 474 Pages, Price 3s. 6d.

The above STANDARD SERIES has already met with a WIDE AND EXTENDED ACCEPTANCE.

Educational Directories.

Infant Treatment under Two Years of Age. 122 *Pages, Price* 1s.

Infant Education from Two to Six Years. 192 *Pages, 8 Wood-cuts.* 1s. 6d.

English Reading and Composition.

Elementary Lesson Sheets.
In 14 *Sheets,* 23 *by* 18 *inches. Price* 1d. *each.*

First Book of Reading.
16 *Pages,* 6 *Wood-cuts, Price* 1½d., *sewed.*

Second Book of Reading.
48 *Pages,* 12 *Wood-cuts, Price* 3d., *sewed.*

Simple Lessons in Reading.
96 *Pages,* 20 *Wood-cuts, Price* 8d.

Lesson Book of Common Things and Ordinary Conduct. 92 *Pages,* 16 *Wood-cuts, Price* 6d. Edited by R. CHAMBERS, LL.D.

English Composition.
48 *Pages, Price* 6d.

Moral Class Book. 208 *Pages, Price* 1s. 6d. Edited by R. CHAMBERS, LL.D.
... revised and improved.

...nglish Grammar.

English Grammar and Composition. 134 *Pages, Price* 1s. 6d.

Etymology, Exercises on.
208 *Pages, Price* 2s.
☞ *Thoroughly revised and improved.*

Principles of Elocution, Oratory, &c. 432 *Pages, Price* 3s. Edited by DR GRAHAM of Edinburgh.
☞ *Thoroughly revised and improved.*

History of the English Language and Literature. 304 *Pages, Price* 2s. By R. CHAMBERS, LL.D.

Spelling-Book, with Dictation Exercises. 192 *Pages, Price* 1s.
*** *Also published in separate Parts (stiff wrappers) as under.*

Part I.	28 pp.	Price 1½d.
II.	80 "	" 6d.
III.	82 "	" 6d.

EDUCATIONAL COURSE—Continued.

Penmanship.

Headlines written and arranged by the late Mr W. Dickson, Writing Master, Edinburgh.

Chambers's Progressive Post Copy-Books. Post 4to. *Price 6d. each.*

Superfine Post Quarto Copy-Books. *Price 6d. each.*

Ten Copy-Books, Foolscap Quarto Size. *Price 3d. each.*

Ten Copy-Books, Foolscap Quarto Size. *Price 2d. each.*

Chambers's Copy-lines, in Twelve Books. *Books 1 to 11, Price 4d. each. Book 12, Price 6d.*

Graduated Writing Sheets for School Walls. Nos. 1 to 15. *Price 2d. each.*

Drawing and Perspective.

First Drawing-Book.
Consisting of Forty-four Different Subjects. 36 *Pages, Price* 1s.

Second Drawing-Book.
36 *Pages, Price* 1s.

Drawing Lesson Books.
A Series of Progressive Lessons in Drawing and Perspective, with some General Instructions. *In* 13 *Books, paper covers, Price* 1s. 6d. *each.*

Drawing-Books, edited by Robert Scott Burn.
In 14 Nos. Mechanical, Architectural, Isometrical, and other Styles; well suited for those who study Mechanics. *Detailed Lists on application.*

Geography, Maps, and Atlases.

Text-Books.

General Geography. [illegible] Pages, 1 Map, 14 Wood-cuts. [illegible] cloth.

Geographical Text-Book of [illegible] Pages, 1 Map, 14 Wood-cuts, [illegible] cloth.

Geographical Text-Book of Scotland. 76 *Pages*, 1 *Map*, 20 *Wood-cuts, Price* 10d., *stiff cloth.*

Geographical Text-Book of Scripture Lands. 76 *Pages*, 1 *Map*, 8 *Wood-cuts, Price* 10d., *stiff cloth.*

Atlases.

Chambers's Primer Atlas.
Nine Quarto Maps. *Price* 2s. 6d.; *or with an additional Map of India, Price* 3s., *cloth.*

Chambers's Minor Atlas.
Fifty-six Maps. [illegible]

Comprehensive Atlas of Ancient and Modern Geography. Thirty-four Maps. *Price* 10s. 6d., *cloth.*

Maps.

Twelve School-room Maps of England, Scotland, Ireland, Europe, Palestine, Asia, Africa, and North and South America. *Price* 12s. *each, unvarnished, or* 14s. *varnished.* The Hemispheres, including Astronomical Illustrations. *Price* 18s. *unvarnished,* 20s. *varnished.* The entire Series may be obtained in a Wooden Case, at 4l., 16s.

Chambers's Outlines of the Principal Countries and Islands in the World. *Price* 1s. 6d. *in a wrapper. Or separately, folio size,* 2d.; *4to size,* 1d. *each.*

History and Biography.

Ancient History.
288 *Pages*, 21 *Wood-cuts, Price* 2s. 6d.

Mediæval History.
372 *Pages*, 7 *Wood-cuts, Price* 3s.

Modern History.
372 *Pages*, 2 *Wood-cuts, Price* 3s. 6d.

History of Rome.
372 *Pages*, 1 *Map, Price* 2s. 6d.

History of Greece.
361 *Pages*, 1 *Map*, 30 *Wood-cuts, Price* 2s. 6d.

History of the British Empire.
340 *Pages*, 13 *Wood-cuts, Price* 2s. Edited by R. Chambers, LL.D.
☞ *Thoroughly revised and improved.*

Exemplary and Instructive Biography. 290 *Pages, Price* 2s.

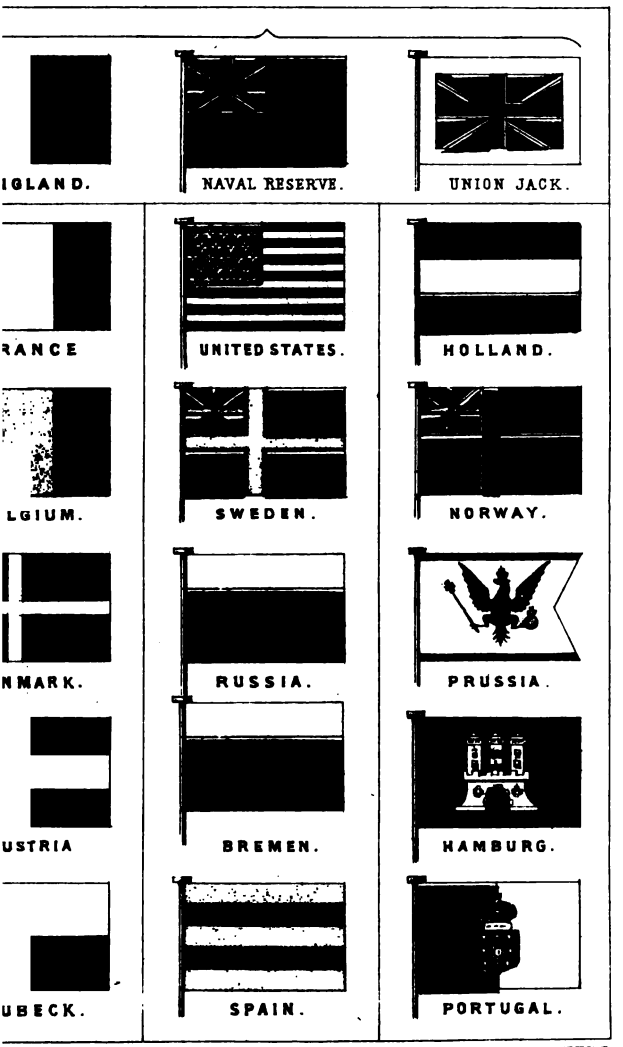
Historical Questions, with Answers. By W. Chambers, F.R.S.E., F.G.S. 210 *Pages*, 53 *Wood-cuts, Price* 2s. 6d.

Historical and Miscellaneous Questions, with Answers. *Price* 4s. 6d.

Questions and Answers in British History. 35 Wood-cuts, Price 1s.

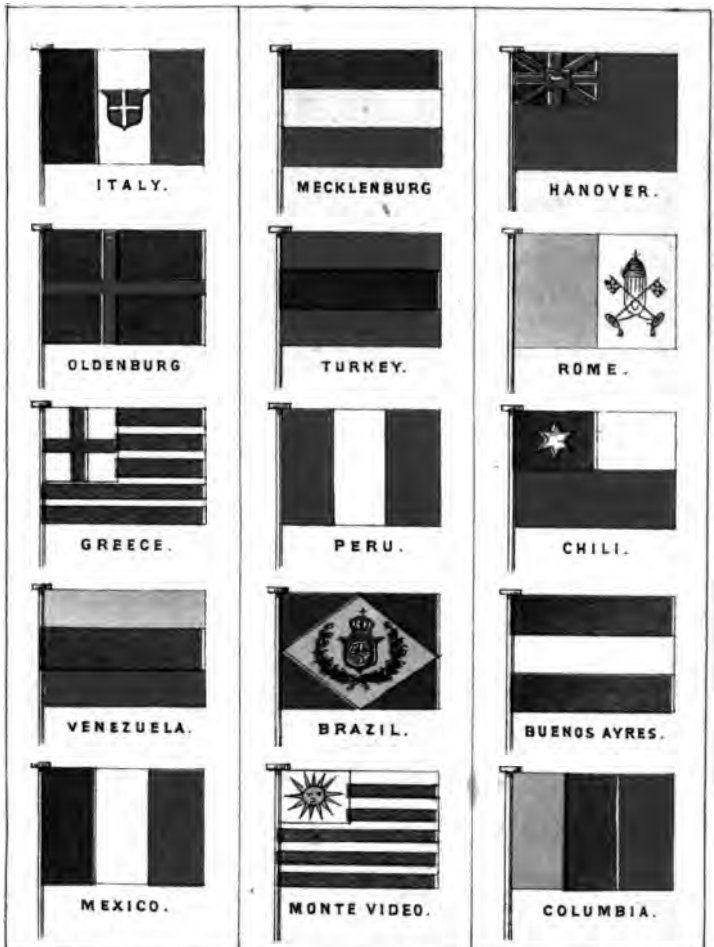
600018046P

National Colours worn by Merchant Vessels of different Countries.



W.H.M.ᶜ Farlane Lithʳ Edinʳ

National Colours worn by Merchant Vessels of different Countries.



W H M.ᵗ Farlane, Lith.ʳ Edin.ʳ

CHAMBERS'S EDUCATIONAL COURSE.

NAVIGATION

BY

JAMES PRYDE

TEACHER OF MATHEMATICS, EDINBURGH

BIBLIOTHECA · ROYAL · NAVAL

W. & R. CHAMBERS

LONDON AND EDINBURGH

1867

231. f. 3.

Edinburgh:
Printed by W. and R. Chambers.

PREFACE.

THOUGH the following Treatise on Navigation has been composed primarily as a text-book for schools, it contains matter which will be found deserving the attention of the experienced mariner.

In it will be found all the problems required by the practical navigator, with the rules for solving them; also the *demonstrations* of these rules, together with examples illustrative of each, and a number of exercises to be solved by the pupil —these exercises *being more numerous than those contained in any other treatise now in use.*

The rules have been given in the most concise form of which they are susceptible, so as not to destroy their perspicuity; and it is hoped that no pupil who has ordinary talent and application will find difficulty in understanding them.

The processes of performing many of the calculations have been considerably shortened, and several problems have been solved in a manner altogether new. Among these may be specially mentioned the rules for Great-circle Sailing, and likewise those for Ex-meridian Altitudes, both of which will be found worthy of the best attention of every mariner, as the solutions are more simple than any hitherto given. The rule

here given for ex-meridian altitudes of the sun, from which to obtain the latitude of the place, is more extensively useful than that commonly given (especially when a good chronometer is on board), being independent of any approximate latitude; but the further the time of observation is from noon, the more accurately should that time be known to obtain correct results.

In order to render this Treatise demonstrative without reference to any other work, sections on Plane and Spherical Trigonometry have been introduced; first, the rules of Right-angled Plane Trigonometry are deduced and illustrated by examples wrought out at length, and followed by numerous exercises for practice by the pupil. The rules for the various cases of Oblique-angled Plane Trigonometry are then deduced, and followed by illustrative examples and appropriate exercises for practice. After these follow the problems in Navigation which depend on these rules, with examples wrought at length, and exercises so numerous as cannot fail to render the pupil expert at their solution.

In like manner, after having given the definitions of Nautical Astronomy, with rules, examples, and exercises in Preparatory Problems, the rules of Right-angled Spherical Trigonometry are demonstrated, and illustrated by examples wrought at length, followed by numerous problems in Nautical Astronomy which can be solved by them. Then follows the investigation of rules for the cases of Oblique-angled Spherical Triangles, which are then applied to the solution of the more advanced problems in Nautical Astronomy, such as finding the time and azimuth from an observed altitude of the sun; and here a *new* rule is given for finding both the *time* and *azimuth* with very little more labour than that used in finding one of them by the rules commonly given.

Rules (followed by examples wrought at length, and arranged in the most convenient form) are also laid down for finding both the latitude and longitude of a place by double altitudes of the sun. A new and very simple process is here explained by which both the latitude and longitude of a place may be found, from two altitudes of the sun, by construction on a Terrestrial Globe; and from it is deduced Sumner's new method of finding a ship's place at sea, followed by two examples of its application wrought out at length.

The demonstrations of four different methods of Clearing the Lunar Distance and of finding the longitude by Lunar Observations are also given, followed by examples shewing the best arrangement of the calculations. These methods are all perfectly accurate, and require only the use of the ordinary tables of natural and logarithmic trigonometrical functions. Those, however, who wish shorter methods, and will be satisfied with near approximations, should consult Thomson's *Lunar and Horary Tables.*

Though the demonstrations of nearly all the rules are given, yet the rules are so enunciated that all the exercises may be solved by them, without mastering their demonstration as is generally done in other treatises; but every navigator will derive a special pleasure from knowing the reason of the operations he performs, and will be thereby much better qualified for advancement in his profession, since the knowledge thus acquired will enable him to pass his examinations before the Nautical Board with more ease and credit.

The exercises that require the use of the *Nautical Almanac* have all been given for the year 1866, so that the use of that almanac is desirable; but as some may use this work on Navigation who may not find it convenient to obtain that

almanac, the data necessary to be extracted therefrom to obtain the solution, are generally given at the end of each question, so that it may be wrought intelligibly without its use; but those who have access to the almanac, should use the data there given, as additional exercises in familiarising them with the many important uses of that volume.

The answers to the exercises, which are inserted before the Appendix, have all been carefully checked, and upwards of a hundred have been recalculated, after being printed, without a single error having been discovered.

The Chapter in the Appendix on the use of the Barometer, Weather Forecasts, and the Law of Storms, has been compiled from the best authorities on these subjects; and it is hoped it will be found of no small importance to the intelligent mariner, who should constantly watch how far the rules and laws there stated agree with his own experience, that he may be the better able to adapt himself to the various and frequently trying positions in which his ship may be placed.

The other subjects contained in the Appendix are all of interest to seamen, who, it is hoped, will derive much improvement from their careful perusal. The Commercial Chapter was furnished by Mr James Bolam of the Board of Trade Nautical School, Leith.

For further information as to the subjects treated of in the Work, the reader is referred to the Table of Contents.

JAMES PRYDE.

EDINBURGH, 1867.

CONTENTS.

	Page
DECIMAL ARITHMETIC	1
Promiscuous Exercises in Decimals	9
ALGEBRAIC AND ANALYTICAL SYMBOLS	11
Exercises	21
Theorems on Proportion	17
Equidifferent and Equirational Series	23
Of Logarithms	25
Properties of Logarithms, and Exercises	28
CONSTRUCTION OF SCALES	34
Gunter's Scale	38
On the Sliding Rule	43
Description and Use of the Sector	45
PLANE TRIGONOMETRY	50
Of Oblique-angled Triangles	59
ASTRONOMY AND GEOGRAPHY	66
Exercises	70
INSTRUMENTS USED IN NAVIGATION	74
Of the Compass	78
Variation of the Magnetic Needle	81
On Leeway and Deviation of the Compass	84
Exercises	87

Page

ON PLANE SAILING 90
Promiscuous Exercises on Plane Sailing 109

TRAVERSE SAILING 113
Exercises in Traverse Sailing 117
PARALLEL SAILING 120
Exercises in Parallel Sailing 129
MIDDLE-LATITUDE SAILING 131
MERCATOR'S SAILING 150
Workman's Table for correcting Middle Latitude 153
Exercises in Middle-latitude and Mercator's Sailing 169

ELEMENTS OF TRIGONOMETRY CONTINUED 174
Exercises in Trigonometry 185

CURRENT SAILING 188
Exercises 193

ON PLYING TO WINDWARD 195
Exercises 201
GREAT-CIRCLE SAILING 202

NAUTICAL ASTRONOMY 218
Definitions, &c. 218
Preparatory Problems in Nautical Astronomy 221
To correct the Observed Altitude of Celestial Objects 230
To deduce the Latitude 237
Exercises in finding the Latitude 240

SPHERICAL TRIGONOMETRY 242
Napier's Rules for the Circular Parts 244
Exercises on Right-angled Trigonometry 249
Oblique-angled Spherical Trigonometry 250
Exercises in Oblique-angled Spherical Trigonometry 256
Latitude by Ex-meridian Altitudes 258
To find the Time at a Given Place 261
To find the Apparent Time and Azimuth by the Sun 264
Exercises on finding the Time and Azimuth 266
Investigation of the Method of Double Altitudes 269

Page

Finding the Latitude by Double Altitudes....271
Exercises in Double Altitudes....276
Latitude and Longitude by Construction....278
Sumner's Method of Double Altitudes....280
Equation of Equal Altitudes....284
Error of Chronometer by Equal Altitudes....286
To calculate the Altitude of a Celestial Body....291
To clear the Lunar Distance....295
To find the Time and Longitude by Lunar Distance....305
Exercises in Lunar Distances....310

ANSWERS....315

APPENDIX....339
On Weather Forecasts and Storms....339
Sextant, its Construction and Use....361
Of a Ship's Journal or Log-book....371
Commercial Code of Signals....383
Mortar and Rocket Apparatus for Saving Life....390
Regulations for preventing Collisions at Sea....392
Diagrams to illustrate the use of Lights....397
Ordinary Examination Papers....400
Astronomical Tables—Description....419
N. A. pages 1 and 2 for October 1866....422
Tables of the Principal Fixed Stars....424
Commercial Chapter....426
Glossary of Nautical Terms....440

CORRECTIONS AND ADDITIONS.

Page.

90, Art. 179, after the first two lines, read 'problem: Of the four quantities, the distance, the course, the difference of latitude, and the departure, having given any two whatever, to find the remaining two.'

94, Art. 188, line 1, for 'latitude,' read 'difference of latitude.'

177, line 2, after 'draw,' insert 'AF perpendicular to BC; and.'

191, 4th line of Solution, after 'produce PO to A;' insert 'and draw CA perpendicular to AP.'

219, Art. 287, add 'And the prime vertical is a great circle passing through the observer's zenith and the east and west points of his rational horizon.'

" Art. 289, line 7, for 'become,' read 'trace out, when their planes are produced.'

220, Art. 295, line 2, for 'a *solar* or an *apparent day*,' read 'an *apparent solar day*.'

221, Art. 298, for the first clause substitute the following: 'MEAN TIME is that which would be shewn by an imaginary *mean* sun revolving in the plane of the equator with the *mean* angular velocity with which the true sun revolves in the ecliptic; the difference,' &c.

265, line 3, read as follows: 'tangent of half the true azimuth, measured from the north, when the observation was taken in north latitude, and from the south when it was taken in south latitude. If the observed azimuth was to the *right* of this true azimuth, the variation is *westerly*, but if to the *left*, the variation is *easterly;* and its amount is the difference between the two. In the same manner may the variation be determined when the object observed is the moon, a star, or a planet.'

270, line 1, for 'ZBP,' read 'ZPB.'

277, Exercise (549), line 10, for '52° 2′ N.,' read '48° 2′ N.'

278, 4th line from the bottom, for 'place of observation' read 'second place of observation.'

285, Art. 362, line 11, beginning with 'log. B,' read as follows: 'log. B is + when the declination is increasing, and — when the declination is decreasing.'

336, Ans. (549), read 'Lat. in 48° 3′ 19″ N., watch 1 m. 43 s. slow for apparent time.'

which are of constant recurrence, and without a competent knowledge of which no progress can be made. These necessary branches are Decimal Fractions, nature and use of Algebraic or Analytical Symbols, nature and use of Logarithms and Logarithmic Arithmetic, and Plane and Spherical Trigonometry. The pupil who has mastered these will be in a position to understand all the principles and processes employed in the theoretical part of Navigation. The elementary principles of Geometry will also be necessary; but for a knowledge of that subject, the student is referred to the *Explicit Euclid* in CHAMBERS'S EDUCATIONAL COURSE.

TREATISE

ON

NAVIGATION.

CHAPTER I.

DECIMAL ARITHMETIC.

1. As nearly all the calculations in Navigation are performed by means of decimals, it is of the utmost importance that the principles and processes of decimals should be thoroughly mastered. For this purpose the following article should be studied as preparatory to the calculations that follow.

2. A decimal fraction is one that has unity with zeros annexed for its denominator, thus $\frac{375}{1000}$ is a decimal fraction, and in the decimal notation is written ·375, without a denominator, but the denominator of every decimal is 1, with as many zeros annexed as there are figures after the decimal point; thus ·4825 means $\frac{4825}{10000}$, ·3475 means $\frac{3475}{10000}$, ·425 means $\frac{425}{1000}$, &c.

3. The first figure to the left of the decimal point is the place of units, the first to the right of the point means tenths of a unit, the second to the right of the point means hundredth parts

of a unit, the third to the right means thousandth parts of a unit, and so on; thus 4·3645, the 4 to the left of the point means four units, the 3 in the first decimal place means $\frac{3}{10}$, the 6 in the second decimal place means $\frac{6}{100}$, the 4 in the third decimal place means $\frac{4}{1000}$, and the 5 in the fourth decimal place means $\frac{5}{10000}$.

Again, $\frac{3}{10}$ is, in the decimal notation, ·3;

$\frac{6}{100}$ " " " ·06;

$\frac{4}{1000}$ " " " ·004;

and, $\frac{5}{10000}$ " " " ·0005.

Adding both sides of these we obtain the following, $\frac{3}{10}+\frac{6}{100}+\frac{4}{1000}+\frac{5}{10000}=$·3645 in the decimal notation; from which it is evident that every zero between the decimal point and a figure diminishes the value of that figure tenfold. Decimals are commonly divided into the following classes—namely, terminating decimals, repeating decimals, circulating decimals, mixed repeaters, mixed circulates, and interminating decimals. When, in reducing a vulgar fraction, or the lower denominations of a compound quantity to a decimal of the higher, the decimal terminates, it is called a *terminating* decimal; when, in performing the same operation, the result gives always the same figure, beginning from the decimal point, it is called a *pure repeater;* but when there are several figures, and then the same figures in the same order, it is called a *pure circulate;* but when several figures intervene between the decimal point and the repeating or circulating part, it is called a *mixed repeater,* or a *mixed circulate;* lastly, when the decimal, though carried ever so far, neither circulates nor repeats, it is called an *interminate* decimal.

4. Since a figure in the sixth decimal place means, by the

above, millionth parts of a unit, and in the seventh place, ten-millionth parts of a unit, if the unit of measure is not very large, these places may generally be neglected without affecting the practical accuracy of the results; and hence in Navigation sufficient accuracy is obtained by carrying repeating and circulating decimals to six or seven places of decimals, and then treating them as terminates. But in this case care should be taken to increase the last figure retained by unity, if the next figure be greater than 5; by attending to this the error is always less than one-half of a unit in the last place retained.

5. CASE I.—To change a vulgar fraction to an equivalent decimal.

RULE.—Annex zeros to the numerator, and divide by the denominator, the quotient will be the required decimal, and will contain as many figures after the point as there were zeros used in the division; if there are not as many figures, the defect must be supplied by prefixing zeros to the quotient, and placing the point before the zeros.

Examples.

Change to decimals the following fractions: $\frac{1}{4}$; $\frac{1}{3}$; $\frac{3}{11}$; $\frac{5}{12}$; and $\frac{9}{28}$.

$\frac{1}{4} = \frac{4)100}{25} = \cdot 25$ .. .. .. a terminating decimal.

$\frac{1}{3} = \frac{3)100}{33} = \cdot\dot{3}$ .. .. .. a repeating decimal.

$\frac{3}{11} = \frac{11)30000}{2727} = \cdot\dot{2}\dot{7}$ .. .. .. a circulating decimal.

$\frac{5}{12} = \frac{12)50000}{4166} = \cdot 41\dot{6}$ .. .. .. a mixed repeater.

$\frac{9}{28} = \frac{28)900000000}{32142857} = \cdot\dot{3}2\dot{1}42857\dot{}$ .. .. a mixed circulate.

Exercises 1.

(1.) Change into equivalent decimals, $\frac{3}{8}$; $\frac{5}{6}$; $\frac{7}{16}$; $\frac{11}{15}$; $\frac{13}{66}$; and $\frac{11}{32}$.

(2.) Change into equivalent decimals the following fractions: $\frac{7}{18}$; $\frac{11}{17}$; $\frac{12}{19}$; $\frac{5}{27}$; $\frac{420}{886}$; and $\frac{113}{355}$; carrying interminants to seven decimal places.

(3.) Express the following mixed numbers as decimals: $7\frac{3}{4}$; $4\frac{3}{8}$; $19\frac{5}{16}$; $186\frac{3}{5}$; $493\frac{11}{33}$; and $714\frac{5}{7}$.

6. CASE II.—To change a lower denomination to a decimal of the next higher.

RULE.—Divide the lowest denomination by the number of that denomination which makes one of the next higher, and write the decimal quotient after the next higher; repeat this operation on the result, and so on to the highest denomination.

Example.

Express as degrees and decimals of a degree the following quantity—namely, 27° 18′ 35″.

60	35″
60	18′·583
	27°·30972 *Ans.*

Here, for conveniency of arrangement, we write the 35″ uppermost, and the 18′ and 27° directly under it, and draw a vertical line to the left of these numbers, and to the left of the line opposite these numbers write for divisors the number of that denomination which makes one of the next higher—namely, 60 opposite the seconds, since 60″ = 1′, and 60 opposite the minutes, since 60′ = 1°. Then dividing 35 by 60, we get ·583, which we write after the minutes, which gives 18′·583; this again divided by 60, the number of minutes in a degree, gives the quotient ·30972, which, being annexed to the degrees 27, gives the answer 27°·30972.

Exercises.

(4.) Express as decimals of a pound the following mixed quantities: £5, 17*s.* 6*d.*; £11, 12*s.* 7½*d.*; £118, 7*s.* 9½*d.*; and £218, 15*s.* 10½*d.*

(5.) Express as decimals of a degree the following quantities: 12° 11′ 15″; 17° 40′ 45″; 108° 16′ 7½″; and 85° 19′ 30″.

(6.) Express as decimals of a day the following quantities: 11 days 14 h. 13 m. 12 s.; 28 days 17 h. 11 m. 45 s.; 15 days 17 m. 54 s.; and 113 days 5 h. 19 m. 15 s.

To find the value of a Decimal.

7. RULE.—Multiply the decimal by the number of the next lower denomination that makes one of the denomination of which it is a decimal, point off the same number of decimals from the product as was in the given decimal; the figures to the left of the decimal point will be the number of the next lower denomination which the decimal contains. If there be still a decimal, repeat the operation as before.

Example.

Given ·785425 days, to find its value in hours, minutes, and seconds.

days.
·785425
24
3141700
1570850
18·850800
60
51·012000
60
·720000

Here we multiply the given decimal of a day by 24, the number of hours in a day, and pointing off six decimal places, we obtain 18 hours and ·8502 of an hour, which, being multiplied by 60, the minutes in an hour, gives 51 m. and ·012 of a minute; this again being multiplied by 60, the seconds in a minute, gives ·72 of a second, from which it appears that ·785425 days is equal to 18 h. 51 m. ·72 s.

Exercises.

(7.) Find the value of £3·84625; £·952765; £·075; and £11·6325.

(8.) Find the value of 7°·85425; 123°·8725; 64°·3825; and 57°·2957795.

(9.) Find the value of 3·854625 weeks; 7·82675 days; 3·115 h.; and 11·7825 h.

ADDITION AND SUBTRACTION.

8. Addition and Subtraction of decimals are performed in the same way as in whole numbers; it being only necessary to place the numbers in such a manner that the decimal points in all the numbers may stand under one another, as in the following

Example.

It is required to find the sum of 35·325, 728·25, 39·4265, ·12325, and ·0035; and to find the difference of 754·125 and 42·63425.

35·325
728·25
39·4265
·12325
·0035
803·12825 Sum.

From 754·125
Take 42·63425
711·49075 Difference.

MULTIPLICATION OF DECIMALS.

9. RULE I.—Multiply as in whole numbers, and from the product point off as many decimals as there are decimals in both factors.

10. RULE II.—When a limited number of decimals only is required, invert the multiplier and place it under the multiplicand, in such a manner that its units figure may stand under that decimal place in the multiplicand which is the same as the last figure to be retained in the product; then in multiplying by any figure begin by multiplying the figure of the multiplicand immediately above it, observing to carry from the figure next to the right, and place the products under each other, without coming in a figure, then add in columns.

Example.

Multiply 34·15286 by 7·3425 first in whole, and second so as to be true to *two* decimal places.

By Rule I.:

34·15286
7·3425
17076430
6830572
13661144
10245858
23907002
250·767374550

Here we multiply the multiplicand 34·15286 by the multiplier, the same as in whole numbers, and then point off from the product as many figures for decimals as there are decimals in both factors, and obtain the product 250·76737455.

By Rule II. :

34·15286	Here we invert the multiplier, and place its units figure under
5243·7	the 5 in the second decimal place of the multiplicand; then we
23907	begin by multiplying the 5 immediately above it, and observing to
1024	carry 2, which would be the carriage to that place if the figures to
137	the right, viz., 286, had been multiplied by 7, we thus obtain the
7	line of figures 23907; next, in multiplying by 3, we begin by multi-
2	plying the 1 immediately above it, and carrying 1 from the figures
250·77	to the right, which gives the second line of the product 1024, which

is written under the former line, without coming in a place; in the same manner the other lines are obtained; the last line 2 is got by observing that if the 3 were multiplied by the 5, and the 2 carried to the product which arises from multiplying the 4, we would obtain 17, which being nearer to 20 than 10, we carry 2. Adding then, the sum is 250·77, if in the former entire product we omit all the figures after the second decimal place, since the first figure omitted is 7, which is greater than the half of 10, we increase the last figure by 1, and thus obtain the same result.

Exercises.

(10.) Multiply 378·54 by 4·275; 849·325 by 4·8125; ·7854 by 4·625; and ·0387425 by 39·1425.

(11.) Find the product of 3·1416 by 3·1416; 87·425 by ·0385; 71·125 by 1·2625; and ·78539 by 3·14159, each true to three decimal places.

(12.) Find the product of ·842675 by 2·8425; ·03258 by 7·8245; 35·6285 by ·03183; and ·06285 by 1·8764, each true to two decimal places.

DIVISION OF DECIMALS.

11. RULE I.—Divide as in whole numbers, and point off for decimals from the quotient, as many figures as there are decimals in the dividend more than in the divisor.

12. If the decimals in the dividend be less than those in the divisor, supply the defect by annexing zeros to the end of the dividend.

13. If in carrying on the work zeros have been annexed to the several remainders, they require to be counted along with the decimals of the dividend in determining the decimal point in the quotient.

14. RULE II.—When a limited number of figures in the quotient is sufficient, as soon as the number of figures in the divisor exceeds the additional figures required in the quotient, instead of bringing down a new figure to the remainder, cut off a figure from the divisor, and so on to the end.

15. If, in dividing by the second Rule, the divisor at first contain more figures than there are figures required in the quotient, cut off the excess before commencing the division, and at each succeeding step cut off a figure from the divisor.

Examples.

(*a.*) Divide 738·25017 by 8·37625 to six decimal places.

```
8·37625)738·25017(88·136119 Ans.
        6701000
         6815017
         6701000
          1140170
           837625
           3025450
           2512875
            5125750
            5025750
             1000000
              837625
              1623750
               837625
               7861250
               7538625
                322625
```

Here the decimals in the dividend are the same as those in the divisor, but in carrying on the work there were 6 zeros annexed to the remainders; hence there are 6 decimals in the quotient.

(*b.*) Divide the same by Rule II., so as to obtain the quotient true to three decimal places.

```
8·,3.7.6.2.5)738·25017(88·136
          670100
           68150
           67010
            1140
             838
             302
             251
              51
              50
               1
```

Here, since the integral part of the dividend 738, being divided by the integral part of the divisor 8, gives two integral figures in the quotient, and there are three decimals required, there will be five figures in the quotient; hence we begin with five figures of the divisor, cutting off the 5, and at each succeeding division cutting off another figure, as seen in the margin.

Exercises.

(13.) Divide 34·82725 by 39·5275; 897·25 by ·98725; 42·9365 by 387·25; and 3·1415926 by 180, true to seven decimal places in the quotient.

(14.) Divide ·86253 by 65·125; 9·387625 by 13·425; ·837$\dot{6}$ by ·03; and 1 by 3·1415926, true to six decimal places in the quotient.

(15.) Divide 11·684626 by 3·1482; 9·587625 by ·003525; 73·4286 by 173·25, and 8·475 by 3·83, true to two decimal places in the quotient.

PROMISCUOUS EXERCISES IN DECIMALS.

(16.) Reduce to decimals, $\frac{3}{80}$; $\frac{5}{72}$; $\frac{43}{115}$; $\frac{15}{64}$; $\frac{17}{75}$; and $\frac{41}{125}$; extending the answer to six places of decimals where it is interminate.

(17.) When the radius of a circle is 1, its semicircumference is 3·14159265, and the semicircumference is also divided into 180°, it is required to find the degrees in an arc equal in length to the radius.

(18.) The length of an arc equal in length to the radius is 57°·2957795; express the length of the radius in seconds.

(19.) The circumference of a circle when the diameter is 1 is expressed by the fraction $\frac{355}{113}$ nearly; express this fraction as a decimal.

(20.) There are two thermometers in common use, Fahrenheit's and the Centigrade, 180° of Fahrenheit's equal 100° of the Centigrade; express by decimals a degree of Fahrenheit in terms of the degree of the Centigrade, and a degree of the Centigrade in terms of a degree of Fahrenheit.

(21.) A sidereal day is 23 h. 56 m. 4·09 s.; express this as a decimal of a common day—that is, of 24 hours—and carry the result to nine decimal places.

(22.) A nautical mile is 6082·66 feet, and an imperial mile is 5280 feet; express each of these miles as decimals of the other.

Also, find how near the results are to the decimal values of $\frac{36}{355}$, and $\frac{33}{355}$.

(23.) A mètre is 39·37079 English inches, a kilomètre is 1000 mètres; express as decimals of each other, a kilomètre and an English mile.

(24.) Divide the sum of ·075 and ·0075 by the difference of ·75 and ·075.

(25.) Multiply the sum of 7·345, 8·375, and ·055, by the difference of 12·395 and 11·344.

(26.) If the length of the year be taken at $365\frac{1}{4}$ days instead of 365·2422419 days, its true value, what will the error amount to in four centuries?

(27.) Multiply ·0235 by 8·08; divide ·0625 by ·25; and find the value of ·8435416 of £5.

(28.) If 90 degrees correspond to 100 French grades, how many degrees and grades are there in the sum of 42·45 degrees and 42·45 grades?

(29.) In the Centigrade thermometer the freezing point is zero, and the boiling point is 100°; in Fahrenheit's the freezing point is 32°, and the boiling point is 212°; what degree C. corresponds to 68° F.? And what degree F. corresponds to 42·25° C.?

CHAPTER II.

ON ALGEBRAIC AND ANALYTICAL SYMBOLS.

16. Whatever can be increased or diminished by addition, subtraction, multiplication, or division, is called quantity.

17. When quantities are considered generally or algebraically, they are represented by the letters of the alphabet; the letters a, b, c, &c. at the beginning of the alphabet, are generally used to represent quantities whose values are already known, and the letters x, y, z, &c. towards the end of the alphabet, are generally used to represent quantities whose numerical values are unknown, and which it is the object of the calculator to determine.

18. Besides the letters which indicate quantities, another set of very important symbols is used to indicate operations to be performed upon them; these symbols of operation are chiefly the following:

$+$ (called *plus*) indicates that the quantity after it is to be added to that before it; thus $a+b$ shews that b is to be added to a, and is read a plus b. If in the above the value of a was 7, and that of b was 3, $a+b$ would be $7+3=10$.

19. $=$ (read *is equal to*) means that the quantities between which it stands are of equal values; and quantities so connected are said to form an Equation. It is not a sign of operation.

20. $-$ (read *minus*) indicates that the quantity after it is to be subtracted from that before it; thus, $a-b$ means that the value of b is to be taken away from a. Using the same values as in (18), $a-b=7-3$, which $=4$; therefore, for these values, $a-b=4$.

21. $\sim$ means the difference of the quantities between which it stands, without reference to which is the greater; thus, $a \sim b$ means either $a-b$, or $b-a$; hence $a \sim b = b \sim a$.

22. $\times$ means *into*, or *multiplied by*, and indicates that the quantities between which it stands are to be multiplied together; thus, $a \times b$ means a taken b times, or b taken a times. This operation is, however, sometimes denoted by a point placed between the letters to be multiplied together, as $a \cdot b$; and still more frequently by the letters being placed close together, as the letters of the same word; wherefore, $a \times b = a \cdot b = ab$. If the same numerical values of a and b be taken as in (18), then $a \times b = a \cdot b = ab = 7 \times 3 = 21$.

23. When a letter is to be multiplied by itself, instead of repeating the letter, it is customary to write it only once, with a small figure after it to indicate the number of times it enters as a factor; thus, aa is written a^2, and read a square; so also aaa is written a^3, and is read a cube or a the third power. In the same manner $aaaaa = a^5$, $a^3 \times a^4 = a^7$; and the superscribed small figure is called an *exponent*.

24. $\div$ means *by* or *divided by*, and always indicates that the quantity before it is to be divided by the numerical value of the letter or number after it; thus, $a \div b$ directs us to divide the value of a by the numerical value of b; but this operation is more frequently denoted by writing the quantity to be divided above a line and the divisor below it; thus, $a \div b = \frac{a}{b}$, and taking again the values as in (18), $a \div b = \frac{a}{b} = \frac{7}{3} = 2\frac{1}{3}$.

25. ( ), { }, [ ] enclosing, or ——— drawn over several quantities, denotes that *collectively* they are to be considered as one quantity, and the enclosing symbols are called *brackets*.

26. $\surd$ (called the radical sign), when by itself, denotes the square root; thus, $\surd\overline{36}$ denotes that a number is to be found that, when multiplied by itself, will produce 36, this number is evidently 6, for $6 \times 6 = 36$; and therefore $\surd\overline{36} = 6$.

When a 3 is placed in the radical sign, as $\sqrt[3]{a}$, it denotes the cube root; when a 4 is placed in the radical sign, as $\sqrt[4]{a}$, it

denotes the fourth root; and generally $\sqrt[n]{}$ indicates the nth root of the quantity over which it is placed.

The same thing is also very frequently indicated by means of fractional exponents; thus $\sqrt{a} = a^{\frac{1}{2}}$, $\sqrt[3]{a} = a^{\frac{1}{3}}$, $\sqrt[4]{a} = a^{\frac{1}{4}}$, and $\sqrt[n]{a} = a^{\frac{1}{n}}$.

27. $\pm$ (read *plus or minus*) indicates that the quantity placed after it is first to be added to that before it, and then subtracted from that before it, thus giving two values of the same quantity; thus, $x = a \pm c$, means that c is to be added to a, which gives one value of x, and then subtracted from it, which gives another value. If in the above $a = 12$, $c = 5$, then $x = a \pm c$, means first $x = 12 + 5$, or $x = 17$; and also $x = 12 - 5 = 7$, wherefore x has two values, the one being 17 and the other 7.

28. When a number stands before a letter, or combination of letters, it is called a coefficient, and indicates the number of times the quantity before which it stands is to be taken; thus, $7a$ means that a is to be taken 7 times, and $13(abc + d)$ means that the product of a into b into c increased by d is to be taken 13 times.

29. In applying the principles of notation stated above, the following AXIOMS will be assumed as true:

(*a.*) Quantities that are equal to the same quantity are equal to each other.

(*b.*) If equal quantities are added to equal quantities, the sums are equal.

(*c.*) If equal quantities are added to unequal quantities, the sums are unequal; the greater sum being that which contains the greater of the unequal quantities.

(*d.*) If equal quantities are taken from equal quantities, the remainders are equal.

(*e.*) If equal quantities are taken from unequal quantities, the remainders are unequal; the greater remainder being that which remains of the greater of the unequal quantities.

(*f.*) If unequal quantities are taken from equal quantities, the remainders are unequal; the greater remainder being the excess above the less of the two unequal quantities.

(*g.*) If equal quantities are multiplied by equal multipliers, the products are equal.

(*h.*) If equal quantities are multiplied by unequal multipliers, the products are unequal; the greater product arising from the greater multiplier.

(*i.*) If equal quantities are divided by equal divisors, the quotients are equal.

(*k.*) If equal quantities are divided by unequal divisors, the quotients are unequal; the greater quotient arising from the less divisor.

(*l.*) If unequal quantities are divided by equal divisors, the quotients are unequal; the greater quotient being that of the greater dividend.

(*m.*) The quotient of any quantity divided by itself, is equal to the quotient of any other quantity divided by itself; for each quotient is one.

(*n.*) The whole is equal to all its parts, and therefore greater than any number of the parts less than them all.

(*p.*) If two quantities be equal, and one of them be $\left\{\begin{matrix}\text{greater}\\\text{less}\end{matrix}\right\}$ than a third, the other is also $\left\{\begin{matrix}\text{greater}\\\text{less}\end{matrix}\right\}$ than the third.

(*q.*) If the first of three quantities be greater than the second, and the second greater than the third, still more is the first greater than the third.

(*r.*) Any supposition which contains in it an absurdity, or leads to an impossibility, is false.

30. From (29, *g*), the squares of equal quantities are equal. For if $a=b$, then $a\times a=b\times b$; but $a\times a=aa=a^2$, and $b\times b=bb=b^2$; and therefore $a^2=b^2$, when $a=b$.

31. From (29, *h*), the square roots of equal quantities are equal. For if $a=b$, the square root of a, that is $a^{\frac{1}{2}}$, $=b^{\frac{1}{2}}$, if not, let $a^{\frac{1}{2}}$ be $>$ (read *greater than*) $b^{\frac{1}{2}}$, then since $a^{\frac{1}{2}}>b^{\frac{1}{2}}$, multiplying both by $a^{\frac{1}{2}}$, gives $a>a^{\frac{1}{2}}b^{\frac{1}{2}}$; again, since $a^{\frac{1}{2}}>b^{\frac{1}{2}}$, multiply both by $b^{\frac{1}{2}}$, $a^{\frac{1}{2}}b^{\frac{1}{2}}>b$; hence $a>a^{\frac{1}{2}}b^{\frac{1}{2}}$, and $a^{\frac{1}{2}}b^{\frac{1}{2}}>b$, wherefore (29, *q*) $a>b$; but a is given equal to b, which is impossible. Therefore $a^{\frac{1}{2}}$ is

not greater than $b^{\frac{1}{3}}$, and in the same manner it may be shewn that it is not less, hence $a^{\frac{1}{3}} = b^{\frac{1}{3}}$ when $a = b$.

32. Cor.—In the same manner it may be demonstrated that the cube roots of equal quantities are equal; that is, that if $a = b$, $a^{\frac{1}{3}} = b^{\frac{1}{3}}$; and generally that if $a = b$, $a^{\frac{1}{n}} = b^{\frac{1}{n}}$, whatever n may be.

Exercises 2.

If $a = 4$, $b = 6$, $c = 8$, $d = 2$, $e = 1$, and $f = 0$, find the numerical values of the following expressions:

(30.) $a+b+c-e$; $3a+7b-5c+3d$.

(31.) $ab+4ad-3ce$; $5b+12e+8bc-24ae$.

(32.) $7a+4(c-d)+8b$; $11a(c-b)-3d(a+b)$.

(33.) $a^2+b^2+c^2+d^2-e^2$; $a^2+4ab^2+3a^2c-11a^2e^2$.

(34.) $\frac{ab}{c}+\frac{a^2d^2}{c}-\frac{5ab}{d^2}$; $\frac{a^2+b^2+3d^2-c^2}{a^2+e^2}$.

(35.) $a^3+b^3+4c^3-8a^2b^2$; $a^4+b^4-3abcd+e^5$.

(36.) $(a+b)(c+d)+(c-d)(b-a)$; $\frac{a^2+b^2+d^2+c^2}{c^2-3a^2}$.

(37.) When $x = 12$, find the value of the following:

$$5x^3+3x^2-100x+17; \text{ and } \frac{x^3}{64}+\frac{x^2}{16}+\frac{x}{4}-10.$$

(38.) When $a = 5$, and $x = 11$, find the value of

$$\frac{7a^2x^2}{22}+\frac{10a^3x}{3}+\frac{a}{5}; \text{ and } 15a^3x-3ax^2+7ax.$$

(39.) Find the value of $x\sqrt{(x^2-8y)}+y\sqrt{(x^2+8y)}$, when $x = 5$, and $y = 3$.

(40.) Find the value of $a+(x+b)\sqrt{(x+y)}-(a+x+b)\sqrt[3]{(x-y)}$, when $a = 10$, $b = 8$, $x = 12$, and $y = 4$.

(41.) Find the value of $\sqrt[3]{\{(a+b)^2y\}}+\sqrt[3]{\{(a+x)(y-2a)\}}+\sqrt[3]{\{(y-b)^2a\}}$, when $a = 2$, $b = 3$, $x = 6$, and $y = 5$.

Prove that in the following equations the two sides are equal,

whatever value is given to the letters; only giving the same value to the same letter on both sides.

$$(a+b)^2 = a^2 + 2ab + b^2.$$

$$(a-b)^2 = a^2 - 2ab + b^2.$$

$$(a+b)(a-b) = a^2 - b^2.$$

$$a^3 + x^3 = (a+x)(a^2 - ax + x^2).$$

$$a^3 - x^3 = (a-x)(a^2 + ax + x^2).$$

$$a^2 + x^2 + y^2 - ax - ay - xy = \frac{(a-x)^2}{2} + \frac{(a-y)^2}{2} + \frac{(x-y)^2}{2}.$$

$$a^2 + x^2 + y^2 + ax + ay + xy = \frac{(a+x)^2}{2} + \frac{(a+y)^2}{2} + \frac{(x+y)^2}{2}.$$

$$(a+x+y)^2 = a(a+x+y) + x(a+x+y) + y(a+x+y).$$

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4,$$

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3.$$

33. Ratio is the quotient arising from dividing one quantity by another; the quantity divided is called the antecedent, and the divisor is called the consequent of the ratio; thus, $\frac{a}{b}$ represents the ratio of a to b, a is called the antecedent, and b the consequent of the ratio.

34. When two ratios are equal, they constitute an analogy, or a proportion; that is, if $\frac{a}{b}$ be equal to $\frac{c}{d}$, $\frac{a}{b} = \frac{c}{d}$, or $a : b = c : d$ is called an analogy, or a proportion, and is read in the first form a divided by b is equal to c divided by d, and in the second form a is to b as c is to d; the first form is generally the most convenient in analytical researches, and consequently will most frequently be used.

35. The first and last terms of analogy are called the *extremes*, and the second and third the *means;* thus, in the analogy $\frac{a}{b} = \frac{c}{d}$, a and d are the extremes, and c and b are the means.

36. THEOREM.—In any analogy the product of the extremes is equal to the product of the means.

For if $\frac{a}{b}=\frac{c}{d}$, multiplying both sides by b,

gives $a=\frac{bc}{d}$ (29, *g*); and again, multiplying both sides by d, gives $ad=bc$, which was to be proved.

37. THEOREM.—In any analogy or proportion the antecedents may be changed into consequents, and the consequents into antecedents. This change is called *Inversion.*

For if $\frac{a}{b}=\frac{c}{d}$, then (36) $ad=bc$; dividing these equals by a, and then by c, gives (29, *i*) $\frac{d}{c}=\frac{b}{a}$, or $\frac{b}{a}=\frac{d}{c}$.

38. THEOREM.—If four quantities of the same kind be proportional, then shall the first antecedent be to the second antecedent as the first consequent is to the second consequent. This change is called *Alternation.*

Given $\frac{a}{b}=\frac{c}{d}$, to prove that $\frac{a}{c}=\frac{b}{d}$.

Multiply both sides of this equation by $\frac{b}{c}$ (29, *g*), and cancel the common factors from numerators and denominators on both sides, which gives $\frac{a}{b}\times\frac{b}{c}=\frac{c}{d}\times\frac{b}{c}$, whence $\frac{a}{c}=\frac{b}{d}$.

39. THEOREM.—If four quantities be proportional, then shall the sum of the first and second have to the second the same ratio that the sum of the third and fourth has to the fourth. This is called *Composition.*

Given $\frac{a}{b}=\frac{c}{d}$, or $a:b=c:d$; to prove $\frac{a+b}{b}=\frac{c+d}{d}$, or $a+b:b=c+d:d$.

Since $\frac{a}{b}=\frac{c}{d}$, and $\frac{b}{b}=\frac{d}{d}$ (29, *m*); therefore by (29, *b*) $\frac{a+b}{b}=\frac{c+d}{d}$, or $a+b:b=c+d:d$.

40. THEOREM.—If four quantities be proportional, the difference of the first and second is to the second as the difference of the third and fourth is to the fourth. This is called proportion by *Division.*

Given that $\frac{a}{b}=\frac{c}{d}$, to prove that $\frac{a-b}{b}=\frac{c-d}{d}$; or if b be greater than a, that $\frac{b-a}{b}=\frac{d-c}{d}$.

For $\frac{a}{b}$ is given $=\frac{c}{d}$, and $\frac{b}{b}=\frac{d}{d}$ (29, *m*); and therefore (29, *e*) $\frac{a-b}{b}=\frac{c-d}{d}$, or $\frac{b-a}{b}=\frac{d-c}{d}$.

41. THEOREM.—If four quantities be proportional, the sum of the first and second is to their difference as the sum of the third and fourth is to their difference. This change is called proportion by *Mixing.*

Given that $\frac{a}{b}=\frac{c}{d}$; to prove that $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

Since $\frac{a}{b}=\frac{c}{d}$, by (39) $\frac{a+b}{b}=\frac{c+d}{d}$, and by (40) $\frac{a-b}{b}=\frac{c-d}{d}$; therefore by (29, *i*) $\frac{a+b}{b}\div\frac{a-b}{b}=\frac{c+d}{d}\div\frac{c-d}{d}$, or $\frac{a+b}{b}\times\frac{b}{a-b}$ $=\frac{c+d}{d}\times\frac{d}{c-d}$, and therefore $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

42. THEOREM.—If there be any number of quantities which are proportional, as one antecedent is to its consequent, so is the sum of all the antecedents to the sum of all the consequents.

Given $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$; to prove that $\frac{a}{b}=\frac{a+c+e}{b+d+f}$.

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=r$, say.

Then since $\frac{a}{b}=r$, $\therefore a=rb$;

and since $\frac{c}{d}=r, \therefore c=rd;$

also since $\frac{e}{f}=r, \therefore e=rf.$

Hence by addition $a+c+e=r(b+d+f)$, (29, *b*);

and dividing both sides by $b+d+f$, (29, *i*),

gives $\frac{a+c+e}{b+d+f}=r$; but $r=\frac{a}{b}$;

therefore (29, *a*) $\frac{a}{b}=\frac{a+c+e}{b+d+f}$

43. Cor.—Since it has been proved in the above that if $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then $\frac{a}{b}=\frac{a+c+e}{b+d+f}$, it follows that, *if there be any number of equal fractions, the sum of all the numerators, divided by the sum of all the denominators, is a fraction equal in value to either of them.* Or, if there be any number of equal ratios, the sum of all the antecedents, divided by the sum of all the consequents, is a ratio equal to either of the given ratios.

44. If four quantities be proportional, and other four quantities also be proportional; then the products, arising from multiplying the corresponding terms together, shall also be proportional.

Let $\frac{a}{b}=\frac{c}{d}$, and also $\frac{e}{f}=\frac{g}{h}$; to prove that $\frac{ae}{bf}=\frac{cg}{dh}$.

Since $\frac{a}{b}=\frac{c}{d}$, and $\frac{e}{f}=\frac{g}{h}$, by (29, *g*) $\frac{ae}{bf}=\frac{cg}{dh}$.

45. If there be three quantities and other three, such that taken two and two in order they have the same ratio; then, the first shall have to the third of the first rank, the same ratio that the first has to the third of the second rank.

Let *a*, *b*, *c*, be three quantities, and *d*, *e*, *f*, other three, such that

$$\frac{a}{b}=\frac{d}{e}, \text{ and } \frac{b}{c}=\frac{e}{f}; \text{ to prove that } \frac{a}{c}=\frac{d}{f}$$

Since $\frac{a}{b}=\frac{d}{e}$, and $\frac{b}{c}=\frac{e}{f}$, by (29, *g*) $\frac{a}{b}\times\frac{b}{c}=\frac{d}{e}\times\frac{e}{f}$;

hence $\frac{a}{c}=\frac{d}{f}$.

46. If there be three quantities and other three, such that, being taken two and two in a cross order, they are proportional; then, the first shall be to the third of the first class, as the first to the third of the second class.

Let *a*, *b*, *c*, be three quantities, and *d*, *e*, *f*, other three, such that

$$\frac{a}{b}=\frac{e}{f} \text{ and } \frac{b}{c}=\frac{d}{e};$$ to prove that $\frac{a}{c}=\frac{d}{f}$.

Since $\frac{a}{b}=\frac{e}{f}$, and $\frac{b}{c}=\frac{d}{e}$, by (29, *g*) $\frac{a}{b}\times\frac{b}{c}=\frac{e}{f}\times\frac{d}{e}$; cancelling the common factors from numerator and denominator of both sides, it becomes $\frac{a}{c}=\frac{d}{f}$.

47. If three terms of an analogy or proportion be given, the fourth can always be found; for either extreme is equal to the product of the means, divided by the other extreme; and either mean is equal to the product of the extremes divided by the other mean.

Let $a:b=c:d$; to prove that $a=\frac{bc}{d}$, that $d=\frac{bc}{a}$, that $b=\frac{ad}{c}$, and that $c=\frac{ad}{b}$.

Since $a:b=c:d$, by (36) $ad=bc$, dividing both sides first by *d* and then by *a*, gives $a=\frac{bc}{d}$, and $d=\frac{bc}{a}$.

Again, since $ad=bc$, $bc=ad$, and dividing both sides first by *c*, and then by *b*, gives $b=\frac{ad}{c}$, and $c=\frac{ad}{b}$.

48. When three quantities are in continued proportion (that is, when the first is to the second as the second to the third) the second is said to be a mean proportional between the other

two, and the third is said to be a third proportional to the other two.

49. THEOREM.—When three quantities are in continued proportion, either extreme is equal to the square of the mean divided by the other extreme; and the mean is equal to the square root of the product of the extremes.

Let $a : b = b : c$; to prove that $a = \frac{b^2}{c}$, $c = \frac{b^2}{a}$, and $b = \sqrt{ac}$.

Since $\frac{a}{b} = \frac{b}{c}$, $ac = b^2$ (36); and dividing both sides first by c, and then by a, gives $a = \frac{b^2}{c}$ and $c = \frac{b^2}{a}$.

Again, since $ac = b^2$, $b^2 = ac$, and taking the square root of both sides, $b = \sqrt{ac}$.

50. If three quantities be in continued proportion, the first is to the third as the square of the first to the square of the second.

Let $a : b = b : c$; to prove that $a : c = a^2 : b^2$.

Since $\frac{a}{b} = \frac{b}{c}$, $\frac{b}{c} = \frac{a}{b}$; multiply both sides by $\frac{a}{b}$ (29, *g*), which gives $\frac{b}{c} \times \frac{a}{b} = \frac{a}{b} \times \frac{a}{b}$, therefore $\frac{a}{c} = \frac{a^2}{b^2}$, or $a : c = a^2 : b^2$.

In this case a is said to have to c, the duplicate ratio of that which a has to b, or which b has to c.

Exercises 3.

(42.) Find the ratios arising (1) from increasing each of the terms of the ratio 13 : 18 or $\frac{13}{18}$ by 7; (2) from diminishing each of the same by 3; (3) find the ratio of the former resulting ratio to the latter.

(43.) Find the ratio of $4\frac{1}{2}$ inches to 1 foot; of $7\frac{1}{2}d.$ to half-a-crown; and of $4\frac{1}{2}$ degrees to 360.

(44.) The last three terms of a proportion are 16, 6, and 8, find the first term; the proportion formed by inverting it; and that formed by alternating it.

(45.) Which is the greater ratio, $\frac{7}{10}$ or $\frac{50}{70}$; $\frac{11}{15}$ or $\frac{16}{20}$; $\frac{108}{151}$ or $\frac{11}{15}$?

(46.) Given the first three terms of a proportion, 12, 8, and 30, to find the fourth; also given the first term 12, the third 8, and fourth 30, to find the second.

(47.) Find a third proportional to 4 and 6; and a mean proportional between 15 and 540.

(48.) $\frac{16}{9}=\frac{48}{27}$, shew by (Art. 39, 40, and 41), that, $\frac{25}{9}=\frac{75}{27}$; $\frac{7}{9}=\frac{21}{27}$; and that $\frac{25}{7}=\frac{75}{21}$.

(49.) Shew that if three terms of a proportion are given, there may be three values of the fourth, according to the order in which they are arranged.

(50.) Three terms of a proportion are 12, 16, and 36; find the three possible values of the fourth.

(51.) Find a third proportional to 6 and 18; and a mean proportional between 11 and 396.

(52.) Find which is the greater ratio, $\frac{3}{5}$ or $\frac{37}{60}$; $\frac{11}{9}$ or $\frac{45}{37}$; $\frac{12}{31}$ or $\frac{49}{124}$.

(53.) What must be added to each term of the ratio 3 to 4, that it may become the ratio of 15 to 16; and what must be subtracted from each term of the same ratio, to reduce it to the ratio of 5 to 12?

(54.) What must be added to each term of the ratio of $a:b$, that it may be changed to the ratio of $c:d$? And what must be subtracted from each term of the same ratio that it may be changed to the ratio of $\frac{e}{f}$?

(55.) Find a fourth proportional to ·0014, 1·4, and ·02; also a mean proportional between ·5 and ·005.

(56.) Find a fourth proportional to $\frac{2}{7}$, $\frac{3}{4}$, and $\frac{5}{6}$; and also a mean proportional between ·17 and ·0153.

(57.) What number must be added to a, and subtracted from b, so that the sum may be to the difference as m is to n?

EQUIDIFFERENT AND EQUIRATIONAL SERIES.

51. When there are any series of numbers such that each is formed from the previous by the addition or subtraction of the same number, they are said to be in Equidifferent progression, or in Arithmetical progression; thus 5, 7, 9, 11, &c. are in equidifferent progression, since each number is found by adding 2 to the immediately preceding.

52. Again, the numbers 3, 6, 12, 24, &c. are in Equirational or Geometrical progression, for each number is formed from the immediately preceding by multiplying by 2. If we take the following series of powers, 3^1, 3^2, 3^3, 3^4, 3^5, 3^6, &c., we find that the exponents proceed in equidifferent progression, and the quantities themselves in equirational progression. Writing now the exponents in one line and the result of the powers in another, gives two series, the first composed of the exponents, and the second of the powers of the number 3, which will possess some interesting properties, which it will be necessary here to note.

Exponents.....1, 2, 3, 4, 5, 6, 7, 8, 9.
Powers of 3.....3, 9, 27, 81, 243, 729, 2187, 6561, 19683.

53. It may be observed, in the first place, that if any two numbers in the first line, as 3 and 5, be added together, their sum, 8, will point to a number in the second line, namely, 6561, which is the product of the two numbers 27 and 243, under the 3 and 5 in the first line. Again, the difference of any two numbers in the first line, such as 9 and 5, leads to 4, under which

will be found 81, which is the quotient of the numbers in the second line under 9 and 5 in the first, for $19683 \div 243$ gives for quotient 81, the number under 4 in the first line. Hence within the limits of this little table, multiplication can be performed by addition, and division by subtraction.

54. If a number in the first line be multiplied by 2, the product will have under it the square of the number under the figure so multiplied; also, if multiplied by 3, the product will have under it the cube of the number under the figure so multiplied; hence multiplication corresponds to the raising of powers.

Again, if any number in the upper line be divided by 2, the quotient will point to a number under it which is the square root of the number under the one at first divided; and in the same manner, if a number in the upper line be divided by three, the quotient will point to the cube root of the number under the exponent first divided.

55. Hence it is evident that the multiplication of exponents corresponds to the raising of powers, and the division of exponents to the extraction of roots.

56. If the table (in 52) be continued towards the left, since beginning at the right and proceeding towards the left, the exponents diminish by 1, and the numbers by the division by 3, it will become,

-2,	-1,	0,	1,	2,	3,	4,	5,	6,	7,	8,	9	Exponents.
$\frac{1}{9}$,	$\frac{1}{3}$,	1,	3,	9,	27,	81,	243,	729,	2187,	6561,	19683	Numbers.

From this it is easily seen that $3^0 = 1$, $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{3^2}$; putting any letter as a, which may represent any number, it may be shewn in the same manner that $a^0 = \frac{a}{a} = 1$, $a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$, $a^{-3} = \frac{1}{a^3}$, and generally $a^{-n} = \frac{1}{a^n}$.

OF LOGARITHMS.

57. Logarithms are numbers in an equidifferent or arithmetical progression, which correspond term by term to another series of numbers in equirational or geometrical progression, which being calculated and arranged in Tables, enable us to perform the operations of multiplication and division by that of the addition and subtraction of their logarithms; and the operation of raising powers and extracting roots of numbers, by multiplying and dividing their logarithms.

58. Logarithms may be calculated to any base; in (52) and (56) the exponents are the logarithms of the numbers over which they are placed, calculated to the base 3. The logarithms in common use are calculated to the base 10; and the logarithm of any number in that system is the exponent of 10 which produces the given number. Thus $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, $10^4 = 10{,}000$; the log. of 10 therefore is 1; that of 100 is 2; that of 1000 is 3; and that of 10,000 is 4, &c.

Now 10^3, 10^2, 10^1, 1, $\frac{1}{10}$, $\frac{1}{100}$, &c., form a continually decreasing geometrical series; their logarithms therefore form a continually decreasing arithmetical series.

Numbers.....	1000,	100,	10,	1,	·1,	·01,	·001,	&c.
Logarithms.....	3,	2,	1,	0,	−1,	−2,	−3,	&c.

Hence the log. of 1 is 0, and the logs. of all numbers less than 1 have their indices negative; the log. of every number between 1 and 10 is between 0 and 1; the log. of every number between 10 and 100 is between 1 and 2; the log. of every number between 100 and 1000 is between 2 and 3, &c.

59. The numbers 1, 2, 3, &c., called the indices of the logarithms, are therefore always less by one than the digits contained in the integral part of the number whose logarithm is

taken. Also, it is evident that the logs. of 10 and all its integral powers will be whole numbers; but to determine the logs. of the intermediate numbers is a task still of considerable labour, and at the time when logs. were first calculated it involved very great labour indeed. The following example will shew how logarithms may be calculated by the aid of arithmetic alone.

Example.

Let it be required to calculate the logarithm of 9.

Here the proposed number lies between 1 and 10.

First, then, the log. of 10 is 1, and the log. of 1 is 0; therefore $(1+0)\div 2=\frac{1}{2}=\cdot 5$ is the equidifferent mean, and $\sqrt{10\times 1}=\sqrt{10}=3\cdot 1622777$ is the equirational mean; hence the log. of $3\cdot 1622777$ is $\cdot 5$.

Secondly, the log. of 10 is 1, and the log. of $3\cdot 1622777$ is $\cdot 5$; therefore $(1+\cdot 5)\div 2=\cdot 75$ is the equidifferent mean, and $\sqrt{10\times 3\cdot 1622777}=5\cdot 6234132$ is the equirational mean; hence the log. of $5\cdot 6234132$ is $\cdot 75$.

Thirdly, the log. of 10 is 1, and the log. of $5\cdot 6234132$ is $\cdot 75$; therefore $(1+\cdot 75)\div 2=\cdot 875$ is the equidifferent mean, and $\sqrt{10\times 5\cdot 6234132}=7\cdot 4989422$ is the equirational; hence the log. of $7\cdot 4989422$ is $\cdot 875$.

Fourthly, the log. of 10 is 1, and the log. of $7\cdot 4989422$ is $\cdot 875$; therefore $(1+\cdot 875)\div 2=\cdot 9375$ is the equidifferent mean, and $\sqrt{10\times 7\cdot 4989422}=8\cdot 6596431$ is the equirational mean; hence the log. of $8\cdot 6596431$ is $\cdot 9375$.

Fifthly, the log. of 10 is 1, and the log. of $8\cdot 6596431$ is $\cdot 9375$; therefore $(1+\cdot 9375)\div 2=\cdot 96875$ is the equidifferent mean, and $\sqrt{10\times 8\cdot 6596431}=9\cdot 3057204$ is the equirational mean; hence the log. of $9\cdot 3057204$ is $\cdot 96875$.

Sixthly, the log. of 8·6596431 is ·9375, and the log. of 9·3057204 is ·96875; therefore $(\cdot 9375 + \cdot 96875) \div 2 = \cdot 953125$ is the equidifferent mean, and $\sqrt{8\cdot 6596431 \times 9\cdot 3057204} = 8\cdot 9768713$ is the equirational mean; hence the log. of 8·9768713 is ·953125.

Proceeding in this manner, taking one factor greater than 9 and the other less, after 25 extractions, it will be found that the log. of 8·9999998 is ·9542425, which may be taken as the log. of 9, and is that which is found in the tables as its logarithm.

60. In a similar manner, with vast labour, were the logarithms of the prime numbers at first calculated; there are, however, now much easier methods known, which have been discovered by analysis; some of these may be found in the *Algebra* of Chambers's Educational Course. If M = ·434294481903, then,

$$\text{log. } 9 = \text{log. } 10 - M\left(\frac{1}{10} + \frac{1}{2(10)^2} + \frac{1}{3(10)^3} + \frac{1}{4(10)^4} + \frac{1}{5(10)^5} + \&c.\right)$$

the calculation of which is as follows:

			Differences.
10)·434294481903			
10)·043429448190		=	·043429448190
10)	4342944819 ÷ 2	=	2171472410
10)	434294482 ÷ 3	=	144764827
10)	43429448 ÷ 4	=	10857362
10)	4342945 ÷ 5	=	868589
10)	434294 ÷ 6	=	72382
10)	43429 ÷ 7	=	6204
10)	4343 ÷ 8	=	543
10)	434 ÷ 9	=	48
	43 ÷ 10	=	4
		−	·045757490559
		+1	
	Log. 9	=	·954242509441

This logarithm of 9 is correct to 10 places of decimals; the last two figures should be 39 instead of 41.

PROPERTIES OF LOGARITHMS.

61. If the power a^x be equal to the number N, or $a^x = N$; then the index x, of the power a^x, is called the logarithm of the number N to the base a, and this is usually written $x = \log._a N$.

62. Since $a^1 = a$, therefore $\log._a a = 1$; and since $e^1 = e$, therefore $\log._e e = 1$; hence in every system of logarithms the logarithm of the base is 1.

63. Since $a^0 = 1$ and $e^0 = 1$, therefore in every system of logarithms the logarithm of 1 is 0. Also since $a^{-\infty} = \frac{1}{a^{\infty}} = 0$; therefore the logarithm of $0 = -\infty$; that is, equals − infinity.

64. *The logarithm of the product of any number of factors is equal to the sum of the logarithms of those factors.*

Let P, Q, R, S, be any quantities, and x, y, z, v, their logarithms to any base a; then

$$a^x = P,\ a^y = Q,\ a^z = R, \text{ and } a^v = S;$$

$$\therefore x = \log._a P,\ y = \log._a Q,\ z = \log._a R, \text{ and } v = \log._a S;$$

$$\therefore PQRS = a^x.a^y.a^z.a^v = a^{x+y+z+v}; \text{ hence}$$

$$\log._a PQRS = x + y + z + v = \log._a P + \log._a Q + \log._a R + \log._a S.$$

Example.

Find by logarithms the continued product of 4·3825, 73·85, 42·25, and ·07285.

Calculation.

Log. 4·3825	=	0·6417219	} to be added.
Log. 73·85	=	1·8683505	
Log. 42.25	=	1·6258267	
Log. ·07285	=	2·8624296	
Log. 996·159	=	2·9983287	sum = log. product.

$\therefore$ 4·3825 × 73·85 × 42·25 × ·07285 = 996·159 *Ans.*

Exercises 4.

Find by logarithms the product

(58.) Of 48·26 × 3·026.
(59.) Of 247·55 × 56·721.
(60.) Of 498·256 × 41·2467.
(61.) Of 4·02674 × ·0123456.
(62.) Of 21·879605 × 1·056385.
(63.) Of 2019·225 × 6·3447.
(64.) Of ·007461 × ·3351767.
(65.) Of 56·3426 × ·023579.
(66.) Of 4·0025 × 608·27 × ·0425839.
(67.) Of 25603·07 × ·00072575 × 63·87009.
(68.) Of 287·69 × 1·0674 × ·098762 × ·0031598.
(69.) Of 498·256 × 41·2467 × 4·02674 × ·0123456.

65. *The logarithm of a fractional quantity is equal to the logarithm of the numerator diminished by the logarithm of the denominator.*

Let $a^x = P$ and $a^y = Q$, then $x = \log._a P$ and $y = \log._a Q$; hence

$$\frac{P}{Q} = \frac{a^x}{a^y} = a^{x-y};$$

$$\therefore \log._a \frac{P}{Q} = x - y = \log._a P - \log._a Q.$$

66. Cor.—*The logarithm of the quotient is equal to the logarithm of the dividend minus the logarithm of the divisor.*

Example.

Find by logarithms $73 \times 54 \div 527$, or $\frac{73 \times 54}{527}$.

Calculation.

Log. 73	=	1·8633229	} to be added.
Log. 54	=	1·7323938	}
		3·5957167	= log. dividend or numerator.
Log. 527		2·7218106	to be subtracted.
Log. 7·480078	=	0·8739061	

$$\therefore 73 \times 54 \div 527 = \frac{73 \times 54}{527} = 7{\cdot}480078 \quad \textit{Ans.}$$

Exercises 5.

Find by logarithms the quotient

(70.) Of $37{\cdot}149 \div 523{\cdot}76$.
(71.) Of $\cdot06314 \div \cdot0007241$.
(72.) Of $76{\cdot}437 \div \cdot056789$.
(73.) Of $\cdot563426 \div \cdot023579$.
(74.) Of $252{\cdot}74 \div 56{\cdot}789$.
(75.) Of $248{\cdot}25 \div 364{\cdot}87$.
(76.) Of $5{\cdot}6949 \div 53{\cdot}058$.
(77.) Of $\cdot085375 \div \cdot07425$.
(78.) Of $69{\cdot}7565 \div \cdot97564$.
(79.) Of $\cdot048869 \div \cdot0071698$.

(80.) Find the value of $\dfrac{67{\cdot}038 \times \cdot010705 \times 4{\cdot}1525}{\cdot7854 \times 3{\cdot}1416 \times \cdot086725}$.

(81.) Find the value of $\dfrac{28{\cdot}045 \times 1{\cdot}3564 \times \cdot0942537}{48{\cdot}375 \times 2{\cdot}71828 \times \cdot52359}$.

(82.) Find the value of $\dfrac{72{\cdot}125 \times 5{\cdot}8675 \times \cdot04875}{4{\cdot}2675 \times \cdot052125 \times \cdot8459}$.

(83.) Find the value of $\dfrac{7\frac{2}{7} \times 8\frac{5}{8} \times 9\frac{7}{9} \times 12\frac{11}{12}}{7\frac{1}{7} \times 8\frac{3}{8} \times 9\frac{2}{9} \times 12\frac{5}{12}}$.

(84.) Find a fourth proportional to $7{\cdot}285$, $68{\cdot}255$, and $\cdot14235$.
(85.) " " " " $8{\cdot}352$, $36{\cdot}9$, and $3{\cdot}057$.
(86.) " " " " $735{\cdot}09$, $5101{\cdot}8$, and $\cdot03125$.
(87.) " third " " $\cdot7118$ and $4{\cdot}3125$.
(88.) " " " " $9{\cdot}0625$ and $17{\cdot}125$.

67. *The logarithm of any power of a quantity is equal to the logarithm of the quantity, multiplied by the exponent or index of the power.*

Let $a^x = P$, then $x = \log._a P$, and $a^{nx} = P^n$; therefore

$$\log._a P^n = nx = n \log._a P.$$

Example.

Find by logarithms the 5th power of $57{\cdot}29577$.

Calculation.

$$\text{Log. } (57{\cdot}29577)^5 = 5 \text{ Log. } (57{\cdot}29577) = 1{\cdot}7581225 \times 5$$

$$\text{Log. } 617465300 \quad = 8{\cdot}7906125$$

$$\therefore (57{\cdot}29577)^5 = 617465300 \quad \textit{Ans.}$$

Exercises 6.

INVOLUTION BY LOGARITHMS.

(89.) Find the squares of 7·5285; 97·125; ·0128; and 41·25.
(90.) " cubes of 52·38; 9·3425; 6·5125; and ·08475.
(91.) " fourth power of 3·1416; or ·5236; and 53·175.

68. *The logarithm of any root of a quantity is equal to the logarithm of the quantity divided by the index or exponent of the root.*

Let $a^x = P$, then $x = \log._a P$, and $a^{\frac{x}{n}} = P^{\frac{1}{n}}$; therefore

$$\log._a P^{\frac{1}{n}} = \frac{x}{n} = \frac{1}{n} \log._a P.$$

In the same manner it may be proved that,

$$\log._a P^{\frac{m}{n}} = \frac{m}{n} \log._a P.$$

Example.

Find by logarithms the 5th root of 3·1416.

Calculation.

$$\text{Log. } (3{\cdot}1416)^{\frac{1}{5}} = \tfrac{1}{5} \text{ Log. } 3{\cdot}1416 = 0{\cdot}4971509$$

$$\text{Log. } 1{\cdot}257275 \qquad = \overline{\tfrac{1}{5}} \quad = 0{\cdot}0994302$$

$$\therefore (3{\cdot}1416)^{\frac{1}{5}} = 1{\cdot}257275 \text{ } _a \textit{Ans.}$$

Exercises 7.

EVOLUTION BY LOGARITHMS.

(92.) Find the square root of 3·1416; 2·71828; and 528·25.
(93.) " " " ·31416; ·271828; and ·052825.
(94.) " cube " 11·425; 99·527; and 8547·5.
(95.) " fourth " 12579; 3·1416; and ·13579.
(96.) Find the square root of 2, 3, 5, 6, 7, 8, and 10.
(97.) Find the value of

$$\frac{2^3 \times 3^2 \times 5^4}{7^2 \times 11^3}, \quad \frac{4^3 \times 5^2 \times 11^3}{7^3 \times 3^4}, \quad \text{and} \quad \sqrt{\frac{15 \times 23^2 \times 7^3}{46 \times 161}}.$$

(98.) Find the value of

$$\frac{3^6 \times 4^{-5}}{7^2}, \quad \frac{45 \times 7^{-2}}{23 \times \sqrt{3}}, \text{ and } \frac{(7920)^3 \times 3{\cdot}1416}{6}.$$

(99.) Find the value of

$$\frac{9^3 \times 10^4 \times 11^2}{15^2 \times 12^3}, \quad \frac{15^4 \times 17^4 \times 18^4}{11^2 \times 9^4}, \text{ and } \frac{(\cdot 03)^2 \times (\cdot 05)^3 \times (\cdot 09)^4}{(\cdot 005)^4 \times (\cdot 06)^3}.$$

(100.) Find the value of

$$\frac{3^4 \times 5^4 \times 7^4}{8^4 \times 25^4}, \quad \frac{5^4 \times 7^4 \times 11^4}{16^4 \times 81^4}, \text{ and } \frac{11^4 \times 13^4 \times 17^4}{27^4 \times 16^4}$$

69. *The logarithms of any quantity in two different systems are inversely as the logarithms of their bases in that system.*

Let N denote any number or quantity, and a, b, any two bases; hence if $a^x = N$ and $b^y = N$; then $a^x = b^y$, and the logarithms of both members to any, the same base, give

$$x \log. a = y \log. b, \text{ hence } (36)\ x : y = \log. b : \log. a;$$

but $$x = \log._a N, \text{ and } y = \log._b N;$$

consequently, $$\log._a N : \log._b N = \log. b : \log. a.$$

These general properties of logarithms are of the highest importance, and (69) will enable us to convert one system of logarithms into another system with facility. Thus, since

$$\log._a N : \log._b N = \log._a b : \log._a a,$$

and $\log._a a = 1$; therefore $\log._b N = \frac{1}{\log._a b} . \log._a N.$

Hence, if a system of logarithms be calculated to the base a, a system of logarithms to the base b may readily be found by means of the preceding equation, which expresses the relation between the two systems. The factor $\frac{1}{\log._a b}$, by which the system whose base is a is transformed to the system whose base is b, is called the *modulus* of the system whose base is b.

NOTE.—It is often necessary to change expressions from the

common to the logarithmic form, and conversely; a few examples are given below, which should be carefully studied by the pupil.

Examples.

1. Let $x = \frac{5ab}{2c}$; this changed into the logarithmic form, becomes

$$\log. x = \log. 5 + \log. a + \log. b - \log. 2 - \log. c;$$

or the sum of the logarithms of the factors in the numerator, diminished by the sum of the logarithms of the factors in the denominators.

2. Let $x = \sqrt{a^2(b-c)^3}$; this changed into the logarithmic form, becomes

$$\log. x = \tfrac{1}{2}\{2 \log. a + 3 \log. (b-c)\};$$

or multiplying both sides by 2,

$$2 \log. x = 2 \log. a + 3 \log. (b-c);$$

or again, dividing both sides by 2,

$$\log. x = \log. a + \tfrac{3}{2} \log. (b-c).$$

Again, changing this latter form from the logarithmic to the common form, it becomes

$$x = a(b-c)^{\frac{3}{2}} = \sqrt{a^2(b-c)^3}, \text{ as first given.}$$

3. Let $$x = \frac{a^3b^2c}{a+b+c};$$

then $\log. x = 3 \log. a + 2 \log. b + \log. c - \log. (a+b+c)$.

4. Let $$x = \frac{\sqrt[3]{a^2b^2c}}{\sqrt[4]{d^3}};$$

then $\log. x = \frac{2}{3} \log. a + \frac{2}{3} \log. b + \frac{1}{3} \log. c - \frac{3}{4} \log. d$.

5. Let $$x = \frac{a^{\frac{1}{2}}bc^{\frac{1}{3}}}{(b+c)^{\frac{1}{2}}};$$

then $\log. x = \frac{1}{2} \log. a + \log. b + \frac{1}{3} \log. c - \frac{3}{2} \log. (b+c)$.

6. Given $a^x = \frac{b^{(nx-m)}}{c^{rx}}$, to find x.

Taking the logarithms of both sides, gives

$$x \log. a = (nx - m) \log. b - rx \log. c$$
$$= nx \log. b - m \log. b - rx \log. c;$$
$$\therefore (r \log. c + \log. a - n \log. b)x = -m \log. b;$$

hence
$$x = \frac{m \log. b}{n \log. b - r \log. c - \log. a}.$$

CONSTRUCTION OF THE PLANE SCALE.

70. With the radius you intend for your scale, describe a semicircle ADB, and from the centre C draw CD perpendicular to AB, which will divide the semicircle into two quadrants, AD, BD; continue CD towards S, draw BT perpendicular to CB, and join BD and AD.

71. Divide the quadrant BD into nine equal parts, then will each of these be 10 degrees; subdivide each of these parts into single degrees, and if your radius will admit of it, into minutes or some aliquot parts of a degree greater than minutes.

72. Set one foot of the compasses in B, and transfer each of the divisions of the quadrant BD to the right line BD, then will BD be a line of chords.

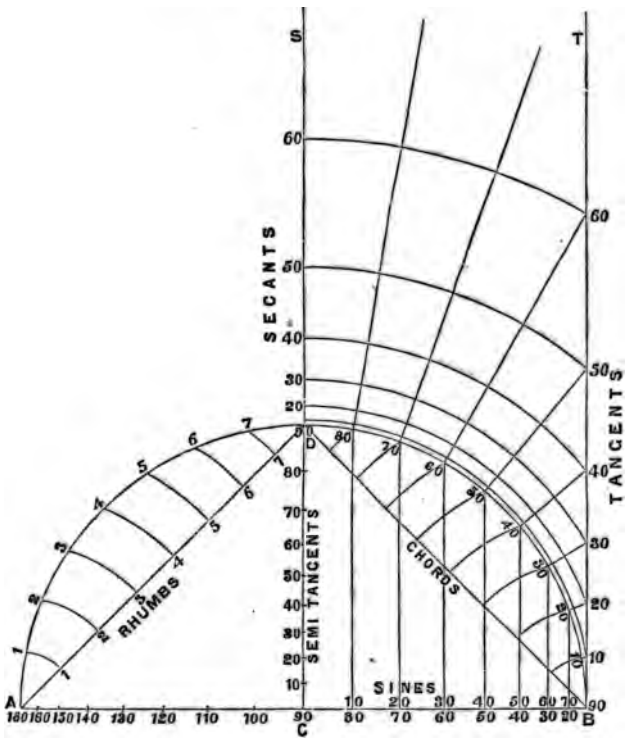
73. From the points 10, 20, 30, &c., in the quadrant BD draw right lines parallel to CD, to cut the radius CB, and they will divide that line into a line of sines which must be numbered from C towards B.

74. If the same line of sines be numbered from B towards C, it will become a line of versed sines, which may be continued to 180°, if the same divisions be transferred on the same line on the other side of the centre C.

75. From the centre C, through the several divisions of the quadrant BD, draw right lines till they cut the tangent BT, so will the line BT become a line of tangents.

76. Setting one foot of the compasses in C, extend the other to the several divisions, 10, 20, 30, &c., in the tangent line BT, and transfer these extents severally to the right line CS, then will that line be a line of secants.

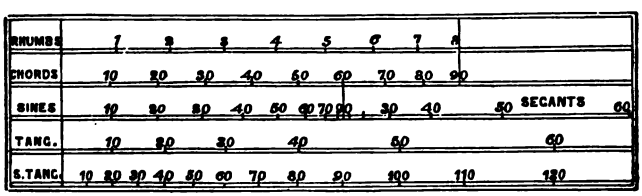
77. Right lines drawn from A to the several divisions, 10, 20,



30, &c., in the quadrant BD, will divide the radius CD into a line of semi-tangents.

78. Divide the quadrant AD into eight equal parts, and from A as a centre transfer these divisions severally into the line AD, then will AD be a line of rhumbs, each division answering to

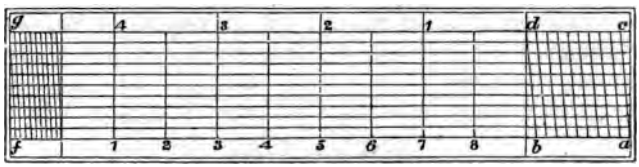
11° 15′ upon the line of chords. The use of this line is for protracting and measuring angles, according to the common division of the mariner's compass. If the radius AC be divided into 100 or 1000, &c. equal parts, and the lengths of the several sines, tangents, and secants, corresponding to the several arches of the quadrant, be measured thereby, and these numbers be set down in a table, each in its proper column, you will by these means have a collection of numbers by which the several cases in trigonometry may be solved. Right lines graduated as above, being placed severally upon a ruler, form the instrument called the Plane Scale, by which the lines and angles of



all triangles may be measured. All right lines, as the sides of plane triangles, &c., when they are considered simply as such, without having any relation to a circle, are measured by scales of equal parts, one of which is subdivided equally into 10, and this serves as a common division to all the rest. In most scales an inch is taken for a common measure, and what an inch is divided into is generally set at the end of the scale. By any common scale of equal parts, divided in this manner, any number less than 100 may be readily taken; but if the number should consist of three places of figures, the value of the third figure cannot be exactly ascertained, and in this case it is better to use a diagonal scale, by which any number consisting of three places of figures may be exactly found. The figure of this scale is given in next page; its construction is as follows:

79. Having prepared a ruler of convenient breadth for your scale, draw near the edges thereof two right lines, *af*, *eg*, parallel to each other; divide one of these lines, as *af*, into equal parts,

according to the size of your scale; and through each of these divisions draw right lines perpendicular to *af*, to meet *cg*, then



divide the breadth into ten equal parts, and through each of these divisions draw right lines parallel to *af* and *cg*; divide the lines *ab*, *cd*, into ten equal parts, and from the point *a* to the first division in the line *cd*, draw a diagonal line; then parallel to that line, draw diagonal lines through all the other divisions, and the scale is complete. Then, if any number, consisting of three places of figures, as 256, be required from the larger scale *gd*, you must place one foot of the compasses on the figure 2 on the line *gd*, then the extent from 2 to the point *d* will represent 200. The second figure being 5, count five of the smaller divisions from *d* towards *e*, and the extent from 2 to that point will be 250. Move both points of the compasses downwards till they are on the sixth parallel line below *gd*, and open them a little till the one point rests on the vertical line drawn through 2, and the other on the diagonal line drawn through 5; the extent then in the compasses will represent 256. In the same way the quantities 25·6; 2·56; 0·256, &c. are measured.

80. Besides the lines already mentioned, there is another on the plane scale marked ML, which is joined to a line of chords; and shews how many miles of easting or westing correspond to a degree of longitude in every latitude.* These several lines

* As it would confuse the figure (page 35) to describe on it the line of longitudes, it is neglected, but the construction is as follows: Divide the line CB into sixty equal parts (if it can be done), and through each point draw lines parallel to CD to intersect the arch BD; about B, as a centre, transfer the several points of intersection to the line BD, and then number it from D, towards B, from 0 to 60, and it will be the line of longitudes.

are generally put on one side of a ruler, two feet long; and on the other side is laid down a scale of the logarithms of the sines, tangents, and numbers, which is commonly called Gunter's Scale, and as it is of general use, it requires a particular description.

GUNTER'S SCALE.

On Gunter's scale are eight lines—namely,

81. Sine rhumbs, marked S.R., corresponding to the logarithms of the natural sines of every point of the mariner's compass, numbered from the left hand towards the right, with 1, 2, 3, 4, 5, 6, 7, to 8, where is a brass pin. This line is also divided, where it can be done, into halves and quarters.

82. Tangent rhumbs, marked T.R., correspond to the logarithms of the tangents of every point of the compass, and are numbered 1, 2, 3, to 4, at the right hand where there is a pin, and thence towards the left hand with 5, 6, 7; it is also divided, where it can be done, into halves and quarters.

83. The line of numbers, marked Num., corresponds to the logarithms of numbers, and is marked thus: Near the left hand it begins at 1, and towards the right hand are 2, 3, 4, 5, 6, 7, 8, 9; and 1 in the middle, at which is a brass pin, then 2, 3, 4, 5, 6, 7, 8, 9, and 10 at the end, where there is another pin. The values of these numbers and their intermediate divisions depend on the estimated values of the extreme numbers 1 and 10; and as this line is of great importance, a particular description of it will be given. The first 1 may be counted for 1, 10, 100, or 1000, &c., and then the next 2, will be 2, 20, 200, or 2000, &c., respectively. Again, the first 1 may be reckoned 1 tenth, 1 hundredth, or 1 thousandth part, &c., then the next will be 2 tenths, or 2 hundredths, or 2 thousandth parts, &c., so that if the first 1 be esteemed 1, the middle 1 will be 10; 2 to its right, 20; 3, 30; 4, 40; and 10 at the end, 100; again, if the first 1 is 10, the next 2 is 20, 3 is 30, and so on, making the

middle 1, 100, the next 2 is 200, 3 is 300, 4 is 400, and 10 at the end is 1000. In like manner, if the first 1 be esteemed 1 tenth part, the next 2 will be 2 tenth parts, and the middle 1 will be 1; the next 2, 2; and 10 at the end will be 10. Again, if the first 1 be counted 1 hundredth part, the next 2 hundredth parts, the middle 1 will be 10 hundredth parts, or 1 tenth part, and the next 2, 2 tenth parts, and 10 at the end will be but one whole number or integer.

84. As the figures are increased or diminished in their value, so in like manner must all the intermediate strokes or subdivisions be increased or diminished; that is, if the first 1 at the left hand be counted 1, then 2 (next following it) will be 2, and each subdivision between them will be 1 tenth part, and so all the way to the middle 1, which will be 10, the next 2, 20, and the longer strokes between 1 and 2 are to be counted from 1 thus, 11, 12 (where is a brass pin), then 13, 14, 15, sometimes a longer stroke than the rest, then 16, 17, 18, 19, 20, at the figure 2; and in the same manner, the short strokes between the figures 2 and 3, 3 and 4, 4 and 5, &c., are to be reckoned as units. Again, if 1 at the left hand be 10, the figures between it and the middle 1 will be common tens; and the subdivisions between each figure will be units; from the middle 1 to 10 as the end, each figure will be so many hundreds; and between these figures each longer division will be 10. From this description it will be easy to find the divisions representing any given number, thus: Suppose the point representing the number 12 were required; take the division at the figure 1 in the middle, for the first figure of 12; then for the second figure count two tenths, or longer strokes to the right hand, and this will be the point representing 12, where the brass pin is.

85. Again, suppose the number 22 were required; the first figure 2 is to be found on the scale, and for the second figure 2, count two-tenths onwards, and that is the point representing 22.

86. Again, suppose 1728 were required; for the first figure 1, take the middle 1; for the second figure 7, count onwards as before, and that will be 1700. And as the remaining figures

are 28 or nearly 30, note the point which is nearly three-tenths of the distance between the marks 7 and 8, and this will be the point representing 1728.

87. If the point representing 435 were required; from the 4 in the second interval count towards 5 on the right, three of the larger divisions and one of the smaller (this smaller division being midway between the marks 3 and 4), and that will be the division expressing 435. In a similar manner other numbers may be found.

88. All fractions found in this line must be decimals; and if they are not, they must be reduced into decimals, which is easily done by extending the compasses from the denominator to the numerator; that extent laid the same way, from 1 in the middle or right hand, will reach to the decimal required.

Example.

Required the decimal fraction equal to $\frac{3}{4}$. Extend from 4 to 3; that extent will reach from 1 on the middle to ·75 towards the left hand. The like may be observed of any other vulgar fraction.

89. Multiplication is performed on this line, by extending from 1 to the multiplier; that extent will reach from the multiplicand to the product.

Suppose, for example, it were required to find the product of 16 multiplied by 4, extend from 1 to 4; that extent will reach from 16 to 64, the product required.

90. Division being the reverse of multiplication, therefore extend from the divisor to unity; that extent will reach from the dividend to the quotient.

Suppose 64 to be divided by 4; extend from 4 to 1, that extent will reach from 64 to 16, the quotient.

91. Questions in the Rule of Three are solved by this line as follows: Extend from the first term to the second, that extent will reach from the third term to the fourth. And it ought to be particularly noted, that if you extend to the left, from the

first number to the second, you must also extend to the left, from the third number to the fourth; and the contrary.

Example.

If the diameter of a circle be 7 inches, and the circumference 22, what is the circumference of another circle, the diameter of which is 14 inches? Extend from 7 to 22, that extent will reach from 14 to 44 the same way.

92. The superficial content of any parallelogram is found by extending from 1 to the breadth; that extent will reach from the length to the superficial content.

Example.

Suppose a plank or board, 15 inches broad and 27 feet long, the content of which is required. Extend from 1 to 1 foot 3 inches (or 1·25); that extent will reach from 27 feet to 33·75 feet, the superficial content; or extend from 12 inches to 15, &c.

93. The solid content of any bale, box, chest, &c., is found by extending from 1 to the breadth; that extent will reach from the depth to a fourth number, and the extent from 1 to that fourth number will reach from the length to the solid content.

Examples.

1. What is the content of a square pillar, whose length is 21 feet 9 inches, and breadth 1 foot 3 inches? The extent from 1 to 1·25 will reach from 1·25 to 1·56, the content of one foot in length; again, the extent from 1 to 1·56 will reach from the length 21·75 to 33·9, or 34, the solid content in feet.

2. Suppose a square piece of timber, 1·25 feet broad, ·56 deep, and 36 long, be given, to find the content. Extend from 1 to 1·25; that extent will reach from ·56 to ·7; then extend from 1 to ·7; that extent will reach from 36 to 25·2, the solid content. In like manner may the contents of bales, &c. be found, which divided by 40 will give the tonnage.

94. The line of sines, marked Sin., corresponding to the log. sines of the degrees of the quadrant, begins at the left hand, and is numbered to the right thus: 1, 2, 3, 4, 5, &c., to 10; then 20, 30, 40, &c., ending at 90°, where is a brass centre-pin, as there is at the right end of all the lines.

95. The line of versed sines, marked V.S., corresponding to the log. versed sines of the degrees of the quadrant, begins at the right hand against 90° on the sines, and from thence is numbered towards the left hand thus: 10, 20, 30, 40, &c., ending at the left hand at about 169°; each of the subdivisions, from 10 to 30, is in general two degrees, from thence to 90 is single degrees, from thence to the end, each degree is divided into 15 minutes.

96. The line of tangents, marked Tang., corresponding to the log. tangents of the degrees of the quadrant, begins at the left hand, and is numbered towards the right thus: 1, 2, 3, &c., to 10, and so on, 20, 30, 40, and 45, where is a brass pin under 90° on the sines; from thence it is numbered backwards, 50, 60, 70, 80, &c., to 89, ending at the left hand where it began at 1°. The subdivisions are nearly similar to those of the sines. When you have any extent in your compasses, to be set off from any number less than 45° on the line of tangents, towards the right, and it is found to reach beyond the mark of 45°, you must see how far it extends beyond that mark, and set it off from 45° towards the left, and see what degree it falls upon, which will be the number sought, which must exceed 45°; if, on the contrary, you are to set off such a distance to the right from a number greater than 45°, you must proceed as before, only remembering that the answer must be less than 45°, and you must always consider the degrees above 45°, as if they were marked on the continuation of the line to the right hand of 45°.

97. The line of the meridional parts, marked Mer., begins at the right hand, and is numbered thus: 10, 20, 30, &c., to the left hand, where it ends at 87°. This line, with the line of equal parts, marked E.P., under it, are used together, and

only in Mercator's sailing. The upper line contains the degrees of the meridian, or latitude in a Mercator's chart, corresponding to the degrees of longitude on the lower line.

98. The use of this scale in solving the usual problems of Trigonometry, Plane Sailing, Middle Latitude Sailing, and Mercator's Sailing, will be given in the course of this work; but it will be unnecessary to enter into an explanation of its use in calculating the common problems of Nautical Astronomy, as it is much more accurate to perform those calculations by logarithms.

ON THE SLIDING RULE.

99. The Sliding Rule consists of a *fixed part* and a *slider*, and is of the same dimensions, and has the same lines marked on it as on a common Gunter or plane scale, which may be used with a pair of compasses in the same manner as those scales; and as a description of those lines has already been given, it will be unnecessary to repeat it here; it being sufficient to observe, that there are two lines of numbers, a line of log. sines and a line of log. tangents on the slider, and that it may be shifted so as to fix any face of it on either side of the fixed part of the scale, according to the nature of the question to be solved.

100. In solving any problem in Arithmetic, Trigonometry, Plane Sailing, &c., let the proportion be so stated that the first and second terms may be alike, and of course the third and fourth terms alike; then *bring the first term on the slider against the third on the fixed part, and against the second term on the slider will be found the fourth term on the fixed part. Or, if necessary, the first and second terms may be found on the fixed part, and the third and fourth on the slider.* Multiplication and Division are performed by this rule, in considering unity as one of the terms of the analogy.

101. Thus, to perform Multiplication, set 1 on the line of numbers of the fixed part against one of the factors on the line

of numbers of the slider, then against the other factor on the fixed part will be found the product on the slider.

Example.

To find the product of 4 by 12, draw out the slider till 1 on the fixed part coincides with 4 on the slider; then opposite 12 on the fixed part will be found 48 on the slider.

102. To perform Division, set the divisor on the line of numbers of the fixed part against 1 on the slider; then against the dividend, on the fixed part, will be found the quotient on the slider.*

Example.

To divide 48 by 4: Set 4 on the fixed part against 1 on the slider, then against 48 on the fixed part will be found 12 on the slider.

Examples in the Rule of Three.

1. If a ship sail 25 miles in 4 hours, how many miles will she sail in 12 hours at the same rate?

Bring 4 on the line of numbers of the fixed part against 25 on the line of numbers of the slider, then against 12 on the fixed part will be found 75 on the slider, which is the answer required.

2. If 3 pounds of sugar cost 21 cents, what will 27 pounds cost?

Bring 3 on the line of numbers of the fixed part, against 21 on the line of numbers of the slider; then against 27 on the fixed part will be found 189 on the slider.*

* For full information on this subject, see Mackay on the Sliding Gunter.

DESCRIPTION AND USE OF THE SECTOR.

103. This instrument consists of two rules or legs, movable round an axis or joint, as a centre, having several scales drawn on the faces, some single, others double; the single scales are like those upon a common Gunter's Scale, the double scales are those which proceed from the centre, each being laid twice on the same face of the instrument—namely, once on each leg. From these scales, dimensions or distances are to be taken, when the legs of the instrument are set in an angular position.

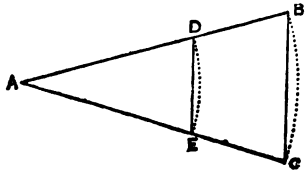
The single scales being used exactly like those on the common Gunter's Scale, it is unnecessary to notice them particularly; we shall therefore only enumerate a few of the uses of the double scales, the number of which is seven—namely, the scale of Lines, marked Lin. or L.; the scale of Chords, marked Cho. or C.; the scale of Sines, marked Sin. or S.; the scale of Tangents to 45°, and another scale of Tangents from 45° to about 76°, both of which are marked Tan. or T.; the scale of Secants, marked Sec. or S.; and the scale of Polygons, marked Pol.

104. The scale of lines, chords, sines, and tangents under 45°, are all of the same radius, beginning at the centre of the instrument, and terminating near the other extremity of each leg—namely, the lines at the division 10, the chords at 60°, the sines at 90°, and the tangents at 45°; the remainder of the tangents, or those above 45°, are on other scales, beginning at a quarter of the length of the former, counted from the centre, where they are marked with 45°, and extend to about 76°. The secants also begin at the same distance from the centre, where they are marked with 0, and are from thence continued to 75°. The scales of polygons are set near the inner edge of the legs, and where these scales begin they are marked with 4, and from thence are numbered backward or towards the centre to 12.

105. In describing the use of the sector, the terms *lateral distance* and *transverse distance* often occur. By the former is

meant the distance taken with the compasses on one of the scales only, beginning at the centre of the sector; and by the latter, the distance taken between any two corresponding divisions of the scales of the same name, the legs of the sector being in an angular position.

The use of the sector depends upon the proportionability of the corresponding sides of similar triangles (Euc. IV. 6). For if in the triangle ABC we take AB = AC and AD = AE, and draw DE, BC, it is evident that DE and BC will be parallel; therefore by the above-mentioned proposition AB : BC :: AD : DE; so that whatever part AD is of AB, the same part DE will be of BC; hence, if DE be the chord, sine, or tangent of any arc to the radius AD, BC will be the same to the radius AB.



Use of the line of Lines.

106. The line of lines is useful to divide a given line into any number of equal parts, or in any proportion, or to find third and fourth proportionals, or mean proportionals, or to increase a given line in any proportion.

Examples.

1. To divide a given line into any number of equal parts, as suppose 9: Make the length of the given line a transverse distance to 9 and 9, the number of parts proposed; then will the transverse distance of 1 and 1 be one of the parts, or the ninth part of the whole; and the transverse distance of 2 and 2 will be two of the equal parts, or two-ninths of the whole line, &c.

2. If a ship sail 52 miles in 8 hours, how much would she sail in 3 hours at the same rate?

Take 52 in your compasses as a transverse distance, and set it off from 8 to 8, then the transverse distance 3 and 3 being

measured laterally, will be found equal to 19½, which is the number of miles required.

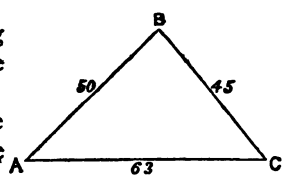
3. Having a chart constructed upon a scale of 6 miles to an inch, it is required to open the sector, so that a corresponding scale may be taken from the line of lines.

Make the transverse distance 6 and 6, equal to 1 inch, and this position of the sector will produce the given scale.

4. It is required to reduce a scale of 6 inches to a degree, to another of 3 inches to a degree.

Make the transverse distance 6 and 6, equal to the lateral distance 3 and 3; then set off any distance from the chart laterally, and the corresponding transverse distance will be the reduced distance required.

5. One side of any triangle being given, of any length, to measure the other two sides on the same scale.



Suppose the side AB of the triangle ABC measures 50, what are the measures of the other two sides?

Take AB in your compasses, and apply it transversely to 50 and 50; to this opening of the sector apply the distance AC in your compasses to the same number on both sides of the rule transversely; and where the two points fall will be the measure on the line of lines of the distance required; the distance AC will fall against 63, 63, and BC against 45, 45, on the line of lines.

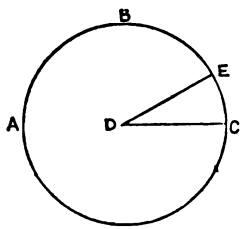
Use of the line of Chords on the Sector.

107. The line of chords upon the sector is very useful for protracting any angle, when the paper is so small that an arc cannot be drawn upon it with the radius of a common line of chords.

Suppose it were required to set off an arc of 30°, from the point C of the small circle ABC.

Take the radius DC in your compasses, and set it off transversely from 60° to 60° on the chords, then take the transverse

extent from 30° to 30° on the chords; and place one foot of the compasses in C, the other will reach to E, and CE will be the arc required. And by the converse operation any angle or arc may be measured—namely, with any radius describe an arc about the angular point; set that radius transversely from 60° to 60°; then take the distance of the arc, intercepted between the two legs, and apply it transversely to the chords, which will shew the degrees of the given angle.



108. NOTE.—When the angle to be protracted exceeds 60°, you must lay off 60°, and then the remaining part; or if it be above 120°, lay off 60° twice, and then the remaining part. And in a similar manner any arc above 60° may be measured.

Uses of the lines of Sines, Tangents, and Secants.

109. By the several lines disposed on the sector, we have scales of several radii, so that,

1. Having a length or radius given, not exceeding the length of the sector when opened, we can find the chord, sine, &c., of an arc to that radius; thus, suppose the chord, sine, or tangent of 20° to a radius of 2 inches be required: Make 2 inches the transverse opening from 60° to 60° on the chords; then will the same extent reach from 45° to 45° on the tangents, and from 90° to 90° on the sines; so that to whatever radius the line of chords is set, to the same are all the others set also. In this disposition, therefore, if the transverse distance between 20° and 20° on the chords be taken with the compass, it will give the chord of 20°; and if the transverse of 20° and 20° be in like manner taken on the sines, it will be the sine of 20°; and lastly, if the transverse distance of 20° and 20° be taken on the tangents, it will be the tangent of 20° to the same radius of 2 inches.

2. If the chord or tangent of 70° were required: For the

chord you must first set off the chord of 60° (or the radius) upon the arc, and then set off the chord of 10°. To find the tangent of 70°, to the same radius, the scale of upper tangents must be used, the under one only reaching to 45°; making therefore 2 inches the transverse distance to 45° and 45° at the beginning of that scale, the extent between 70° and 70° on the same will be the tangent of 70° to 2 inches radius.

3. To find the secant of any arc: Make the given radius the transverse distance between 0 and 0 on the secants; then will the transverse distance of 20° and 20°, or 70° and 70°, give the secant of 20° or 70° respectively.

4. If the radius and any line representing a sine, tangent, or secant, be given, the degrees corresponding to that line may be found by setting the sector to the given radius, according as a sine, tangent, or secant is concerned; then taking the given line between the compasses, and applying the two feet transversely to the proper scale, and sliding the feet along till they both rest on like divisions on both legs; then the divisions will shew the degrees and parts corresponding to the given line.

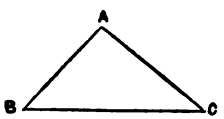
Use of the line of Polygons.

110. The use of this line is to inscribe a regular polygon in a circle. For example, let it be required to inscribe an octagon in a circle. Open the sector till the transverse distance 6 and 6 be equal to the radius of the circle; then will the transverse distance of 8 and 8 be the side of the inscribed polygon.

Use of the Sector in Trigonometry.

111. All proportions in trigonometry are easily worked by the double lines on the sector; observing that the sides of triangles are taken off the line of lines, and the angles are taken off the sines, tangents, or secants, according to the nature of the proportion. Thus, if in the triangle ABC we have given AB = 56, AC = 64, and the angle ABC = 46° 30′, to find the rest. In this

case we have (132) the following proportions, as AC (64) : AB (56) :: sine ∠ B : sine ∠ C, and as sine B : sine A :: AC : BC. Therefore to work these proportions by the sector, take the lateral distance 64 = AC from the line of lines, and open the sector to make this a transverse distance at 46° 30′ = ∠ B on the sines; then take the lateral distance 56 = AB on the lines, and apply it transversely on the sines, which will give 39° 24′ = ∠ C. Hence the sum of the angles B and C is 85° 54′, which, taken from 180°, leaves the angle A = 94° 6′. Then to work this second proportion, the sector being set at the same opening as before, take the transverse distance of 94° 6′ = the angle A on the sines, or, which is the same thing, the transverse distance of its supplement 85° 54′; then this applied laterally to the lines, gives the sought side BC = 88. In the same manner we might solve any problem in trigonometry, where the tangents and secants occur, by only measuring the transverse distances on the tangents or secants, instead of measuring them on the sines, as in the preceding example. All the problems that occur in Nautical Astronomy may be solved by the sector, but as the calculation by logarithms is much more accurate, it will be useless to enter into a further detail on this subject.



PLANE TRIGONOMETRY.

112. Plane Trigonometry is the science which shews how to find the measures of the sides and angles of plane triangles, some of them being already known. It is divided into two parts, right-angled and oblique-angled; in the former case, one of the angles is a right angle or 90°; in the latter, they are all oblique.

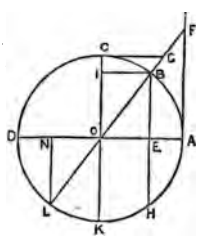
113. In trigonometry, for the purposes of calculation, the circumference of a circle, and also all the angles round a point,

are each divided into 360 equal parts, called *degrees;* each degree is subdivided into 60 equal parts, called *minutes;* and each minute is again subdivided into 60 equal parts, called *seconds:* degrees, minutes, and seconds are thus indicated—7° 13′ 43″; which is read—seven degrees, thirteen minutes, and forty-three seconds.

114. In every triangle there are *six* parts—the three sides, and the three angles; any three of which being given, *except the three angles*, the other three may be found by various methods—by Gunter's scale, by geometrical construction, or by arithmetical calculation, the latter of which is by far the most accurate.

Definitions.

115. The *complement of an arc* is its difference from a quadrant, either in defect or excess; and that of an angle is its difference from 90°, either in excess or defect. The *supplement of an arc* is its difference from a semicircumference, and that of an angle is its difference from 180°.



116. In the adjoining figure, let AB be an arc, and AOB the corresponding angle at the centre; then CB is the complemental arc, and COB is the complemental angle; also BCD is the supplemental arc, and BOD is the supplemental angle.

117. The *sine of an arc* is a straight line drawn from one extremity of an arc perpendicular to the radius passing through the other extremity of the arc; thus, BE is the *sine* of the arc AB, or of the angle AOB, to radius OB.

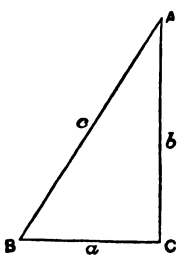
118. The *tangent of an arc* is a straight line touching the arc at one extremity, and limited by the radius produced through the other extremity; thus, AF is the *tangent* of the arc AB, or of the angle AOB, to radius OA = OB.

119. The *secant of an arc* is a straight line joining the centre of the circle, and the further extremity of the tangent drawn at the

origin of the arc; thus, OF is the *secant* of the arc AB, or of the angle AOB, to radius OA or OB.

120. The *sine, tangent, and secant of the complement of an arc,* are called the cosine, cotangent, and cosecant of the arc itself; thus, BC is the complement of the arc AB, and BOC, the angle which BC subtends at the centre, is the complement of the angle AOB. Now BI = OE is the sine of the arc BC, or the angle BOC; it is therefore called the *cosine* of the arc AB, or of the angle AOB. In the same manner, CG is the *tangent* and OG the *secant* of the arc BC, or the angle BOC; and therefore CG is called the *cotangent* and OG the *cosecant* of the arc AB, or of the angle AOB.

In the above figure AE = AO − OE = Rad. − cosine, is called the versed sine; and CI = OC − IO = OC − BE = Rad. − sine, is called the coversed sine of the arc AB, or the angle AOB.



121. The following are also frequently now adopted as the definitions of the trigonometrical ratios, and are often found more convenient in the investigation of rules and formulæ. Let ABC be a triangle right-angled at C, and let the length of the side opposite to the angle at A, be called a; that opposite the angle at B, b; and that opposite the angle at C, c; a, b, and c all being measured in the same kind of units: then,

Definitions.		Consequences.
122. Sin. A $= \frac{a}{c}$;	∴ multiplying by c,	$a = c$ sin. A.
123. Cos. A $= \frac{b}{c}$;	∴ " "	c, $b = c$ cos. A.
124. Tan. A $= \frac{a}{b}$;	∴ " "	b, $a = b$ tan. A.
125. Cot. A $= \frac{b}{a}$;	∴ " "	b, $b = a$ c

Definitions. Consequences.

126. Sec. $A = \frac{c}{b}$; $\therefore$ multiplying by b, $c = b$ sec. A.

127. Cosec. $A = \frac{c}{a}$; $\therefore$ " " a, $c = a$ cosec. A.

128. By applying to these definitions and their consequences the analytical and logarithmic principles already established, all the parts of any right-angled plane triangle may be calculated, when there are given either two sides, or a side and one of the acute angles.

129. If two sides are given:

RULE.—Use the definitions to find one of the acute angles, then the other is found by applying the fact, that *all the angles of every plane triangle are together equal to two right angles*, or 180° (Euc. I. 32).

130. When a side and one of the acute angles are given:

RULE.—Apply the consequences of the definitions.

Example I.

In a right-angled triangle, the hypotenuse, or side opposite the right angle, is 425, and the angle at the vertex is 42° 15′; find the other angle and the two sides that contain the right angle.

Solution.

By Art. (129) the triangle being right-angled, the third angle = 180° − 90° − 42° 15′ = 90° − 42° 15′ = 47° 45′, the angle at the base.

Again, by the consequences in Arts. (122, 123), are obtained the perpendicular $b = c$ cos. A, and the base $a = c$ sin. A.

These calculated by natural sines and cosines are as follows:

Nat. cos. 42° 15′	=	·7402181	Nat. sin. 42° 15′	=	·6723668
		425			425
		37010905			33618340
		14804362			13447336
		29608724			26894672
∴ perp.	=	314·5926925	Base	=	285·7558900

BY LOGARITHMS.

Log. b = log. c + log. cos. 42° 15′ − 10; and
Log. a = log. c + log. sin. 42° 15′ − 10.

Perpendicular.			Base.		
Log. 425	=	2·6283889	Log. 425	=	2·6283889
Log. cos. 42° 15	=	9·8693597	Log. sin. 42° 15′	=	9·8276063
314·59	=	2·4977486	285·75	=	2·4559952
		49			862
26	=	37	59	=	90
∴ AC = 314·5926			and BC = 285·7559		

131. NOTE.—In adding the above logarithms, 10 has been omitted from the sum of the indices; the reason is that all the logarithms of the trigonometrical functions are inserted in the tables 10 of an index greater than the true logarithms of the natural quantities, to avoid minus indices. So in calculating from the definitions by logarithms, the logarithm of the numerator has to be increased by 10 before subtracting the logarithm of the denominator.

BY CONSTRUCTION.

Draw AC and AB (fig., Arts. 121–127), containing an angle of 42° 15′, and from a scale of equal parts lay off AB = 425; from B draw BC perpendicular to AC, apply the length of BC to the same scale of equal parts from which AB was measured, and the length will be found to be 285·7 nearly; in the same way apply the length of AC, and it will measure 314·6 nearly.

Example II.

In a right-angled triangle the perpendicular is 328, and the angle at the vertex = 48° 17′; find the third angle, the base, and the hypotenuse.

The third angle = 90 − (48° 17′) = 41° 43′.

Also (Art. 124), $\frac{BC}{AC} = \text{tan. A}$;

$\therefore$ BC = AC × tan. A = 328 tan. 48° 17′.

And (Art. 126) $\frac{AB}{AC} = \text{sec. A}$;

$\therefore$ AB = AC × sec. A = 328 sec. 48° 17′.

BY NATURAL TANGENTS AND SECANTS.

$BC = 328 \times 1{\cdot}1217183 = 367{\cdot}9236024.$

$AB = 328 \times 1{\cdot}5027481 = 492{\cdot}9013768.$

BY LOGARITHMS.

To find the base BC.

Log. 328	=	2·5158738
Log. tan. 48° 17′	=	10·0498838
∴ BC = 367·92		2·5657576
		34
36		= 42
∴ BC = 367·9236		

To find the hypotenuse AB.

Log. 328	=	2·5158738
Log. sec. 48° 17′	=	10·1768862
492·90		2·6927600
		588
13		12
AB = 492·9013		

and

BY CONSTRUCTION.

Draw AC and AB containing an angle = 48° 17′, from a scale of equal parts lay off AC = 328; draw CB perpendicular to AC, and produce it to meet AB in B; then apply the lengths of AB and BC to the same scale of equal parts, and AB will measure 492·9, and BC will measure 367·9 nearly.

Example III.

In a right-angled triangle the base is 530, and the vertical angle is 51° 35′ 25″; find the third angle, the hypotenuse, and the perpendicular.

Solution.

The third angle = 90° − (51° 35′ 25″) = 38° 24′ 35″.

Also (Art. 127), $\frac{AB}{BC} = \text{cosec. A};$

$\therefore AB = BC \times \text{cosec. A} = 530 \text{ cosec. } 51° 35′ 25″.$

And (Art. 125), $\frac{AC}{BC} = \text{cot. A};$

$\therefore AC = BC \text{ cot. A} = 530 \text{ cot. } 51° 35′ 25″.$

BY NATURAL TANGENTS AND SECANTS.

$AB = 530 \times 1{\cdot}2761808 = 676{\cdot}375824.$

$AC = 530 \times {\cdot}7928666 = 420{\cdot}219298.$

BY LOGARITHMS.

To find AB.			To find AC.		
Log. 530	=	2·7242759	Log. 530	=	2·7242759
Log. cosec. 51° 35′ 25″	=	10·1059121	Log. cot. 51° 35′ 25″	=	9·8992000
676·37		2·8301880	420·21		2·6234759
		43			664
57	=	37	91	=	95

$\therefore AB = 676{\cdot}3757$ and $AC = 420{\cdot}2191$

BY CONSTRUCTION.

Draw BC (fig. to Arts. 121–127), and from a scale of equal parts make it = 530, and draw CA perpendicular to it; take the complement of the vertical angle, namely, 38° 24′ 35″, and at the point B make the angle ABC = that complement; ABC will be the triangle required, and its sides AB and AC, measured on the same scale of equal parts, will be found to be 676·4 and 420·2 respectively.

Example IV.

The base and perpendicular of a right-angled triangle are 360 and 420 respectively; required the two acute angles and the hypotenuse.

Solution.

Here, referring to the figure (Arts. 121–127), AC = 420, BC = 360; therefore (Art. 124),

$$\text{tan. } A = \frac{BC}{AC} = \frac{360}{420} = \frac{6}{7} = {\cdot}8571428.$$

But $\text{tan. } 40° 36′ = {\cdot}8571037$

391 difference.

Now the difference between the tan. 40° 37′ and that of 40° 36′ is found by subtraction to be 5047, and therefore

$$5047 : 391 :: 60'' : 4''\cdot6.$$

The angle at the vertex is therefore = 40° 36′ 4″·6, and hence the angle at the base = 90° − (40° 36′ 4″·6) = 49° 23′ 55″·4.

The angles being now calculated, the hypotenuse can be found as follows,

$$\text{(Art. 126)}\ \frac{AB}{AC} = \text{sec. A.}$$

$$\therefore AB = AC \times \text{sec. A} = 420 \times \text{sec. } 40° 36' 4''\cdot6$$
$$= 420 \times 1\cdot3170775 = 553\cdot17255.$$

BY LOGARITHMS.

$$\text{Log. tan. A} = 10 + \log. 360 - \log. 420.$$

Log. 360 + 10	=	12·5563025
Log. 420	=	2·6232493
		9·9330532
Log. tan. 40° 36′	=	9·9330334
		198 × 60 ÷ 2556 = 4″·6.

Hence angle A = 40° 36′ 4″·6, and by Art. (126),
AB = AC sec. A ∴ log. AB = log. 420 + log. sec. 40° 36′ 4″·6.

Log. 420	=	2·6232493
Log. sec. 40° 36′ 4″·6	=	10·1196113
Log. 553·17		2·7428606
		586
		20
25		

$$\therefore AB = 553\cdot1725$$

BY CONSTRUCTION.

Draw AC and CB at right angles to each other, and from a scale of equal parts lay off AC = 420, and CB = 360; then lay off on the same scale the measured length of AB, and it will be found to be 553·2 nearly.

To measure the angles from a scale of chords, as found on

Gunter's scale or the common sector, measure off the chord of 60° with a pair of compasses, and from the centre, A, and radius equal the chord of 60°, describe a circle cutting the sides or the sides produced, then measure with the compasses the distance of the points of section of the arc with the sides; this distance applied to the scale of chords will give the angle BAC = 40° 36′ nearly. In the same manner, by taking the angle at B as a centre, the other acute angle may be measured and found to be 49° 24′.

Exercises 8.

(101.) The hypotenuse of a right-angled triangle is 400, and the angle at the base is 32° 11′; find the other parts.

(102.) The hypotenuse of a right-angled triangle is 256, and the vertical angle is 43° 23′; find the other parts.

(103.) The hypotenuse of a right-angled triangle is 555, and the base is 444; find the acute angles and the perpendicular.

(104.) The hypotenuse of a right-angled triangle is 290·35, and the perpendicular is 187·2; find the acute angles and the base.

(105.) The base of a right-angled triangle is 630, and the angle at the base is 51° 30′; find the other parts.

(106.) The base of a right-angled triangle is 339, and the vertical angle is 48° 57′; find the other parts.

(107.) The base of a right-angled triangle is 427, and the hypotenuse is 609; find the acute angles and the perpendicular.

(108). The base of a right-angled triangle is 247·48, and the perpendicular is 347·95; find the hypotenuse and the acute angles.

(109.) The perpendicular of a right-angled triangle is 410, and the vertical angle is 45° 37′; find the other parts.

(110). The perpendicular of a right-angled triangle is 963, and the angle at the base is 87° 11′; find the other parts.

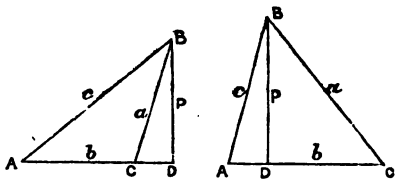
(111.) The perpendicular of a right-angled triangle is 113, and its hypotenuse is 355; find the acute angles and the base.

(112.) The perpendicular of a right-angled triangle is 113, and the base is 355; find the acute angles and the hypotenuse.

OF OBLIQUE-ANGLED TRIANGLES.

132. Let it now be required to find a relation between the sides of any plane triangle and the trigonometrical ratios of its angles.

Let ABC be any oblique-angled triangle, and call the length of the sides opposite the angles A, B, and C respectively, a, b, and c; also, let BD, a perpendicular from B on AC, or AC produced, be called p; then, since ABD is a right-angled triangle,



by (122), $$\text{sin. A} = \frac{p}{c};$$

and since BCD is a right-angled triangle,

by (122), $$\text{sin. C} = \frac{p}{a}.$$

Dividing now the first of these results by the second, side by side; then

(29, *i*), $$\frac{\text{sin. A}}{\text{sin. C}} = \frac{p}{c} \times \frac{a}{p} = \frac{a}{c};$$

$$\therefore \frac{\text{sin. A}}{\text{sin. C}} = \frac{a}{c}, \text{ or sin. A : sin. C} = a : c.$$

In the same manner, if perpendiculars be drawn from the other angles, it may be demonstrated that

$$\frac{\sin.\ A}{\sin.\ B}=\frac{a}{b},\ \text{or that sin. A : sin. B} = a : b;$$

and

$$\frac{\sin.\ B}{\sin.\ C}=\frac{b}{c},\ \text{or that sin. B : sin. C} = b : c.$$

Hence also

$$\frac{a}{\sin.\ A}=\frac{b}{\sin.\ B}=\frac{c}{\sin.\ C}.$$

133. *Hence the sines of the angles of a triangle are proportional to the opposite sides.*

By this principle just proved the sides and angles of a triangle can all be found when there are given either *two sides and an angle opposite to one of them, or two angles and a side,* for then the third angle is also known, since the three angles are together equal to two right angles, or 180°.

134. From the formulæ proved in Art. (132) are easily derived the following:

$$\sin.\ A=\frac{a}{b}\sin.\ B=\frac{a}{c}\sin.\ C.$$

$$\sin.\ B=\frac{b}{a}\sin.\ A=\frac{b}{c}\sin.\ C.$$

$$\sin.\ C=\frac{c}{a}\sin.\ A=\frac{c}{b}\sin.\ B.$$

Also,

$$a=\frac{\sin.\ A}{\sin.\ B}b=\frac{\sin.\ A}{\sin.\ C}c.$$

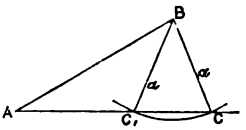
$$b=\frac{\sin.\ B}{\sin.\ A}a=\frac{\sin.\ B}{\sin.\ C}c.$$

$$c=\frac{\sin.\ C}{\sin.\ A}a=\frac{\sin.\ C}{\sin.\ B}b.$$

135. All triangles included in the data of (133) can be solved either by (133) or by (134); only there is one case which admits of a double solution, and is therefore called *ambiguous.*

It is as follows: When there are given two sides, and the

angles opposite the *less* of the given sides, there can be constructed two triangles having the same data, but different in other respects.



Thus, let AB and the angle at A be given, and also the length of BC, *less than* AB; take B as a centre, and BC as radius, and describe an arc CC_1, it will cut the line AC in two points, or touch it in one; join BC and BC_1, and there are two triangles, ABC and ABC_1, each of which have the side AB and the angle at A, while their third sides BC and BC_1 are equal. In solving this triangle by

$$(132),\ \frac{\sin.\ C}{\sin.\ A}=\frac{c}{a};$$

$$\therefore \sin.\ C=\frac{c}{a}\sin.\ A.$$

But sin. C is either the sine of BCC_1 or of BC_1C, for these angles are equal (Euc. I. 5), but the sine of BC_1C = the sine of BC_1A, for *the sine of an angle is equal to the sine of its supplement.* Hence, in working for the angle at C, we get the acute angle ACB, but this being subtracted from 180°, we get the obtuse angle AC_1B; the two angles at A and C or C_1 being now known, the third angles ABC or ABC_1 can readily be found, and the base AC or AC_1. If the arc CC_1 should only touch the line AC, then the angle at C would be found a right angle; hence, it and its supplement would be the same, and the points C and C_1 would coincide.

136. Solution of this case of oblique-angled triangles.

RULE.

Write the quantity sought for the numerator of a fraction, and the given quantity of the same kind for its denominator, make this fraction equal to its value by Art. (132), *which gives an equation; multiply both sides by the denominator of the unknown quantity,*

and its value will be found.—Or use the formulæ in Art. (134), which is the same thing.

Example I.

In an oblique-angled triangle ABC, given AB = 536, BC = 385, and the angle at C = 49° 13′; to find the angles at A and B, and the third side AC.

Solution.

$$\frac{\text{sin. A}}{\text{sin. C}} = \frac{a}{c} = \frac{\text{BC}}{\text{AB}} = \frac{385}{536};\ \text{multiply by sin. C};$$

$$\therefore \text{sin A} = \frac{385}{536}\ \text{sin. } 49^\circ\ 13'.$$

BY LOGARITHMS.

Log. sin. A = log. sin. 49° 13′ + log. 385 − log. 536, or

= log. sin. 49° 13′ + log. 385 + ar. co. log. 536 − 10

To find ∠ A.			Or, by ar. co. log. 536.		
Log. sin. 49° 13′	=	9·8792021	Log. sin. 49° 13′	=	9·8792021
Log. 385	=	2·5854607	Log. 385	=	2·5854607
		12·4646628	Ar. co. log. 536	=	7·2708352
Log. 536	=	2·7291648	Log. sin. 32° 56′ 51″·8	=	9·7354980
Log. sin. 32° 56′ 51″·8	=	9·7354980	Hence ∠ A = 32° 56′ 52″ nearly.		

∠ B = 180° − (∠ A + ∠ C) = 180° − 82° 9′ 52″ = 97° 50′ 8″

To find AC by (136), $\frac{\text{AC}}{\text{AB}} = \frac{\text{sin. B}}{\text{sin. C}}$ ∴ $\text{AC} = \frac{\text{sin. B}}{\text{sin. C}} \times \text{AB}$

$$= \frac{\text{sin. } 97^\circ\ 50'\ 8''}{\text{sin. } 49^\circ\ 13'} \times 536$$

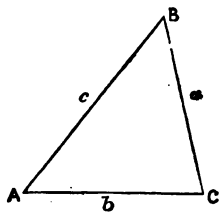
∴ Log. AC = log. sin. 97° 50′ 8″ + log. cosec. 49° 13′ + log. 536 − 20

Log. sin. 97° 50′ 8″	=	9·9959261
Log. cosec. 49° 13′	=	10·1207979
Log. 536	=	2·7291648
Log. 701·2756	=	2·8458888

∴ AC = 701·2756

Example II.

In the triangle ABC there are given the side BC = 198, the angle at B = 51° 58′, and the angle at C = 75° 2′; required the third angle at A, and the sides AB and AC.



Solution.

The $\angle A = 180° - (51° 58' + 75° 2') = 180° - 127° = 53°$.

To find AC.

$$\frac{AC}{BC} = \frac{\text{sin. B}}{\text{sin. A}}$$

$$\therefore AC = \frac{\text{sin. B}}{\text{sin. A}} \times BC$$

$$= \text{sin. B cosec. A} \times BC.$$

Log. sin. 51° 58′	=	9·8963346
Log. cosec. 53°	=	10·0976514
Log. (BC = 198)	=	2·2966652
Log. 195·277		2·2906512

$\therefore AC = 195·277.$

To find AB.

$$\frac{AB}{BC} = \frac{\text{sin. C}}{\text{sin. A}}$$

$$\therefore AB = \frac{\text{sin. C}}{\text{sin. A}} \times BC$$

$$= \text{sin. C cosec. A} \times BC.$$

Log. sin. 75° 2′	=	9·9850114
Log. cosec. 53°	=	10·0976514
Log. (BC = 198)	=	2·2966652
Log. 239·5124	=	2·3793280

$\therefore AB = 239·5124.$

Example III.

Given the side AB = 530, the side BC = 325, and the angle at A = 30°; to find the remaining angles, and the third side.

Solution.

This is an example in the ambiguous case referred to in Art. (135), since the given angle is opposite to the least of the two given sides. Had the side BC been given 265, and the other parts the same, there would have been but one solution; or, if the side AB had been given 650, and the other parts the same, there would have been but one solution; or, lastly, had the angle at A been given that whose sine is 325 ÷ 530, or ·6132075, there would have been but one solution, and the triangle would

have been in all the cases right-angled at C; in the first two cases supposed, the angle at B would have been 60°; and in the latter case, the angle A would = 37° 49′ 18″, and that at B would = 52° 10′ 42″.

$$\frac{\sin.\ C}{\sin.\ A} = \frac{c}{a};\ \therefore \sin.\ C = \frac{c}{a} \sin.\ A.$$

Log.(c = 530)	=	2·7242759
Log. sin. (A = 30°)	=	9·6989700
		12·4232459
Log.(a = 325)	=	2·5118834
Log. sin. 54° 37′ 31″	=	9·9113625

$\therefore \angle C = 54° 37′ 31″$, or $180 - (54° 37′ 31″) = 125° 22′ 29″$

	$\angle C =$ 54° 37′ 31″	or	$\angle C =$ 125° 22′ 29″
and	$\angle A =$ 30°		$\angle A =$ 30°
$\therefore \angle A + \angle C =$	84° 37′ 31″	or	= 155° 22′ 29″
	180°		180°
	$\angle B =$ 95° 22′ 29″	or	$\angle B =$ 24° 37′ 31″

To find AC.

$$\frac{AC}{BC} = \frac{\sin.\ B}{\sin.\ A}\ \therefore AC = \frac{\sin.\ 24° 37′ 31″}{\sin.\ 30°} \times 325\ (A);$$

$$\text{or } AC = \frac{\sin.\ 95° 22′ 29″}{\sin.\ 30°} \times 325\ (B).$$

(A)			(B)		
Log. sin. 24° 37′ 31″	=	9·6198046	Log. sin. 95° 22′ 29″	=	9·9980864
Log. 325	=	2·5118834	Log. 325	=	2·5118834
		12·1316880			12·5099698
Log. sin. 30°	=	9·6989700	Log. sin. 30°	=	9·6989700
Log. 270·8434	=	2·4327180	Log. 647·1423	=	2·8109998

$\therefore$ AC = 270·8434 or 647·1423.

Exercises 9.

(113.) Given the base AC = 1003, AB = 918, and the vertical angle ABC = 60° 43′; to find the other angles and the side BC.

(114.) Given the base AC = 158·7, the side BC = 200, and the angle at A = 71° 37′; to find the other angles and the side AB.

(115.) Given the angle at A = 38° 12′, the angle at C = 48° 24′, and the side AC = 1250; to find the angle at B, and the sides AB and BC.

(116.) Given the angle at A = 56° 15′, that at B = 61° 12′, and the side AC = 1105; to find the angle at C, and the sides AB and BC.

(117.) If AB = 432, BC = 528, and ∠ A = 60°; what are the angles at B and C, and the side AC?

(118.) If AB = 593, AC = 486, and ∠ C = 80° 45′; what are the angles at A and B, and the side BC?

(119.) If AB = 738, BC = 439, and the angle at C = 59° 18′; find the angles at A and B, and the third side AC.

(120.) If AC = 490, BC = 365, and the angle at A = 40° 30′; find the angles at B and C, and the third side AB.

(121.) The sum of the three sides of a triangle is 1105, one side is 420, and the angle opposite it is 63°; find the other two angles, and the other two sides. (Apply Euc. I. 20.)

(122.) The difference of two sides of a triangle is 40, and the third side is 583, also the angle opposite this third side is 76° 30′; find the other angles, and also the other sides.

CHAPTER III.

OF ASTRONOMY AND GEOGRAPHY, SO FAR AS NECESSARY TO THE NAVIGATOR.

137. The land and water of this earth make up a body of a spherical or globular form, or so nearly so that it is called the TERRAQUEOUS GLOBE.

138. This globe moves round its axis from *west* to *east* in about 24 hours, and thereby causes the celestial bodies to revolve *apparently* from east to west in the same time; whence the vicissitudes of day and night.

139. The two points in which the axis, on which the earth revolves, meets the surface, are called the *poles* of the earth. If the same axis be produced both ways to meet the celestial sphere, it will meet it in two opposite points, called the CELESTIAL POLES. The pole of the earth lying towards the north is called the NORTH or ARCTIC POLE, and that towards the south is called the SOUTH or ANTARCTIC POLE.

140. If a plane be supposed to pass through the earth's centre at right angles to its axis, it will intersect its surface in a great circle, called the EQUATOR; and if the plane of the equator be produced to meet the celestial sphere, it will trace out on it a circle, called the EQUINOCTIAL, which will divide the earth and heavens into two halves or hemispheres; that towards the north being called the NORTHERN HEMISPHERE, and that towards the south being called the SOUTHERN HEMISPHERE.

141. A GREAT CIRCLE is the intersection of a plane passing through the centre with the circumference of the sphere.

142. Great circles passing through the poles of the earth are called MERIDIANS, and that which passes through any place is called the *meridian* of that place. These planes being produced to the heavens pass through the celestial poles, and trace out on the celestial sphere the CELESTIAL MERIDIANS. They are called *meridians*, because when the sun is on the meridian of any place, it is noon or mid-day at that place.

143. If a plane be supposed to touch the surface of the earth in any point upon which a spectator stands, and to be produced to the heavens, it will there make a circle, called the HORIZON, which separates the visible from the invisible parts of the celestial sphere. This horizon is, properly speaking, called the *sensible* horizon; the TRUE or RATIONAL HORIZON is traced out by a plane passing through the centre of the earth, and parallel to the former plane, or sensible horizon; it divides the earth and heavens into hemispheres, called the UPPER and LOWER HEMISPHERES. These two horizons, however, when produced to the region of the fixed stars, sensibly coincide; but at the distance of the *Sun*, *Moon*, and *Planets*, their distance causes what is called PARALLAX. (See Note on page 73.)

144. Since the earth revolves on its axis from *west* to *east*, it is plain that a spectator upon its surface, together with his horizon, in the axis of which he always is, must move the same way; consequently those celestial bodies towards the east that were before in the *Lower Hemisphere*, will gradually come into the *Upper Hemisphere* of the spectator, and thereby become visible, his horizon being depressed below them; while those in the west that were at first visible will gradually disappear, the horizon being elevated above them. In this way arises the apparent motion of the heavenly bodies by which they are said to *rise* and *set*, and by which they appear to describe circles round the pole, parallel to the equinoctial, which are greater or less, according as they are more or less distant from the nearest pole.

145. When a celestial body comes first into view on the eastern side of the horizon, it is said to *rise;* and when by

its apparent motion it comes to the *meridian*, it is said to CULMINATE, being at its greatest height; and lastly, when it begins to disappear on the western horizon, it is said to *set*.

146. If through the centre of the earth there be drawn a line perpendicular to the plane of the horizon, and produced on both sides to the heavens, it will there mark out the two poles of the horizon; the one which is directly over the spectator's head is called the ZENITH, and the opposite one, which is invisible to him, being directly under his feet, is called the NADIR.

147. Vertical or azimuth circles are great circles passing through the *zenith* and *nadir*. Among these there are two principal ones—namely, the *meridian*, which passes through the *zenith* and *nadir* and the *poles* of the celestial sphere, and so cuts both the horizon and equinoctial at right angles; the points in which it cuts the horizon are called the NORTH and SOUTH points. The other principal vertical is called the PRIME VERTICAL, and cuts at right angles the meridian in the *zenith* and *nadir*, and the horizon in two opposite points, called the EAST and WEST points. The *north*, *south*, *east*, and *west* are called the four CARDINAL POINTS.

148. The MERIDIAN of any place is a semicircle passing through that place, and terminating at the poles of the equator.

149. The LATITUDE of any place is that portion of the meridian which is contained between the *equator* and the *given place*, and is either *north* or *south*, according as the given place is in the *northern* or *southern* hemisphere.

150. The PARALLEL of LATITUDE of any place is a circle passing through that place parallel to the equator.

151. The DIFFERENCE of LATITUDE between any two places is an arc of a meridian intercepted between their corresponding *parallels* of *latitude*. Hence, if the places lie between the equator and the same pole, their difference of latitude is found by subtracting the less latitude from the greater; but if they are on opposite sides of the equator, their difference of latitude is equal to the sum of the latitudes of both places.

152. The FIRST MERIDIAN is an imaginary semicircle passing

through any remarkable place, and is therefore arbitrary. The meridian of the Royal Observatory at Greenwich is by the British esteemed the first meridian. In other countries, the first meridian is usually that which passes through the principal observatory or chief city of that country.

153. The LONGITUDE of any place is that portion of the equator which is contained between the first meridian and the meridian of the given place. The longitude of a place is said to be *east* or *west*, according as the place is on the east or west side of the first meridian, and it cannot exceed 180°.

154. The DIFFERENCE of LONGITUDE between any two places is the intercepted portion of the equator between the meridians of these places.

Problem I.

155. Given the latitudes of two places, to find the difference of latitude between them.

RULE.

Subtract the less latitude from the greater, if they be *both north* or *both south;* but *add* them if the one be *north* and the other *south.*

Example I.

Required the difference of latitude between Edinburgh in latitude 55° 57′ N., and Brest in latitude 48° 23′ N.

Edinburgh, latitude of 55° 57′ N.
Brest, latitude of 48° 23′ N.
} Subtract, since they are both north.

Diff. latitude = 7° 34′
60
= 454 nautical miles.

Example II.

Required the difference of latitude between the Lizard Light in latitude 49° 58′ N., and Bencoolen in latitude 3° 48′ S.

Lizard Light, latitude of 49° 58′ N.
Bencoolen, latitude of 3° 48′ S.
} Add, since the one is north and the other south.

Diff. latitude = 53° 46′
60
= 3226 nautical miles.

its apparent motion it comes to the *meridian*, it is said to CULMINATE, being at its greatest height; and lastly, when it begins to disappear on the western horizon, it is said to *set*.

146. If through the centre of the earth there be drawn a line perpendicular to the plane of the horizon, and produced on both sides to the heavens, it will there mark out the two poles of the horizon; the one which is directly over the spectator's head is called the ZENITH, and the opposite one, which is invisible to him, being directly under his feet, is called the NADIR.

147. Vertical or azimuth circles are great circles passing through the *zenith* and *nadir*. Among these there are two principal ones—namely, the *meridian*, which passes through the *zenith* and *nadir* and the *poles* of the celestial sphere, and so cuts both the horizon and equinoctial at right angles; the points in which it cuts the horizon are called the NORTH and SOUTH points. The other principal vertical is called the PRIME VERTICAL, and cuts at right angles the meridian in the *zenith* and *nadir*, and the horizon in two opposite points, called the EAST and WEST points. The *north*, *south*, *east*, and *west* called the four CARDINAL POINTS.

148. The MERIDIAN of any place is a semicircle passing through that place, and terminating at the poles of the equator, and is that portion of the meridian which is contained between the *equator* and the *place*, and is either *north* or *south*, according as the place is in the *northern* or *southern* hemisphere.

150. The PARALLEL of LATITUDE of any place is a circle passing through that place parallel to the equator.

151. The DIFFERENCE of LATITUDE between any two places is an arc of a meridian intercepted between the *parallels* of *latitude*. He[illegible] equator and the same [illegible] by subtracting the [illegible] on opposite sides [illegible] equal to the [illegible]

152. [illegible]

Exercises 10.

Find the difference of latitude between the two places A and B in the following exercises:

(123.) A in latitude 70° 18′ N., and B in latitude 49° 45′ N.
(124.) A " 36° 50′ N. " B " 29° 11′ N.
(125.) A " 11° 7′ S. " B " 54° 16′ S.
(126.) A " 12° 13′ N. " B " 12° 13′ S.
(127.) A " 11° 51′ S. " B " 6° 50′ N.
(128.) A " 65° 0′ S. " B " 31° 33′ S.
(129.) A " 19° 29′ N. " B " 0° 2′ S.
(130.) A " 11° 50′ N. " B " 61° 15′ N.

Problem II.

156. Given the longitudes of two places, to find the difference of longitude between them.

RULE.

If the longitudes be of the *same name*—that is, both west or both east—subtract the less from the greater, and the remainder will be the difference of longitude; but if the longitudes be of *contrary names*, add them and their sum, if less than 180° is the difference of longitude; but if greater than 180°, subtract it from 360°, and the remainder will be the difference of longitude.

Example I.

Required the difference of longitude between Edinburgh in longitude 3° 12′ W., and Brest in longitude 4° 29′ W.

Edinburgh, longitude of 3° 12′ W.
Brest, longitude of 4° 29′ W.
} Subtract, for they are both of the same name.

Diff. of longitude = 1° 17′
60
= 77 nautical miles.

Example II.

Required the difference of longitude between Lizard Light in longitude 5° 12′ W., and Bencoolen in longitude 102° 28′ E.

Lizard Light, longitude of	5° 12′ W.	}	Add, for they are of contrary names.
Bencoolen, longitude of	102° 28′ E.		
Diff. of longitude	= 107° 40′		
	60		
	= 6460 nautical miles.		

Example III.

Required the difference of longitude between Porto Bello in longitude 79° 43′ W., and Canton in longitude 113° 14′ E.

Porto Bello, longitude of		79° 43′ W.
Canton, longitude of		113° 14′ E.
Sum	=	192° 57′
Subtract from		360
Diff. longitude	=	167° 3′

Exercises 11.

Find the difference of longitude between the places A and B in the following exercises:

(131.) A in longitude 17° 13′ E., and B in longitude 23° 59′ E.
(132.) A " 28° 12′ W. " B " 16° 12′ W.
(133.) A " 103° 14′ W. " B " 3° 56′ W.
(134.) A " 93° 18′ E. " B " 5° 17′ W.
(135.) A " 104° 0′ E. " B " 10° 19′ W.
(136.) A " 112° 7′ W. " B " 93° 8′ E.
(137.) A " 54° 17′ E. " B " 151° 17′ W.
(138.) A " 34° 57′ E. " B " 145° 3′ W.

Problem III.

157. Given the latitude and longitude of a place sailed from, and the difference of latitude and longitude made, to find the latitude and longitude of the place arrived at.

RULE.

If the difference of latitude or longitude made be of the same name as that sailed from, add the difference, and the sum will be that arrived at; but if they be of contrary names, take their difference, and affix the sign of the greater.

158. *Remark.*—The latitude can in no case exceed 90°, the distance between the equator and poles; and should the sum of the longitudes so obtained exceed 180°, subtract it from 360°, and change the name of the remainder. Longitude is measured westward and eastward from the first meridian up to 180° in each direction; it would have been more convenient had it been measured only westward up to 360°; but as the former method has been universally adopted, except by Sir John Herschel in his *Outlines of Astronomy*, where he recommends it to be always measured westward up to 360°, it is here retained.

Example I.

If a ship sail from Cape Finisterre in latitude 42° 54′ N., longitude 9° 16′ W., towards the south-west till her difference of latitude is 160 miles, and her difference of longitude 136 miles; what is her latitude and longitude in?

Cape Finisterre, lat.		42° 54′ N.	Long.		9° 16′ W.
Diff. lat. 160	=	2° 40′ S.	Diff. long. 136	=	2° 16′ W.
Lat. in	=	40° 14′ N.	Long. in	=	11° 32′ W.

Example II.

If a ship sail from latitude 50° 28′ S., longitude 178° 20′ E. towards the south-east till her difference of latitude is 520, and her difference of longitude 385 miles, what is her latitude and longitude in?

Lat. left		50° 28′ S.	Long. left		178° 20′ E.
Diff. lat. 520	=	8° 40′ S.	Diff. long. 385	=	6° 25′ E.
Lat. in		59° 8′ S.			184° 45′ E.
					360
			Long. in		175° 15′ W.

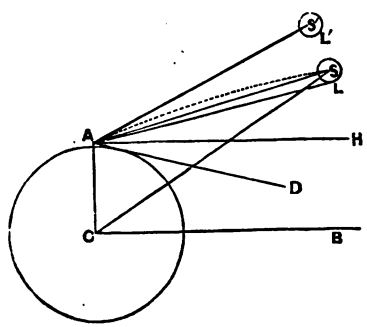
Exercises 12.

In the following exercises the latitude and longitude arrived at are required :

	Lat. left.	Long. left.	Diff. Lat. Miles.	Diff. Long. Miles.
(139.)	47° 36′ N.	9° 17′ W.	218 S.	305 W.
(140.)	35° 16′ S.	49° 18′ E.	136 S.	159 E.
(141.)	27° 54′ S.	85° 14′ E.	186 N.	267 W.
(142.)	65° 14′ N.	12° 34′ W.	212 S.	311 W.
(143.)	29° 38′ N.	27° 36′ W.	156 N.	256 E.
(144.)	20° 19′ S.	34° 21′ E.	259 N.	290 W.
(145.)	5° 18′ N.	71° 5′ W.	318 S.	530 E.
(146.)	41° 16′ S.	3° 24′ E.	176 N.	204 W.
(147.)	39° 19′ N.	15° 7′ W.	250 N.	185 E.
(148.)	56° 10′ S.	178° 40′ E.	260 N.	305 E.

NOTE.—A clear idea may be obtained of parallax (which is the angle subtended by the earth's radius at the centre of the sun or moon), refraction, semidiameter, dip, true and observed altitudes, by examining the following diagram, in which S represents the true place of the sun, and S′ the apparent

∠ ASC = Parallax.
∠ SAS′ = Refraction.
∠ SAL = Semidiameter.
∠ HAD = Dip.
∠ SCB = True altitude.
∠ L′AD = Observed altitude.



place when elevated by atmospheric refraction; AH, a parallel to CB, the rational horizon; and AD the sensible horizon as seen by an eye elevated above the earth's surface.

CHAPTER IV.

INSTRUMENTS USED IN NAVIGATION.

THE LOG-SHIP AND GLASSES.

159. The necessary instruments of Navigation are the LOG, which, with the help of the sand-glasses for measuring small intervals of time, or a watch shewing seconds, gives the velocity or rate of the ship; and the COMPASS, which gives the direction of the ship's track.

Sand-glass.

160. The LOG consists of the *log-ship* and *line*. The log-ship is a thin wooden sector, of about five inches radius, the circular edge being loaded with lead to make it float upright, and present its broad face to the water, so as not to slip through it. The

The Log.

inner end of the *log-line* is fastened to a reel which is kept board, while the outer end is fixed to the log-ship. At

ten or twelve fathoms from the log-ship, on the log-line a bit of bunting rag is placed, to mark off a sufficient quantity of line, called *stray-line*, to let the log-ship be clear of the ship's eddy before the time is counted.

Reel.

161. The log-line is divided into equal portions, called *knots*, at each of which a bit of string, with the number of knots upon it, is put through the strands. The length of a knot depends on the number of seconds that the glass runs, and may be thus determined—The feet in a knot : the feet in a mile = the seconds the glass runs : the seconds in an hour. Or,

$$\frac{\text{the feet in a knot}}{\text{the feet in a mile}} = \frac{\text{the seconds the glass runs}}{\text{the seconds in an hour (3600)}}.$$

The nautical mile is about 6083 feet, from which we obtain, by the proportion above, for the 30 seconds' glass—

$$\text{Length of a knot} = \frac{6083 \times 30}{3600} = \frac{6083}{120} = 50{\cdot}7 \text{ nearly.}$$

And for a 28 seconds' glass—

$$\text{Length of a knot} = \frac{6083 \times 28}{3600} = \frac{42581}{900} = 47{\cdot}3 \text{ feet.}$$

In the same manner may the length of a knot be found for a glass that runs any other number of seconds.

162. It is plain from the above that the ship sails as many miles per hour as there are knots run off the line during the seconds that the glass runs. Both the length of a knot and the seconds that the glass runs are liable to become faulty; and it is necessary to know how to correct the distance that has been measured by a faulty line or a faulty glass, or when both the line and glass are faulty.

In order to make this correction,

Let k = the true length of a knot,
k' = the measured length,
s = the seconds the glass should run,
s' = the seconds it does run,
d = the true distance the ship has sailed,
and d' = the apparent distance found by the faulty glass and line.

Then $\frac{k}{k'} = \frac{d''}{d'}$, or $d = \frac{k'd''}{k}$, if the line only were faulty (A);

and $\frac{s'}{s} = \frac{d''}{d'}$, or $d = \frac{sd''}{s'}$, if the glass only were faulty (B).

But if both the *line* and glass be faulty; then

$k : k' = d' : x =$ the correction for the line only.

$s' : s = x : d =$ the correction for both.

Therefore $ks' : k's = d'' : d$.

Hence $d = \frac{d''k's}{ks'} =$ the true distance when both the glass and line are faulty (C).

Hence, if the line only is faulty, use (A); if the glass only is faulty, use (B); if both glass and line are faulty, use (C).

163. Since we have seen above (Art. 161) that when the glass runs 30 seconds, the true length of a knot may be taken at 50 feet; that is, when

$$s = 30,$$

then $$k = 50;$$

and substituting these values in (C), we get

$$d = \frac{3d''k'}{5s'};$$ wherefore

Problem IV.

164. To correct the distance given by the log and half-minute glass, when the line, the glass, or both are faulty.

RULE.

Multiply three times the given distance by the measured length of a knot, and divide the product by five times the seconds which the glass takes in running out, and the quotient will in any case be the true distance.

Example.

A ship sails 143 miles, but it is found on examination that

the length of the knot on the log-line is $51\frac{1}{2}$ feet, and that the glass runs out in 28 seconds; what is the true distance?

$143 \times 3 = 429$ = three times the given distance.

$$\begin{array}{r} 51\frac{1}{2} \\ \hline 214\frac{1}{2} \\ 429 \\ 2145 \\ \hline 22093\frac{1}{2} \end{array}$$

divide by $28 \times 5 = 140$ $\quad \frac{22093\frac{1}{2}}{140} = \frac{1104{\cdot}675}{7} = 157{\cdot}81$, the true distance.

Exercises 13.

In the following exercises the true distance is required:

	Dist. by Log. Miles.	Length of Knot. Feet.	Seconds by Glass.
(149.)	93	49	31
(150.)	150	52	29
(151.)	212	51	28
(152.)	308	52	32
(153.)	258	48	31
(154.)	290	47	30
(155.)	318	50	33
(156.)	219	48	31
(157.)	305	49	27
(158.)	110	52	28
(159.)	217	48	32

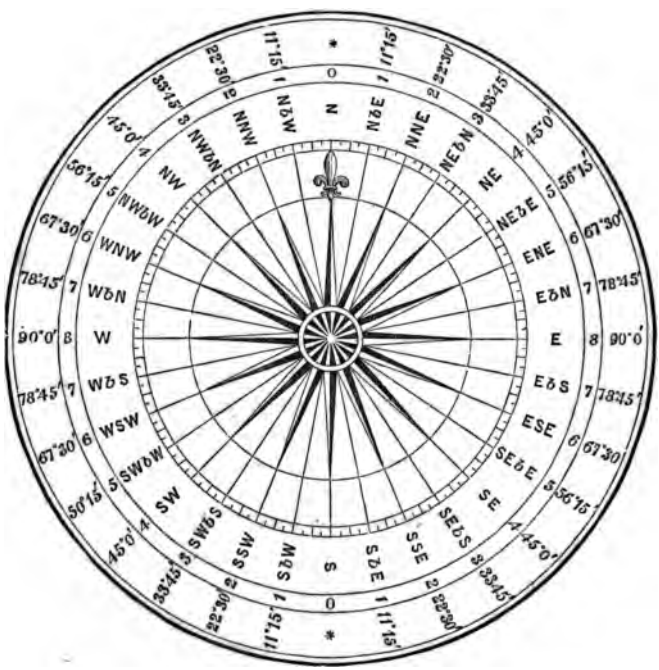
MASSEY'S PATENT LOG,

165. Called after the patentee, Edward Massey, is an instrument for registering the distance a ship has actually gone through the water. It consists of a rotator or fly, which communicates motion to wheels, which register on three dials the distance sailed since it was immersed. It is drawn behind the ship by means of a line varying from 20 to 50 fathoms, according to the size of the vessel, its use being to let the log be so

far astern that its action will not be affected by eddy or the ship's wake. By means of this instrument, 100 miles may be registered without once taking in the log. Every time, however, the course is changed the log must be taken in, and a fresh commencement of the register made.

OF THE COMPASS.

166. The Compass is an instrument which utilises the directive power of the magnet; it consists of a circular card attached to



a magnetised bar of steel, which by its peculiar property points always in a definite direction at any given place; the circum-

ference of this card is divided into 32 points, and these again into half and quarter points, and also sometimes into 360 degrees. The four principal points, or, as they are called, the *cardinal* points, are the North (represented by N.), South (S.), East (E.), and West (W.); the East being to the right, and the West to the left, when facing the North.

All the points of the compass are called by names composed of these four terms.

Thus, the points half-way between the cardinal points are named by the two adjacent cardinal points; hence, the point midway between the north and east is called *north-east* (and represented by NE.); so, midway between the south and east is called *south-east* (written SE.); in like manner, we get *south-west* (written SW.), and *north-west* (written NW.).

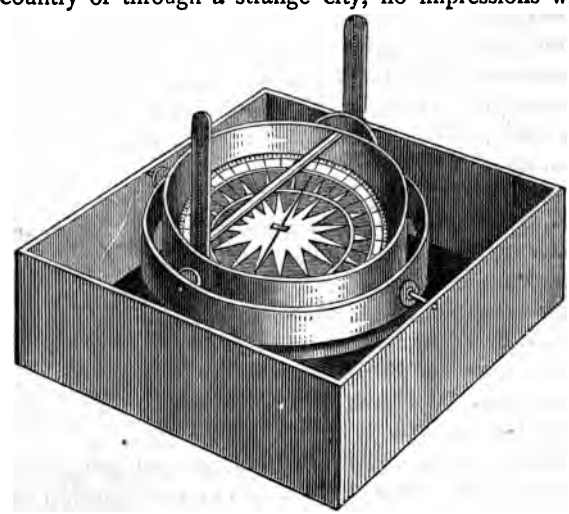
A point half-way between one of these last and a cardinal point is called, in like manner, by a name composed of the nearest cardinal point and the adjacent point NE., SE., SW., and NW.; thus the point midway between the N. and NE. is called *north-north-east* (written NNE.), while that midway between the E. and NE. is called *east-north-east* (written ENE.). In like manner we get the six following: ESE., SSE., SSW., WSW., WNW., and NNW.

The points next the eight principal points—namely, N., NE., E., SE., S., SW., W., NW.—are named by placing *by* between the letter representing the point to which it is adjacent and the next cardinal point in the same direction. Thus the point next to the *north* on the *east* side of it is called *north by east*, and written N. by E., whilst that adjacent to the *north* but towards the *west* is written N. by W.; in this manner we get other sixteen points. We have thus got names to all the thirty-two points of the compass.

A *half-point* is the middle division between two points, and can be named by either of the points between which it lies and the first cardinal point reached after passing it, placed after the ½; thus, the half-point between NE. and NE. by N. can be named either NE. ½ N. or NE. by N. ½ E., so also the half-point

between W. by N. and WNW. may either be written WNW. $\frac{1}{2}$ W. or W. by N. $\frac{1}{2}$ N. The half-points adjacent to the cardinal points, and also those adjacent to NE., NW., SE., and SW., are always named by these letters with the $\frac{1}{2}$ and the cardinal point next after on the same side as the half; thus N. $\frac{1}{2}$ E., N. $\frac{1}{2}$ W., SE. $\frac{1}{2}$ S., SW. $\frac{1}{2}$ W., &c. The same holds for a *quarter*-point, and for *three-quarters* of a point; thus the point of the compass which is $\frac{1}{4}$ of a point nearer to the N. than NE. by N. is also $\frac{3}{4}$ of a point nearer to the E. than NNE., it may therefore be either called NE. by N. $\frac{1}{4}$ N., or NNE. $\frac{3}{4}$ E.; the latter is, for several reasons, the best.

167. Repeating the points of the compass in their proper order and in any direction is called *boxing the compass*, and is one of the first things that a seaman learns; but their utility is not confined exclusively to navigation, for in walking across a new country or through a strange city, no impressions will be



The Azimuth Compass.

more distinct and permanent than those grounded on the points of the compass.

168. The AZIMUTH COMPASS is a compass of superior construction, particularly adapted to observe bearings. It is mounted on a tripod stand, and is fitted with vertical *sight vanes*, for the purpose of observing objects elevated above the horizon; one of these vanes has a long and narrow slit, and in the other there is an opening of the same kind, but wider, with a wire up and down the middle of it, exactly opposite the slit.

VARIATION OF THE MAGNETIC NEEDLE.

169. The needle points to the magnetic north, which in most places does not agree with the true north: the difference between the bearing of the magnetic and true north is called the *variation of the compass.*

The variation is named easterly when the magnetic north is to the east of the true north, and westerly when it is to the west of the true north. The variation is different at different places, and is constantly changing, though slowly, and in a long course of years it changes, at the same place, from easterly to westerly, and conversely.

170. To find how to correct the compass course or bearing, suppose one compass card to be placed over another, the under one being the true one; then, if the compass have two points of easterly variation, the north of the upper compass will be on the NNE. point of the true, and in order to bring it correct it will have to be turned through two points to the *left*, supposing we look from the centre of the compass to the circumference. Again, if the variation of the compass were $2\frac{1}{2}$ points westerly, the north of the upper card would be over the NNW. $\frac{1}{2}$ W. point of the true, and therefore to make it coincide with the true compass, it would be necessary to turn the upper card through $2\frac{1}{2}$ points to the right.

Problem V.

171. When the compass course and the variation of the compass are given, to find the true course.

RULE.

When the variation is *easterly*, apply it to the *right* of the compass course or bearing; when *westerly*, apply it to the *left* of the compass course or bearing, to get the true course or bearing.

Example I.

Given the compass course NW. $\frac{3}{4}$ W., and the variation of the compass $2\frac{3}{4}$ easterly, to find the true course.

Here the compass course is N. $4\frac{3}{4}$ W., and the variation being easterly must be applied to the *right*, thereby bringing it $2\frac{3}{4}$ nearer to the north, that is, within two points of the north; the true course is therefore NNW.

Example II.

Given the compass course WSW. $\frac{1}{2}$ S. and the variation $1\frac{1}{2}$ points westerly, to find the true course.

Here the compass course is S. $5\frac{1}{2}$ W., and the variation is westerly, and hence must be applied to the *left*, thereby bringing it $1\frac{1}{2}$ points nearer to the south, that is, within 4 points of the south; the true course is therefore SW.

Exercises 14.

In the following exercises the true courses are required:

(160.)	Compass course	SW. by W.,	variation	$2\frac{3}{4}$ pts.	E.
(161.)	" "	SSE. $\frac{1}{2}$ E.	"	$3\frac{1}{4}$ pts.	W.
(162.)	" "	NW. $\frac{3}{4}$ W.	"	$1\frac{1}{4}$ pts.	W.
(163.)	" "	NE. by E.	"	$3\frac{3}{4}$ pts.	E.

(164.)	Compass course	NNE. $\frac{1}{2}$ E.,	variation	$2\frac{1}{4}$ pts.	W.
(165.)	"	" SE. by S. $\frac{1}{2}$ S.	"	$1\frac{3}{4}$ pts.	E.
(166.)	"	" S. by W. $\frac{3}{4}$ W.	"	$2\frac{1}{4}$ pts.	E.
(167.)	"	" W. by S. $\frac{1}{2}$ S.	"	3 pts.	W.
(168.)	"	" S. $\frac{3}{4}$ W.	"	2 pts.	E.
(169.)	"	" N. 36° W.	"	15°	W.

Problem VI.

172. Given the true course and the variation of the compass, to find the compass course.

RULE.

If the variation be *westerly*, apply it to the *right;* but if *easterly*, apply it to the *left* of the true course, and the point thus determined will be the compass course.

Example I.

Given the true course E. by S. $\frac{1}{2}$ S. and the variation $1\frac{3}{4}$ points W., to find the compass course.

Here the true course is S. $6\frac{1}{2}$ E., and the variation being westerly, is applied to the right, which gives S. $(6\frac{1}{2} - 1\frac{3}{4})$ E. = S. $4\frac{3}{4}$ E. or SE. $\frac{3}{4}$ E., which is the compass course required.

Example II.

Given the true course WSW. $\frac{1}{2}$ W., and the variation $2\frac{1}{4}$ easterly, to find the compass course.

Here the true course is WSW. $\frac{1}{2}$ W. = S. $6\frac{1}{2}$ W., and the variation being easterly is applied to the left, and therefore subtractive, wherefore the compass course is S. $(6\frac{1}{2} - 2\frac{1}{4})$ W. = S. $4\frac{1}{4}$ W. = SW. $\frac{1}{4}$ W. = the compass course required.

Exercises 15.

In the following exercises the compass course is required:

(170.)	The true course is	SE. $\frac{3}{4}$ E.,	variation	$1\frac{1}{4}$ pts.	E.
(171.)	" "	WSW.	"	$2\frac{3}{4}$ pts.	E.
(172.)	" "	SSE. $\frac{1}{2}$ S.	"	$3\frac{1}{4}$ pts.	W.
(173.)	" "	N. by W.	"	$2\frac{1}{4}$ pts.	W.
(174.)	" "	NW. by W.	"	$1\frac{1}{2}$ pts.	E.
(175.)	" "	ESE. $\frac{3}{4}$ E.	"	$3\frac{1}{2}$ pts.	W.
(176.)	" "	NW. by W.	"	2 pts.	W.
(177.)	" "	S. by W. $\frac{1}{2}$ W.	"	3 pts.	E.
(178.)	" "	S. 74° E.	"	17°	W.
(179.)	" "	N. 29° W.	"	24°	W.

ON LEEWAY AND DEVIATION OF THE COMPASS.

173. Besides the correction of the course of a ship for *variation* of the compass, there are two other corrections often necessary to be attended to; these are *Leeway* and *Deviation* of the compass. The angle which the ship's track, indicated by her wake, makes with her fore-and-aft line or keel, is called LEEWAY.

When a ship is sailing with a side wind, she will not only be pressed forward by the action of the wind on the sails properly set, but she will also be pressed sideways before the wind, thus making leeway, which in this case must be allowed for in estimating her true course.

174. When a ship is sailing so that the wind blows from the *left* side of a person looking in the direction in which she sails, she is said to be on the PORT tack; and when the wind blows from the right of a person so situated, she is said to be on the

STARBOARD tack. Hence it is evident that when a ship is sailing on the *port* tack, the leeway must be allowed to the *right;* and when she is on the *starboard* tack, it must be allowed to the *left.*

The amount of leeway should be estimated by observing the angle which the ship's wake makes with her apparent direction.

175. The DEVIATION of the compass is a term applied to indicate the effect produced on the compass by *local* causes, such as the ship being built of iron, laden with iron, or having a large mass of iron in her vicinity. If the ship be built of iron, a table of the deviations on each course should be made out, for the deviation will be different on different courses, and the amount for each course may be determined by turning the ship's head gradually round, and observing the bearing of her head from some fixed distant object, by compass, and also by means of a sextant, the difference of these angles will give the deviation of the compass at each position. The true bearing of the distant object must be determined by some means which is not affected by the local attraction.

Problem VII.

176. To find the true course made good, when there are given the compass course, the variation of the compass, the leeway, and the deviation.

RULES.

1. If the *variation* and *deviation* be both east or both west, add them together, and their sum is a correction to be applied in the same direction; but if the one is east and the other west, subtract the less from the greater, and their difference is then to be applied in the same direction as the greater; that is, if the result is *easterly*, apply the correction to the *right* of the course; but if *westerly*, apply it to the *left* of the course.

2. Apply the leeway to the *right* of the resulting course when the ship is on the *port tack*, but to the left when the ship is on the starboard tack.

Example I.

A ship sails SW. by W., the variation of the compass being $2\frac{1}{4}$ points E., and the leeway $3\frac{1}{4}$ points, the ship being on the port tack, also the deviation being $1\frac{1}{2}$ points E.; required her true course.

Solution.

Here the ship's course being SW. by W. is		S. 5 pts. W.
The variation and deviation being both east, are to be added and applied to the *right*; hence $2\frac{1}{4} + 1\frac{1}{2}$	=	$+ 3\frac{3}{4}$
	Sum	S. $8\frac{3}{4}$ W.
This being greater than 8 pts. we take it from 16 pts. and get		N. $7\frac{1}{4}$ W.
Lastly, the leeway being made on the *port tack* is applied to the *right*		$- 3\frac{1}{4}$
	Difference	N. 4 W. = N.W. the true course.

Example II.

A ship sailing ENE. on the starboard tack makes $2\frac{1}{2}$ points leeway, the variation of the compass being $1\frac{1}{4}$ W., and the deviation $2\frac{3}{4}$ E.; required her true course.

Solution.

Here the course by compass is ENE. or		N. 6 E.
The variation and deviation are of contrary names, and their difference is $1\frac{1}{4}$ E., and	$\therefore$	$+ 1\frac{1}{4}$
Therefore the course corrected for variation and deviation is	=	N. $7\frac{1}{4}$ E.
The ship being on the starboard tack the leeway is applied to the left, and hence is		$- 2\frac{1}{2}$
	Difference	N. 5 E. = NE. by E. the true course.

Example III.

A ship sailing S. 54° E. on the starboard tack makes 25° of leeway, the variation of the compass being 15° W., and the deviation 7° E.; required her true course.

Solution.

Here the ship's apparent course is		S. 54° E.
The variation of the compass being 15° W., and the deviation 7° E., their difference 8° W.	=	+ 8°
The leeway being made on the starboard tack is applied to the left, and is therefore	=	+ 25°
The true course	=	S. 87° E. Sum.

177. *Remark.*—From the above rules and examples, it will be seen that a ship's course is always measured from the *north* or *south* towards *east* and *west*, and that *easterly* variation and deviation, as also leeway on the port tack, being measured to the *right* of the course, increase it in the NE. and SW. quarters, but diminish it in the SE. and NW. quarters. Whilst *westerly* variation and deviation, as also leeway made on the starboard tack, being applied to the *left* of the course, diminish it in the NE. and SW. quarters, but increase it in the SE. and NW. quarters. By attending to the Rules and the above remark, the necessary corrections of the compass course in the following exercises will easily be made, and the true course thus found.

Exercises 16.

(180.) A ship sails WSW. $\frac{3}{4}$ W. on the starboard tack, and makes $2\frac{1}{4}$ points of leeway, the variation of the compass being 2 points W., and the deviation $1\frac{1}{4}$ points W.; required the true course.

(181.) A ship sails NE. by E. $\frac{1}{4}$ E. on the port tack, and makes $1\frac{1}{2}$ points of leeway, the variation of the compass being $2\frac{1}{2}$ points E., and the deviation $1\frac{1}{4}$ E.; required the true course.

(182.) A ship sailing SW. by W. on the port tack, makes 2 points of leeway, the variation of the compass being 3 points E., and the deviation $1\frac{1}{2}$ points W.; required her true course.

(183.) A ship sailing SSE. ½ E. on the starboard tack makes 3½ points of leeway, the variation of the compass being 2¼ points E., and the deviation 1¼ points W.; required her true course.

(184.) A ship sailing N. on the starboard tack makes 1¾ points of leeway, the variation of the compass being 2¼ points W., the deviation nothing; required the true course.

(185.) A ship sails E. ¾ S. on the port tack, and makes 3¼ points of leeway, the variation of the compass being 1½ points W., and the deviation 1½ points E.; required her true course.

(186.) A ship sails NE. by E. on the port tack, and makes 2¾ points of leeway, the variation of the compass being 2 points W., and the deviation 3½ points E.; required her true course.

(187.) A ship sailing E. ¾ S. on the starboard tack, makes 1¼ points of leeway, the variation of the compass being 3 points E., and the deviation 1½ points E.; required her true course.

(188.) A ship sailing S. on the starboard tack makes 2¾ points of leeway, the variation of the compass being 1½ points E., and the deviation 1¼ points E.; required her true course.

(189.) A ship sails N. by W. on the port tack, and makes 1 point of leeway, the variation of the compass being 2¼ W., and the deviation 1¼ points W.; required her true course.

(190.) A ship sails W. by S. ½ S. on the starboard tack, and makes 2¾ points of leeway, the variation of the compass being ½ point E., and the deviation 2½ points W.; required the true course.

(191.) A ship sailing E. by N. ½ N. on the port tack makes 3½ points of leeway, the variation of the compass being 1½ points W., and the deviation 2½ points E.; required her true course.

(192.) A ship sails N. 36° E. on the port tack, and makes 15° leeway, the variation of the compass being 18° W., and the deviation 7° E.; required her true course.

(193.) A ship sailing S. 53° W. on the starboard tack makes

23° of leeway, the variation of the compass being 12° E., and the deviation 7° E.; required her true course.

(194.) A ship sailing N. 73° W. on the starboard tack makes 26° of leeway, the variation of the compass being 10° E., and the deviation 12° W.; required her true course.

(195.) A ship sails S. 48° W. on the port tack, and makes 28° of leeway, the variation of the compass being 17° E., and the deviation 9° E.; required her true course.

CHAPTER V.

ON PLANE SAILING.

178. In Plane Sailing, the surface of the earth is considered as a plane, the meridians as parallel straight lines, and the parallels of latitude as straight lines cutting the meridians at right angles. It is not, however, strictly correct to consider any part of the earth's surface as a plane, yet when the operations to be performed are confined within the distance of a few miles, the results thus obtained will be sufficiently correct to serve many useful purposes.

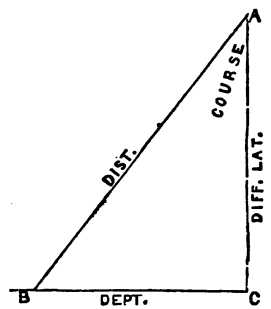
179. Plane Sailing requires a knowledge to some extent of Plane Trigonometry, and proposes the solution of the following problems: 1. Given the latitude and longitude of two places, required the course and distance from one to the other; and 2. Given the course and distance sailed, and the latitude and longitude of the place sailed from, to find the latitude and longitude of the place arrived at.

180. A RHUMB LINE is a curve line on the earth's surface, which cuts all the meridians which it crosses at the same angle; this is therefore the track described by a ship while she continues to sail on the same course, and the length of this rhumb line is called the DISTANCE SAILED.

181. DEPARTURE is the distance due east or due west which a ship has made good from the meridian from which she first started.

182. A ship's COURSE is the angle which her direction makes with a meridian line, and is measured from the *north* or *south* towards the *east* or *west*, either in points of the compass or in degrees.

183. In the figure in the margin, let A represent the place from which a ship sails, AC a portion of a meridian passing through the place, AB the line on which the ship sails, and B the point at which she has arrived; then AB represents the distance sailed, AC the difference of latitude she has made, BC the departure made from the first meridian AC, and the angle BAC is called the course.



184. In constructing a figure to represent the conditions of any problem in plane sailing, it is customary to make the top of the figure, as A, the north, and the bottom, as BC, the south, the line AC being north and south, the right-hand side of AC being the east, and the left-hand side being the west. The figure above therefore represents a ship sailing from the north point A along the line AB towards the SW., making a difference of latitude to the south represented by AC, and a departure to the west represented by BC, and a course to the west of the south represented by the angle BAC contained between the meridian AC and the line AB, along which the ship sails.

185. Applying now the principles laid down in Arts. (122–130), and observing that the *departure* is there represented by *a*, the *difference of latitude* by *b*, and the *distance* by *c*, as in these articles, when any two of the four parts, *dist.*, *course*, *dif. lat.*, and *dept.* are given, we will be enabled to find the other two by Plane Trigonometry.

186. Using now the Navigation names instead of the letters

a, *b*, *c*, we have the Arts. (122–127) changed into the following:

$$(122)\quad \frac{\text{dept.}}{\text{dist.}} = \text{sin. course};$$

multiply by dist.; $\therefore$ dept. = dist. × sin. course.

$$(123)\quad \frac{\text{dif. lat.}}{\text{dist.}} = \text{cos. course};$$

multiply by dist.; $\therefore$ dif. lat. = dist. × cos. course.

$$(124)\quad \frac{\text{dept.}}{\text{dif. lat.}} = \text{tan. course};$$

multiply by dif. lat.; $\therefore$ dept. = dif. lat. × tan. course.

$$(125)\quad \frac{\text{dif. lat.}}{\text{dept.}} = \text{cot. course};$$

multiply by dept.; $\therefore$ dif. lat. = dept. × cot. course.

$$(126)\quad \frac{\text{dist.}}{\text{dif. lat.}} = \text{sec. course};$$

multiply by dif. lat.; $\therefore$ dist. = dif. lat. × sec. course.

$$(127)\quad \frac{\text{dist.}}{\text{dept.}} = \text{cosec. course};$$

multiply by dept.; $\therefore$ dist. = dept. × cosec. course.

Problem VIII.

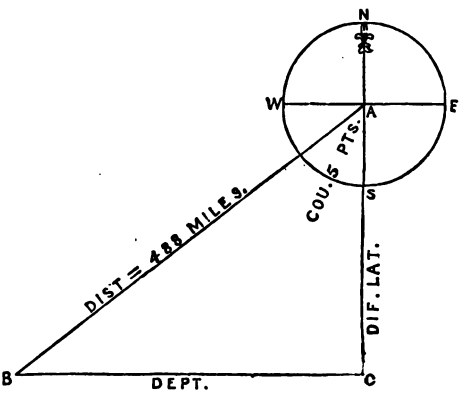
187. Given the course and distance sailed, to find the difference of latitude and the departure from the meridian.

Example.

A ship from latitude 49° 57′ N. sails SW. by W. 488 miles; required the latitude she is in, and the departure she has made from the meridian sailed from.

BY CONSTRUCTION.

Draw the line AC to represent the meridian of the place A sailed from. With the chord of 60° in the compasses, and one foot in A as a centre, describe the compass NWSE. Take five points in the compasses from the line of rhumbs on the plane scale, and set it off on the arc from S towards W. for the course; through this point B and A draw the line AB, which make equal to the distance 488; draw BC parallel to E and W points to cut the meridian in C. Then will AC be the difference of latitude 271·1 miles, and BC the departure = 405·8 miles.



BY LOGARITHMS.

To find the Departure.	To find the Dif. Latitude.
Art. (122) $\frac{\text{dept.}}{\text{dist.}} = \text{sin. course.}$	Art. (123) $\frac{\text{dif. lat.}}{\text{dist.}} = \text{cos. course.}$
∴ Dept. = dist. × sin. course.	∴ Dif. lat. = dist. × cos. course.
Log. dept. = log. dist. + log. sin. course − 10.	Log. dif. lat. = log. dist. + log. cos. course − 10.
Log. 488 = 2·6884198	Log. 488 = 2·6884198
Log. sin. 5 pts. = 9·9198464	Log. cos. 5 pts. = 9·7447390
Log. 405·75 = 2·6082662	Log. 271·11 = 2·4331588
∴ The dept. = 405·75 miles.	∴ The dif. lat. = 271·11.

Now as the ship is in north latitude, and has sailed southward, her difference of latitude made is south; hence

From the latitude left	=	49° 57′ N.
Take the dif. lat. 271 miles	=	4° 31′ S.
And there remains lat. in	=	45° 26′ N.

BY GUNTER'S SCALE.

1. Extend the compasses from radius or 8 points on the line of rhumbs, marked SR, to 5 points; that extent will reach from the distance 488 to the departure 405·7 on the line of numbers.

2. Extend from radius or 8 points to 3 points, the complement of the course, on the line SR; that extent will reach from the distance 488 to the difference of latitude 271 on the line of numbers.

Thus may all the operations be performed in the several cases of Navigation.

188. By this case are calculated the tables of latitude and departure for every degree, point, and quarter-point of the mariner's compass, and for all distances from 1 to 300 miles, and these results being collected together form the Traverse Tables. By the inspection of these tables, a day's work may be calculated in a much more expeditious manner than by logarithms or by Gunter's scale. In consequence of this, the method by inspection is generally used at sea in preference to every other method.

BY INSPECTION.

189. RULE.—Find the given *course* at the top of the page when not greater than 4 points or 45°, but if greater than 4 points or 45° find it at the bottom of the page, and read the dist., lat., and dept. at the top when the course is found at the top, but at the bottom of the page when the course is found at the bottom. Having found the course, look for the distance, and adjacent to it will be found in their proper columns the difference of latitude and departure.

In this question, the distance given being greater than 300

miles, is not found in the tables; hence take out any two or more distances, which together make up the given distance, and the difference of latitude and departure corresponding to each, add them together, and their sum will be the difference of latitude and departure sought; thus 488 = 300 + 188 = 248 + 240.

Or take any part as the half or third part of the distance, and find the difference of latitude and departure corresponding to it, then multiply the results by the divisor; thus 244 of distance corresponds to dif. lat. 135·6, and dept. 202·9, which, being doubled, gives dif. lat. 271·2, and dept. = 405·8.

Dist.		Dif. Lat.	Dept.	Dist.		Dif. Lat.	Dept.
300	=	166·7	249·4	240	=	133·3	199·6
188	=	104·4	156·3	248	=	137·8	206·2
∴ 488	=	271·1	405·7	∴ 488	=	271·1	405·8

Therefore for a course of 5 points and a distance of 488 miles, we find a difference of latitude of 271·1 and a departure 405·7 or 405·8 miles; the calculation gave 405·75.

Exercises 17.

(196.) A ship from latitude 50° 8′ N. sails SSE. 254 miles; required the difference of latitude and departure she has made, and the latitude in.

(197.) A ship from latitude 34° 17′ S. sails SW. by S. 290 miles; required the difference of latitude and departure made, and the latitude now in.

(198.) A ship from latitude 29° 18′ N. sails SSW. $\frac{3}{4}$ W. 379 miles; required the difference of latitude and departure made, and the latitude in.

(199.) A ship from latitude 1° 35′ S. sails N. by W. $\frac{3}{4}$ W. 273 miles; required the difference of latitude and departure made, and the latitude in.

(200.) A ship from latitude 46° 49′ N. sails S. 39° W. 269 miles; required the difference of latitude and departure made, and the latitude in.

(201.) A ship from latitude 55° 57′ S. sails W. by N. ¾ N. 236 miles; required the latitude she is in, and the departure from the meridian she has made.

(202.) A ship from 0° 7′ S. sails N. 43° W. 508 miles; required the latitude she is in, and the departure she has made from the meridian sailed from.

(203.) A ship from 2° 17′ N. sails S. 17° W. 496 miles; required the latitude she is in, and the departure she has made from the meridian sailed from.

Problem IX.

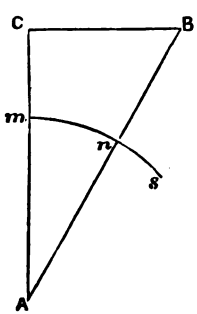
190. Given the course and difference of latitude made, and the latitude sailed from; to find the distance sailed, the departure made, and the latitude now in.

Example.

A ship from latitude 56° 2′ N. sails NE. by N. till her difference of latitude is 205 miles; find the distance she has sailed, the departure from the meridian made, and the latitude now in.

BY CONSTRUCTION.

Draw AC a vertical line, and make it = 205 miles, the difference of latitude; from the centre A, with a radius equal to the chord of 60°, describe the arc *ms*, and from it cut off *mn* with the chord of three points: through A and *n* draw A*n*B to meet CB at right angles to AC; then apply the distance AB to the same scale of equal parts from which AC was taken, and it will be found to measure 246·5 miles; in the same way apply CB, and it will be found to measure 137 miles, whence the distance sailed is 246·5 miles, and the departure from the meridian made is 137 miles.



BY LOGARITHMS.

Here apply (124) of Art. (186) to find BC, and (126) to find AB.

To find the Departure BC.

$\frac{\text{dept.}}{\text{dif. lat.}} = \text{tan. course.}$

$\therefore$ dept. = dif. lat. × tan. course.

Hence log. dept. = log. (dif. lat.) + log. tan. course − 10.

Log. 205	=	2·3117539
Log. 3 pts.	=	9·8248926
Log. 136·97	=	2·1366465

$\therefore$ The departure = 136·97 = 137 miles nearly.

To find the Distance AB.

By (126) $\frac{\text{dist.}}{\text{dif. lat.}} = \text{sec. course.}$

$\therefore$ dist. = dif. lat. × sec. course.

Hence log. dist. = log. dif. lat. + log. sec. course − 10.

Log. 205	=	2·3117539
Log. sec. 3 pts.	=	10·0801536
Log. 246·55	=	2·3919075

$\therefore$ The distance sailed = 246·55 = 246·5 nearly.

BY GUNTER'S SCALE.

On the line marked RS, extend the compasses from 5 points, the complement of 3 points, to 8 points, this distance will extend from 205 up to 246·5 the distance; again, extend the compasses from 5 points to 3 points on the line marked RS, and the same extent will reach from 205 on the line marked Num. back to 137.

BY INSPECTION.

Look for 3 points at the top of the first Traverse Table (page 344), and in the column marked at the top Dif. Lat. will be found 204·5 and 205·4; take the former, which is ·5 too little, then to it add the tenth part of that which corresponds to 5 in the dif. lat. column; thus,

Dist.	Dif. lat.	Dept.
246	204·5	136·7
·6	·5	·3
246·6	205	137

Hence to a dif. lat. 204·5 there corresponds a dist. 246·6 and a departure 137 miles.

Lastly, the latitude sailed from is	=	56° 2′ N.
To which add the dif. lat. 205 miles	=	3° 25′ N.
$\therefore$ the latitude in is	=	59° 27′ N.

Exercises 18.

(204.) A ship from latitude 54° 47′ N. sails SW. ½ W. till she arrives in latitude 51° 51′ N.; required the difference of latitude made, the distance sailed, and the departure from the meridian.

(205.) A ship from latitude 34° 18′ S. sails WSW. ½ W. till her difference of latitude is 151 miles; required the distance sailed, the departure made, and the latitude now in.

(206.) A ship from latitude 20° 53′ N. sails N. by W. ¾ W. till her difference of latitude is 222 miles; required the latitude arrived at, the distance sailed, and the departure made.

(207.) A ship sails SSW. ¾ W. from 2° 18′ N. till her difference of latitude is 207 miles; required the latitude arrived at, the distance sailed, and the departure from the meridian made.

(208.) A ship sails NE. ¾ E. from latitude 47° 18′ N. till her difference of latitude is 213 miles; required the latitude arrived at, the distance run, and the departure from the meridian.

(209.) A ship from latitude 38° 52′ N. sails NW. ½ N. till her difference of latitude is 236 miles; required the latitude arrived at, the distance sailed, and the departure from the meridian made.

(210.) A ship sails SW. by W. from latitude 59° 13′ N. till her difference of latitude made is 178 miles; required the distance run, the departure from the meridian made, and the latitude arrived at.

(211.) A ship from latitude 39° 11′ S. sails SSW. ½ W. till her difference of latitude made is 157 miles; required the latitude arrived at, the distance run, and the departure from the meridian made.

Problem X.

191. Given the course, the departure, and the latitude sailed

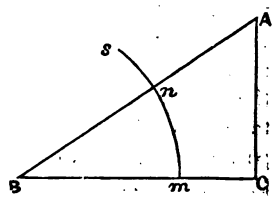
from; to find the distance sailed, the difference of latitude made, and the latitude arrived at.

Example.

A ship from latitude 51° 28′ N. sails SW. by W. ½ W. till her departure is 184 miles; required the distance sailed, the difference of latitude made, and the latitude arrived at.

BY CONSTRUCTION.

From a scale of equal parts lay down the horizontal line BC = 184; from B as a centre with the chord of 60° describe the arc *ms*, and from it, with the chord of 2½ points, the complement of the course BAC; cut off *mn*, and through B and *n* draw BA, and through C draw CA perpendicular to BC, meeting BA in A; then A is the point sailed from, BAC = the course, AB = the distance sailed, and AC = the difference of latitude made. Then AB being applied to the same scale of equal parts from which BC was measured will be found to be 208·6; and AC measured in the same manner will be found to measure 98·4 miles.



BY LOGARITHMS.

To find the Distance sailed.

By (127) $\frac{\text{dist.}}{\text{dept.}} = \text{cosec. course.}$

$\therefore$ dist. = dept. × cosec. course.

Log. dist. = log. dept. + log. cosec. course − 10.

Log. 184	=	2·2648178
Log. cosec. 5½ pts.	=	10·0545702
Log. 208·63	=	2·3193880

$\therefore$ the dist. = 208·6 nearly.

To find the Difference of Latitude.

By (125) $\frac{\text{dif. lat.}}{\text{dept.}} = \text{cot. course.}$

$\therefore$ dif. lat. = dept. × cot. course.

Log. (dif. lat.) = log. dept. + log. cot. course − 10.

Log. 184	=	2·2648178
Log. cot. 5½ pts.	=	9·7279567
Log. 98·35	=	1·9927745

$\therefore$ dif. lat. = 98·4 nearly.

BY GUNTER'S SCALE.

On the line marked SR extend the compasses from $5\frac{1}{2}$ points back to $2\frac{1}{2}$ points, this extent will reach from 184, the departure, back to 98·4 miles, the difference of latitude; again, on the same line extend the compasses from $5\frac{1}{2}$ points up to 8 points, this extent will reach from 184 miles, the departure, upwards to 208·6, the distance sought.

BY INSPECTION.

Enter the first Traverse Table (page 344), with the course $5\frac{1}{2}$ points at the bottom of the page, and in the column marked Dep. *at the bottom* will be found 183·4 and 184·3, the true reading is ·6 greater than the former, and ·3 less than the latter; we can therefore either take the former distance and difference of latitude, and *add* the correction for ·6 (which is the one-hundredth part of 60); or, we can take the latter and subtract from it the correction for ·3 (which is the one-hundredth part of 30); thus,

	Dist.	Dept.	Dif. lat.	or	Dist.	Dept.	Dif. lat.
	208	183·4	98		209	184·3	98·5
Add	·7	·6	·3	Subt.	·3	·3	·2
	208·7	184	98·3		208·7	184	98·3

To find the latitude arrived at,

From the latitude sailed from	=	51° 28′ N.
Subtract the dif. lat. = 98·3	=	1° 38′ S.
∴ The latitude in	=	49° 50′ N.

Exercises 19.

(212.) A ship from latitude 55° 58′ N. sails ENE. till her departure from the meridian is 253 miles; required the distance sailed, and the latitude arrived at.

(213.) A ship from latitude 34° 37′ S. sails NE. by E. till her

departure from the meridian is 413 miles; required the distance sailed, and the latitude arrived at.

(214.) A ship from latitude 17° 15′ N. sails ENE. ½ N. till her departure from the meridian is 217 miles; required the distance sailed, and the latitude arrived at.

(215.) A ship from latitude 4° 16′ S. sails N. by W. ¼ W. till her departure from the meridian is 194 miles; required the distance sailed, and the latitude arrived at.

(216.) A ship from latitude 59° 36′ N. sails S. by E. ¾ E. till her departure from the meridian is 111 miles; required the distance sailed, the difference of latitude made, and the latitude arrived at.

(217.) A ship from latitude 51° 24′ N. sails W. by S. ½ S. till her departure from the meridian is 218 miles; required the distance sailed, the difference of latitude made, and the latitude arrived at.

(218.) A ship from latitude 37° 39′ S. sails WSW. till her departure from the meridian is 219 miles; required the distance sailed, the difference of latitude made, and the latitude arrived at.

(219.) A ship from latitude 2° 7′ S. sails N. by E. till her departure from the meridian is 97 miles; required the distance sailed, the difference of latitude made, and the latitude arrived at

Problem XI.

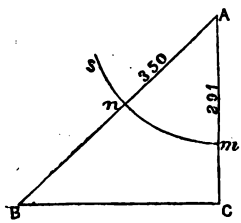
192. Given the distance sailed, and the difference of latitude made, to find the course and departure.

Example.

A ship from Cape St Vincent, in latitude 37° 3′ N., sails between the north and west 350 miles, until her difference of latitude is 291 miles; required her course steered, and departure made good.

BY CONSTRUCTION.

Draw AC to represent the meridian, and from it lay off 291 miles, the difference of latitude; from C draw CB to the left of AC and at right angles to it; then from the centre A with a radius = 350 miles, the distance sailed, cut BC in B, join AB; then from the centre A, with radius A*m* = the chord of 60°, describe the arc *ms*, and measure the intercepted part from *m* to *n*; this distance will be the chord of the course, 3 points. Lastly, apply the distance BC to the scale of equal parts, and the departure will be found to be 194·4 miles.



BY LOGARITHMS.

To find the Course.

(Art. 123) $\frac{\text{dif. lat.}}{\text{dist.}}$ = cos. course;

and therefore

Log. cos. course = log. (dif. lat.) + 10 − log. dist.

Log. (dif. lat. = 291) + 10	=	12·4638930
Log. (dist. = 350)	=	2·5440680
Log. cos. 33° 45′ 15″	=	9·9198250

∴ The course = 3 pts. = NW. by N.

To find the Departure made good.

By Art. (122) $\frac{\text{dept.}}{\text{dist.}}$ = sin. course;

Hence dept. = dist. × sin. course.

∴ Log. dept. = log. 350 + log. sin. 3 pts. − 10.

Log. 350	=	2·5440680
Log. sin. 3 pts.	=	9·7447390
Log. 194·45	=	2·2888070

∴ The departure = 194·4 miles to the west.

BY GUNTER'S SCALE.

Extend the compasses from 350 back to 291, that distance applied to the line marked SR will reach from 8 points back to 5 points, the complement of the course; hence the course is 3 points. Again, extend the compasses on the line SR from 8 points to 3 points, that extent applied to the line of numbers will reach from 350, the distance, back to 194·4, the departure.

BY INSPECTION.

Since 350 is greater than the distances given in the Traverse Tables, take its half, 175, and also the half of the difference of latitude, 145·5, with these enter the Traverse Table, and in the page of 3 points will be found opposite to 175 in the distance 145·5 dif. lat. and 97·2; these being doubled, give the distance 350, the difference of latitude 291, and the departure 194·4, also the course 3 points.

Lastly, to the latitude sailed from		37° 3′ N.
Add the difference of latitude 291	=	4° 51′ N.
∴ the latitude in	=	41° 54′ N.

Exercises 20.

(220.) A ship from latitude 46° 23′ N. sails between the south and west 280 miles till her difference of latitude is 205 miles; required the course steered, and the departure made good.

(221.) A ship from latitude 24° 5′ S. sails between the south and east 410 miles till her difference of latitude is 323 miles; required the course steered, and the departure from the meridian.

(222.) A ship from latitude 57° 13′ N. sails between the south and west 356 miles till her difference of latitude is 310 miles; required the course steered, and the departure made good.

(223.) A ship from latitude 34° 15′ S. sails between the south and east 218 miles till her difference of latitude is 130 miles; required the course steered, and the departure made good.

(224.) A ship from latitude 54° 16′ N. sails between the north and west 294 miles till her difference of latitude is 181 miles; required the course steered, and the departure from the meridian.

(225.) A ship from latitude 1° 1′ S. sails between the north and west 580 miles till her difference of latitude is 388 miles; required the latitude arrived at, the course steered, and the departure made good.

(226.) A ship from latitude 45° 17′ N. sails between north and east 284 miles till her difference of latitude made is 190 miles; required the latitude arrived at, the course steered, and the departure made good.

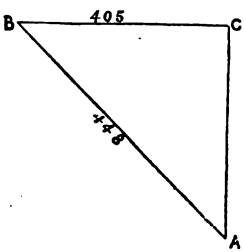
(227.) A ship from latitude 36° 31′ N. sails between the south and west for a distance of 446 miles, till her difference of latitude is 374 miles; required the latitude arrived at, the course steered, and the departure from the meridian.

Problem XII.

193. Given the distance sailed, and the departure from the meridian; to find the course steered, the difference of latitude made, and the latitude arrived at.

Example.

Suppose a ship sails 448 miles between the north and west from the latitude of 32° 25′ N. until her departure is 405 miles; what course has she steered, and what latitude is she in?



BY CONSTRUCTION.

Draw the line BC equal to the departure 405, and perpendicular to it draw CA to represent the meridian; then take the distance 448 in the compasses, and fixing one foot in B as a centre, describe an arc cutting CA in A, join AB, then the angle BAC will be the course, and the line AC will be the difference of latitude.

BY LOGARITHMS.

To find the Course.

By Art. (122) sin. A. $= \frac{BC}{AB} = \frac{\text{dept.}}{\text{dist.}}$

Hence log. sin. course = log. dept. + 10 − log. dist.

Log. 405 + 10	=	12·6074550
Log. 448	=	2·6512780
Log. sin. 64° 42′	=	9·9561770

∴ The course = N. 64° 42′ W. = WNW. ½ N. very nearly.

To find the Difference of Latitude.

By Art. (123) $\frac{AC}{AB} = \frac{\text{dif. lat.}}{\text{dist.}} = \text{cos. course.}$

∴ Dif. lat. = dist. × cos. course.

Hence log. (dif. lat.) = log. dist. + log. cos. course − 10.

Log. (dist. = 448)	=	2·6512780
Log. cos. 64° 42′	=	9·6307917
Log. dif. lat. 191·5	=	2·2820697

∴ The dif. lat. = 191·5 m. = 3° 12′ N. nearly.
Add the lat. sailed from 32° 25′ N.
∴ The latitude arrived at = 35° 37′ N.

BY GUNTER'S SCALE.

1. Extend from the distance, 448, to the departure, 405, on the line of numbers; that distance will reach from 8 points on the line marked SR to $5\frac{3}{4}$ points, the course.

2. Extend from 8 points to $2\frac{1}{4}$ points, the complement of the course; this extent will reach from 448, the distance, back to 191·5, the difference of latitude on the line of numbers.

BY INSPECTION.

Seek in the Traverse Table till against the distance, taken in its column, is found the given departure in one of the following columns; adjoining to it will be found the difference of latitude, which if greater than the departure, the course is to be found at the top; but if less, the course is to be found at the bottom of the page.

Since the distance is greater than any found in the table, take its half, 224, and also the half of the departure, 202·5; with these enter the table, and in that page which has $5\frac{3}{4}$ points at the bottom will be found the given distance and departure, with 95·8 in the dif. lat. column, the double of which, 191·6, is the difference of latitude sought.

Exercises 21.

(228.) If a ship sail 285 miles between the south and west from latitude 47° 51′ N. till her departure is 151 miles; what course has she steered, and what latitude is she in?

(229.) Suppose a ship sails 520 miles between the north and west from latitude 29° 18′ S. till her departure is 426 miles; what course has she steered, and what latitude is she in?

(230.) If a ship sail 356 miles between the south and east from latitude 25° 29′ S. till her departure is 302 miles; what course has she steered, and what latitude is she in?

(231.) If a ship sail 428 miles between the north and west from latitude 39° 19′ N. till her departure is 318 miles; what course has she steered, and what latitude is she in?

(232.) Suppose a ship sails 270 miles between the south and west from latitude 58° 38′ N. till her departure is 229 miles; what course has she steered, and what latitude is she in?

(233.) Suppose a ship sails 580 miles between the south and east from latitude 13° 11′ S. till her departure is 546 miles; what course has she steered, and what latitude is she in?

(234.) If a ship sail 418 miles between the north and east from latitude 37° 57′ N. till her departure is 400 miles; what course has she steered, and what latitude is she in?

(235.) If a ship sail 626 miles between the south and west from latitude 14° 10′ S. till she makes a departure from the meridian of 218 miles; what course has she steered, and what latitude is she in?

Problem XIII.

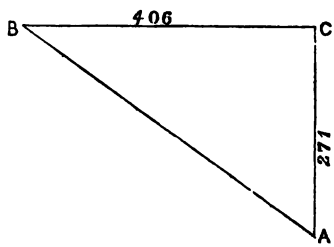
194. Given the difference of latitude, and the departure made good; to find the course, and the distance sailed.

Example.

A ship sails between the north and west till her difference of latitude is 271 miles and her departure is 406 miles; required her course and distance.

BY CONSTRUCTION.

Draw AC = 271, and perpendicular to it on the left; from C draw CB = 406; join A and B; then will the angle BAC be the course = 56° 17′; and AB the distance = 488 miles.



BY LOGARITHMS.

To find the Course.

By Art. (124) $\frac{\text{dept.}}{\text{dif. lat.}}$ = tan. course.

∴ Log. tan. course = log. dept. + 10 − log. dif. lat.

Log. 406 + 10	=	12·6085260
Log. 271	=	2·4329693
Log. tan. 56° 17′	=	10·1755567

Hence the course = 56° 17′ = NW. by W. nearly.

To find the Distance AB.

By Art. (126) $\frac{\text{AB}}{\text{AC}}$ = sec. course.

∴ AB = 271 × sec. 56° 17′

Hence log. AB = log. 271 + log. sec. 56° 17′ − 10.

Log. 271	=	2·4329693
Log. sec. 56° 17′	=	10·2556394
Log. 488·2	=	2·6886087

∴ The distance sailed = 488·2.

BY GUNTER'S SCALE.

1. Extend from 271, the difference of latitude, to 406, the departure, on the line of numbers; that extent will reach from radius or tangent of 45° to 56° 17′, the course on the line of tangents.

2. For the distance we must consider it as radius (unless

there is a line of secants on the scale), and extend from the course 56° 17′ to the radius or sine of 90° on the line of sines; that extent will reach from the departure, 406, to the distance, 488, on the line of numbers.

BY INSPECTION.

Seek in the Traverse Tables till the given difference of latitude and departure are found together in their respective columns; then against them is the distance in its column, and the course will be found at the top of that table, if the departure be less than the difference of latitude, otherwise at the bottom.

Thus, with half the difference of latitude, 135·5, and half the departure, 203, enter the tables; and these numbers will be found to correspond nearly to 5 points, or NW. by W., as a course, and a distance of 244 miles, which, being doubled, gives the distance sought 488 miles.

Exercises 22.

(236.) A ship sails between the south and east from latitude 54° 19′ N. till her difference of latitude is 262 and her departure 140 miles; required the course steered, the distance sailed, and the latitude in.

(237.) A ship from latitude 28° 13′ S. sails between the south and west till her difference of latitude is 175 and her departure 216 miles; required the course steered, the distance sailed, and the latitude in.

(238.) A ship from latitude 11° 19′ N. sails between the north and east till her difference of latitude is 150 and her departure 240 miles; required the course steered, the distance sailed, and the latitude in.

(239.) A ship sails between the north and east from latitude 50° 16′ S. till her difference of latitude is 255 and her departure 103 miles; required the course steered, the distance sailed, and the latitude in.

(240.) A ship from latitude 24° 53′ N. sails between the south and west till her difference of latitude is 167 and departure 137 miles; required her course steered, the distance sailed, and the latitude in.

(241.) A ship from latitude 60° 11′ N. sails between the south and east till her difference of latitude is 290 and departure 203 miles; required the course steered, the distance sailed, and the latitude in.

(242.) A ship from latitude 41° 18′ N. sails between the north and west till her difference of latitude is 357 and departure 279 miles; required the course steered, the distance sailed, and the latitude in.

(243.) A ship from latitude 31° 17′ S. sails between the south and east till her difference of latitude is 198 and departure 205 miles; required the course steered, the distance sailed, and the latitude in.

Promiscuous Exercises on the preceding Problems, 23.

(244.) A ship from latitude 37° 18′ N. sails on the port tack SSE. $\frac{3}{4}$ S. by compass for a distance of 270 miles, the variation of the compass being $2\frac{1}{4}$ W. and the leeway $1\frac{1}{4}$ points; required the true course, the difference of latitude and departure made, and the latitude in.

(245.) A ship from latitude 58° 14′ N. sails on the starboard tack SSE. $\frac{1}{4}$ E. by compass for a distance of 218 miles, the variation of the compass being $1\frac{3}{4}$ E. and the leeway $2\frac{1}{4}$ points; required the true course, the difference of latitude, the departure made and the latitude arrived at.

(246.) A ship sails on the port tack for a distance of 336 miles from latitude 27° 56′ S., her course by compass being NE. $\frac{3}{4}$ E., but the variation of the compass was found to be $1\frac{3}{4}$ points E. and the leeway $2\frac{1}{4}$ points; required her true course, the difference of latitude, the departure made, and the latitude arrived at.

(247.) If a ship sail SW. by W. on the port tack for a distance of 292 miles from latitude 46° 17′ N., and make 1½ points of leeway; required her true course, the difference of latitude, the departure made, and also the latitude arrived at.

(248.) If a ship sail from latitude 53° 31′ N. on the port tack, making 2½ points of leeway, her apparent course being SW. ½ W. till she arrive in latitude 52° 47′ N.; required the true course steered, the distance sailed, and the departure made good.

(249.) If a ship sail from latitude 29° 43′ S. till she arrive in latitude 31° 44′ S., having made a departure of 255·8 to the west; required the course steered, the distance sailed, and the time taken, her rate of sailing being 6 miles an hour.

(250.) A ship sails for 40 hours at 7½ miles per hour between the south and west on the port tack, making 1½ points of leeway, till her difference of latitude is 122 miles; required the apparent course sailed, the distance, and the departure.

(251.) If a ship sail from latitude 46° 51′ N. for 37 hours, at 8 miles an hour, between the north and west, till she arrive in latitude 51° 17′ N.; required the course steered, the distance sailed, and the departure made.

(252.) If a ship sail from latitude 29° 29′ N. on the starboard tack SW. by W. for a distance of 276 miles, making leeway 1¾ points, the variation of the compass being 1¼ W.; required her true course, the latitude arrived at, and the departure made.

(253.) A ship sails on the port tack, making 25° leeway, from latitude 58° 12′ N. on a compass course S. 58° W., the variation of the compass being 18° W. for 30 hours at 9½ miles per hour; required her true course, the departure made, and the latitude arrived at.

(254.) A ship sails by compass SW. by S. on the port tack for a distance of 420 miles from latitude 33° 54′ S., but she makes 2¼ points of leeway, and the variation of the compass is 1¾ E.; required her true course, the departure made, and the latitude in.

(255.) If a ship sail for 52 hours at the rate of $6\frac{1}{2}$ miles an hour on a course by the compass WSW. on the starboard tack, making 2 points of leeway, and find that she has made a difference of latitude of 290; required the course steered, the departure made, and the variation of the compass.

(256.) If a ship sail on the starboard tack between the south and west, making 3 points of leeway, where the variation of the compass is $1\frac{3}{4}$ E. till her distance is 238 and her difference of latitude is 184 miles; required the true course, the apparent course, and the departure.

(257.) A ship from latitude 20° 27′ S. sails between the north and west for 18 hours at 9 miles per hour, and finds she has arrived in latitude 17° 59′ S.; required her true and apparent course, the variation of the compass being 24° W.; also find the departure and distance sailed.

(258.) A ship from latitude 51° 15′ N. sails on the starboard tack, making $2\frac{1}{2}$ points of leeway for a distance of 251 miles, her apparent course being ENE.; required her true course, the departure made, and the latitude now in.

(259.) A ship sails WSW. from latitude 28° 54′ N. till she arrives in latitude 27° N.; find the difference of latitude made, the distance sailed, and the departure made.

(260.) A ship from latitude 38° 48′ S. sails between the north and west till her difference of latitude is 217 and her departure 116 miles, the variation of the compass is $1\frac{1}{4}$ W.; required the compass course, the distance sailed, and the latitude arrived at.

(261.) A ship from latitude 29° 49′ N. sails between the north and east till her difference of latitude is 146 and her departure 141 miles, the variation of the compass is 15° E.; required the compass course, the distance sailed, and the latitude arrived at.

(262.) A ship from latitude 54° 23′ N. sails between the south and west till her difference of latitude is 172 and her departure 191 miles, the variation of the compass being 18° E.; required

the compass course steered, the distance sailed, and the latitude arrived at.

(263.) A ship sails from latitude 41° 29′ N. a distance of 295 miles, and makes a difference of latitude of 143 miles, her course is between the north and west, and the variation of the compass is 21° W.; required the compass course steered, the departure made, and the latitude arrived at.

(264.) A ship sails between the north and east on the port tack from latitude 47° 41′ N., a distance of 274 miles, till her departure is 184 miles; she makes $1\frac{1}{4}$ points leeway, and the variation of the compass is $2\frac{3}{4}$ W.; required her true and apparent course, and the latitude arrived at.

CHAPTER VI.

TRAVERSE SAILING.

195. When a ship sails on different courses, the crooked line which she describes is called a *traverse*, and the method of finding a single course and distance which would have brought the ship to the same place, is called resolving a traverse. For this purpose any of the *four* methods already described in the problems in Plane Sailing may be employed; that, however, in most general use at sea is by inspection, which shall therefore be principally adhered to. To render the subject more clear, however, we shall also shew how to resolve a traverse by construction.

BY INSPECTION.

196. Make a table of such breadth and depth as may be necessary to contain the several courses, &c. Divide this table into six vertical columns; in the first, the several courses steered are to be placed, and the distances sailed on each of these courses are to be placed opposite them in the second column; the third and fourth columns, marked N. and S. at the top, are to contain the differences of latitude, and the last two, marked E. and W. at the top, contain the departures made on the several courses.

197. The several courses and their corresponding distances being properly arranged in the table; find in the first or second

Traverse Table, according as the course is given in points or degrees, the difference of latitude and departure answering to each. The difference of latitude is to be placed in the N. or S. column, according as the course is northward or southward; and the departure is to be put in the E. or W. column, according as the course is easterly or westerly. To avoid the mistake which frequently occurs of interchanging the difference of latitude and departure, observe that the difference of latitude is greater than the departure when the course is less than 4 points or 45°; otherwise it is less.

198. Add up the columns of northing, southing, easting, and westing, and write the sum at the bottom of each; take the difference of the northing and southing, and also of the easting and westing; the former difference will be the difference of latitude, which will be of the same name as the greater; and the latter will be the departure, which will also be of the same name as the greater. With this difference of latitude and departure, the course and distance made good are to be found by Problem XIII., Plane Sailing.

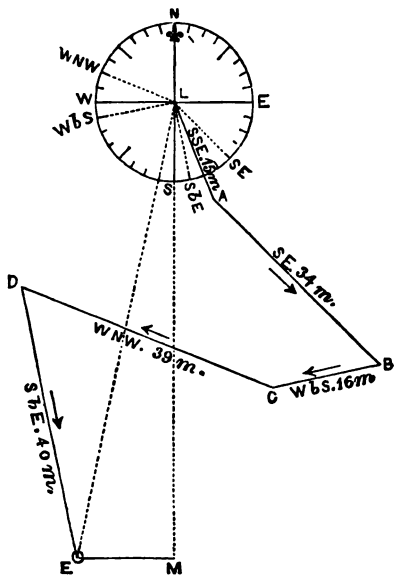
Example.

Suppose a ship takes her departure from Block Island in the latitude of 41° 10′ N., the middle of it bearing NNW., distance by estimation 15 miles, and sails SE. 34 miles, W. by S. 16 miles, WNW. 39 miles, and S. by E. 40 miles; required the latitude she is in, and her bearing and distance from Block Island.

BY CONSTRUCTION.

Let L represent the middle of Block Island: draw the meridian LM, and on L as a centre, with a chord of 60°, describe a circle to represent the compass, on which mark the various courses sailed, and the bearing of the land at the time of taking the departure; opposite to this bearing draw the SSE. line LA, which make equal to 15 miles, the estimated distance of the

land; then will A represent the place of the ship at the time of taking the departure: through A draw AB = 34 miles parallel to the SE. line; then will B be the place of the ship after sailing her first course; in like manner draw BC = 16 miles parallel to the W. by S. line, CD = 39 miles parallel to the WNW. line, and DE = 40 miles parallel to the S. by E. line; then will E represent the place of the ship after sailing her several courses. Join EL, and draw EM perpendicular to LM; then will LE be the distance of Block Island, 66·8 miles, and the angle ELM = 12° 16′ will be the course made good, LM the difference of latitude, and EM the departure.



BY LOGARITHMS.

To find the Dif. Latitude.	To find the Departure.
Art. (123) $\frac{\text{dif. lat.}}{\text{dist.}} = \text{cos. course.}$	Art. (122) $\frac{\text{dept.}}{\text{dist.}} = \text{sin. course.}$
∴ Dif. lat. = dist. × cos. course.	∴ Dept. = dist. × sin. course.

Hence log. dif. lat. = log. dist. + log. cos. course ∽ 10.
and log. dept. = log. dist. + log. sin. course − 10.

FIRST COURSE, SSE. 15 miles.

To find the Difference of Latitude.			To find the Departure.		
Log. 15	=	1·176091	Log. 15	=	1·176091
Log. cos. 2 pts.	=	9·965615	Log. sin. 2 pts.	=	9·582840
Log. dif. lat. = 13·86	=	1·141706	Log. dept. = 5·74	=	0·758931

SECOND COURSE, SE. 34 miles.

Log. 34	=	1·531479	Log. 34	=	1·531479
Log. cos. 4 pts.	=	9·849485	Log. sin. 4 pts.	=	9·849485
Log. dif. lat. = 24·04	=	1·380964	Log. dept. = 24·04	=	1·380964

THIRD COURSE, W. by S. 16 miles.

Log. 16	=	1·204120	Log. 16	=	1·204120
Log. cos. 7 pts.	=	9·290236	Log. sin. 7 pts.	=	9·991574
Log. dif. lat. = 3·12	=	0·494356	Log. dept. = 15·69	=	1·195694

FOURTH COURSE, WNW. 39 miles.

Log. 39	=	1·591065	Log. 39	=	1·591065
Log. cos. 6 pts.	=	9·582840	Log. sin. 6 pts.	=	9·965615
Log. dif. lat. = 14·93	=	1·173905	Log. dept. = 36·03	=	1·556680

FIFTH COURSE, S. by E. 40 miles.

Log. 40	=	1·602060	Log. 40	=	1·602060
Log. cos. 1 pt.	=	9·991574	Log. sin. 1 pt.	=	9·290236
Log. dif. lat. = 39·23	=	1·593634	Log. dept. = 7·80	=	0·892296

This method of finding the difference of latitude and departure is the most accurate, but the calculations may be more easily made and with sufficient accuracy by the Traverse Table, as shewn by inspection in Problem VIII., Plane Sailing.

TRAVERSE TABLE.

Courses.	Dist.	Dif. Lat.		Departure	
		N.	S.	E.	W.
SSE.	15	..	13·9	5·7	..
SE.	34	..	24·0	24·0	..
W. by S.	16	..	3·1	..	15·7
WNW.	39	14·9	..	..	36·0
S. by E.	40	..	39·2	7·8	..
From sum.	..	14·9	80·2	37·5	51·7
Take	..	..	14·9	..	37·5
Remains	..	..	65·3	..	14·2

Place all these courses, distances, &c., in the Traverse Table, then add up all the westings, eastings, northings, and southings separately, and set down their respective sums at the bottom of each column; and as the westing is greater than the easting, subtract the easting therefrom; the difference, 14·2, shews that the ship's departure is so much west of her first meridian.

Again, the southing being greater than the northing, subtract the northing from it, and the remainder, 65·3, shews how far the ship is to the southward of her first place.

To find the Direct Course or Bearing of Block Island from the Ship.

By Art. (124) tan. course $= \frac{\text{dept.}}{\text{dif. lat.}}$

$\therefore$ Log. tan. course = log. dept. + 10 − log. dif. lat.

Log. 14·2 + 10	=	11·1522883
Log. 65·3	=	1·8149132
Log. tan. 12° 16′	=	9·3373751

To find the Distance of the Island.

By Art. (126) $\frac{\text{dist.}}{\text{dif. lat.}} = \text{sec. course} = \text{sec. } 12° 16'$.

$\therefore$ Dist. = 65·3 × sec. 12° 16′.

Log. 65·3	=	1·8149132
Log. sec. 12° 16′	=	10·0100302
Log. 66·83	=	1·8249434

$\therefore$ The distance = 66·83 miles.

Also, since the difference of latitude made is south and the departure west, the ship bears from the island S. 12° 16′ W., and the island bears from the ship N. 12° 16′ E.

BY INSPECTION.

With the difference of latitude and departure made good taken from the above, find the course and distance sailed by Problem XIII., Plane Sailing.

Lastly, to find the latitude arrived at, we have the latitude from 41° 10′ N., and the difference of latitude made is 65·3 south, and hence we have

Latitude from	41° 10′ N.
Take dif. latitude	1° 5′ S.
$\therefore$ Latitude in	= 40° 5′ N.

Exercises 24.

(265.) A ship takes her departure from Cape Clear in latitude 51° 25′ N., the Cape bearing from her NNE. $\frac{1}{2}$ E. 15 miles, and sails WSW. $\frac{1}{2}$ W. 56 miles; SW. by S. 37 miles; W. by S. 28 miles; W. by N. $\frac{1}{2}$ N. 42 miles; S. by E. $\frac{3}{4}$ E. 47 miles; and WNW. $\frac{1}{4}$ N. 29 miles: find the course and distance made good and the latitude now in.

(266.) A ship sails from latitude 43° 18′ S., NNE. 43 miles; W. by S. 27 miles; E. 15 miles; NE. by N. 51 miles; W. by S.

24 miles; NE. 38 miles; N. by E. ½ E. 31 miles; NW. ½ W. 45 miles; and W. by S. ½ S. 23 miles: find the course and distance made good and the latitude arrived at.

(267.) Suppose a ship from latitude 37° 58′ N., sails WNW. 41 miles; N. by E. ¾ E. 33 miles; NW. ½ W. 38 miles; E. by S. 43 miles; N. by W. ¾ W. 35 miles; and NE. by E. ¼ E. 47 miles: find the course and distance made good and the latitude arrived at.

(268.) Suppose a ship from latitude 43° 18′ S. sails S. by E. ½ E. 35 miles; NE. ½ E. 46 miles; W. by N. ½ N. 48 miles; WSW. ¾ W. 49 miles; ESE. ¼ E. 27 miles; NW. by W. 41 miles; and E. by S. 36 miles: find the course and distance made good and the latitude arrived at.

(269.) Suppose that since yesterday at noon the following courses and distances have been run—*1st*, SW. by S. 20 miles; *2d*, W. 16 miles; *3d*, NW. by W. 28 miles; *4th*, SSE. 32 miles; *5th*, ENE. 14 miles; and *6th*, SW. 36 miles: what difference of latitude and departure has the ship made, and what is her direct course and distance made good?

(270.) During the last twenty-four hours we have run the following courses—namely, *1st*, SE. 40 miles; *2d*, NE. 28 miles; *3d*, SW. by W. 52 miles; *4th*, NW. by W. 30 miles; *5th*, SSE. 36 miles; and *6th*, SE. by E. 58 miles: required the difference of latitude, the departure, and the direct course and distance.

(271.) Since leaving latitude 46° 15′ N. we have sailed the following courses and distances—*1st*, NW. by W. 38 miles; *2d*, S. by E. 14 miles; *3d*, NNW. ½ W. 58 miles; *4th*, NW. by W. 52 miles; *5th*, W. by S. 23 miles; *6th*, WNW. 46 miles; and *7th*, NW. ½ N. 39 miles: required the difference of latitude and departure made, the course and distance made good, and the latitude now arrived at.

(272.) Suppose a ship from latitude 40° 29′ N. sails as follows—*1st*, SE. by S. 29 miles; *2d*, NNE. 10 miles; *3d*, ESE. 50 miles; *4th*, ENE. 50 miles; *5th*, SSE. 10 miles; *6th*,

NE. by N. 29 miles; *7th*, W. 25 miles; *8th*, SSE. 10 miles; *9th*, WSW. ½ W. 42 miles; *10th*, N. 110 miles; *11th*, E. ¾ N. 62 miles; *12th*, N. 7 miles; *13th*, W. 62 miles; *14th*, N. 10 miles; *15th*, W. 8 miles; *16th*, S. 10 miles; *17th*, W. 62 miles; *18th*, S. 7 miles; *19th*, E. ¾ S. 62 miles; *20th*, S. 110 miles; *21st*, WNW. ½ W. 42 miles; *22d*, NNE. 10 miles; and *23d*, W. 25 miles: required the ship's place, with the direct course and distance.

(273.) Yesterday at noon we were in latitude 3° 18′ S., and since, the ship has plied the following courses—*1st*, SE. 40 miles; *2d*, NE. 20 miles; *3d*, SW. by S. 45 miles; *4th*, NW. ½ W. 32 miles; *5th*, SSE. ½ E. 34 miles; *6th*, SE. ¼ S. 32 miles; *7th*, WNW. ¼ N. 17 miles; required the dif. latitude, departure, direct course and distance, and the latitude arrived at.

(274.) Since leaving latitude 34° 11′ N., we have sailed the following courses—*1st*, N. 36° W. 27 miles; *2d*, N. 24° E. 30 miles; *3d*, S. 75° W. 47 miles; *4th*, S. 80° W. 29 miles; *5th*, N. 72° W. 42 miles; *6th*, N. 78° W. 34 miles; and *7th*, S. 12° E. 28 miles: required the course and distance made, the latitude arrived at, and the departure made.

(275.) Since leaving latitude 47° 18′ S. we have sailed the following courses—*1st*, S. 40° E. 50 miles; *2d*, S. 49° W. 43 miles; *3d*, 54° W. 54 miles; *4th*, N. 70° W. 35 miles; *5th*, S. 20° E. 48 miles; *6th*, N. 48° W. 36 miles; *7th*, S. 42° E. 44 miles; and *8th*, N. 25° W. 45 miles: required the difference of latitude and departure made, the course and distance made good, and the latitude arrived at.

(276.) Since leaving latitude 55° 58′ N. we have sailed—*1st*, N. 75° E. 30 miles; *2d*, S. 79° E. 38 miles; *3d*, E. 15 miles; *4th*, N. 80° E. 48 miles; *5th*, S. 45° E. 18 miles; and *6th*, N. 38° E. 27 miles: required the difference of latitude and departure made, the course and distance made good, and the latitude arrived at.

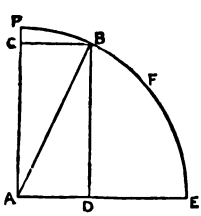
(277.) Since leaving latitude 38° 14′ S. we have sailed the following courses—*1st*, S. 54° E. 47 miles; *2d*, S. 15° W. 84

miles; *3d*, N. 75° W. 49 miles; *4th*, N. 35° W. 64 miles; *5th*, S. 58° E. 29 miles; and *6th*, S. 28° W. 42 miles: required the difference of latitude and departure made, the course and distance made good, and the latitude arrived at.

PARALLEL SAILING.

199. The figure of the earth is spherical; the meridians, therefore, gradually approach each other, and meet at the poles. The *difference of longitude* between any two places is the angle at the pole contained between the meridians of these places, or it is the arc of the equator intercepted between the meridians of the given places; and the *meridian distance* between two places in the same parallel of latitude is the arc of that parallel contained between their meridians. It hence follows that the meridian distance answering to the same difference of longitude, varies with the latitude of the parallel on which it is measured; and the same difference of longitude will not correspond to a given meridian distance when reckoned upon different parallels.

200. PARALLEL SAILING is, therefore, the method of resolving problems, in which the difference of longitude between two places on the same parallel, their meridian distance, and the latitude of the parallel are concerned. This sailing is particularly useful in any trade where the course is near a parallel, or in reaching low or small islands.



201. In order to illustrate the principles of Parallel Sailing, let AEBP represent a section of one-fourth part of the earth, the arc EBP being a portion of a meridian from E, the equator, to P, the pole; AP the polar semi-axis, and AE the equatorial semi-axis. Also let B be the situation of any place on the earth; join AB, and through B draw BD parallel to AP, and BC parallel to AE; then the triangles ADB and ACB are both right-angled, the one

at D, and the other at C. For our present purpose, though not strictly true, it will be sufficiently exact to consider the three lines, AE, AP, and AB as all equal. The arc EB, or the angle EAB, is the measure of the latitude of the place B, and the arc BP, or the angle PAB, is the measure of the co-latitude; and hence BC = the cosine of the latitude. Conceive the whole figure to revolve round the fixed axis AP, and it is plain that the points E and B will describe circles whose radii are AE = AB and CB; now the circumferences of circles are one to another as their radii, and any corresponding portions of circles are in the same ratio; hence any portion of the circle traced out by the point E is to the corresponding portion traced out by B, as AB : BC; but the portion of the circle traced out by E is the measure of the longitude between the two positions of the meridian, and the portion of the circle traced out by the point B is the meridian distance between two places in the same latitude in the two positions of the meridian; hence we have

$$\frac{\text{mer. dist.}}{\text{dif. long.}} = \frac{\text{CB}}{\text{AB}} = \frac{\text{cos. lat.}}{\text{Rad.}}, \text{ or if Rad.} = 1$$

we have $$\frac{\text{mer. dist.}}{\text{dif. long.}} = \text{cos. lat.}$$

$$\therefore \text{ mer. dist.} = \text{dif. long.} \times \text{cos. lat.} \qquad \text{(A)}$$

$$\text{Also } \frac{\text{dif. long.}}{\text{mer. dist.}} = \frac{\text{AB}}{\text{BC}} = \text{cosec. BAC} = \text{cosec. PAB} =$$

$$\text{sec. BAE} = \text{sec. lat.}$$

$$\therefore \text{ dif. long.} = \text{mer. dist.} \times \text{sec. lat.} \qquad \text{(B)}$$

202. If F were any other latitude, we would have as before

$$\text{mer. dist. (at lat. F)} = \text{dif. long.} \times \text{cos. (lat. at F)}$$

$$\therefore \frac{\text{mer. dist. at B}}{\text{mer. dist. at F}} = \frac{\text{dif. long.} \times \text{cos. EB}}{\text{dif. long.} \times \text{cos. EF}} = \frac{\text{cos. EAB}}{\text{cos. EAF}}. \qquad \text{(C)}$$

Hence the meridional distances on two different latitudes are to one another as the cosines of these latitudes respectively.

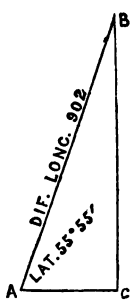
Problem XIV.

203. Given the latitude of a parallel, and the number of miles contained in a portion of the equator, to find the miles contained in a similar portion of that parallel.

Example.

Required the meridian distance between St Abb's Head in lat. 55° 55′ N., long. 2° 10′ W., and Uraniberg in the same latitude, but in long. 12° 52′ E.

Longitude, St Abb's Head	=	2° 10′ W.
Longitude, Uraniberg	=	12° 52′ E.
Difference of longitude	=	15° 2′ = 902 miles.



BY CONSTRUCTION.

Draw the indefinite straight line AC, make at the point A the angle CAB = 55° 55′, the given latitude, and make AB = 902, the difference of longitude; and from B draw BC perpendicular to AC, and AC the meridian distance sought, applied to the scale of equal parts, will measure 505·5 miles.

BY CALCULATION.

By Art. (201, A) mer. dist. = dif. long. × cos. lat. = 902 × cos. 55° 55′.

By Logarithms.			By Natural Numbers.		
Log. mer. dist. = log. dif. long. + log. lat. − 10.			Nat. cos. 55° 55′	=	·5603981
					902
Log. 902	=	2·9559065			1120796 2
Log. cos. 55° 55′	=	9·7484967			504358 29
Log. 505·479	=	2·7037032			505·479086 2

Hence the meridian distance sought = 505·5 miles nearly.

BY INSPECTION.

To 56°, the nearest degree to the given latitude, and 300 as a distance, is found in the column Dif. Lat. 167·8, which being multiplied by 3 gives 503·4, to which add the dif. lat. corresponding to distance 2, which is 1·1, giving 504·5. To correct for the 5′ in the course, the dif. lat. corresponding to 902 and 55° of latitude is 517·4, the difference is 12·9. Then 60′ : 5′ :: 12·9 to 1·1, the correction to be added to the former; hence 504·5 + 1·1 = 505·6 miles, the distance sought.

BY GUNTER'S SCALE.

The extent, from 90° to 34° 5′, the complement of the latitude, on the line of sines, will reach from 902 on the line of numbers, back to 505·5 on that line.

204. REMARK.—By this problem the number of miles contained in a degree of longitude in any parallel is found: a table of meridian distances may be calculated by multiplying the cosine of the latitude by 60; or for any whole degree it will be found by entering the Traverse Table with the latitude as a course and 60 as a distance; opposite to 60 in the column Dif. Lat. will be found the meridian distance required.

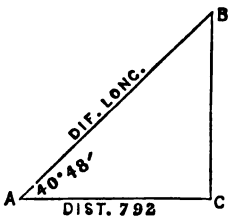
Problem XV.

205. Given the number of miles contained in a portion of a known parallel, to find the number of miles contained in the corresponding portion of the equator.

Example.

A ship from New York, in latitude 40° 48′ N. and longitude 74° 10′ W., sails due east till her meridian distance is 792 miles; required the difference of longitude made, and the longitude now in?

BY CONSTRUCTION.



Draw the straight line AC = 792 miles; make the angle BAC = 40° 48′, the given latitude; draw CB perpendicular to AC, meeting AB in B; then AB will represent the required longitude in miles, and its length being applied to the scale of equal parts will measure 1046 miles.

BY CALCULATION.

By Art. (201, B) dif. long. = mer. dist. × sec. lat. = 792 × sec. 40° 48′.

By Logarithms.			By Natural Numbers.		
Log. sec. 40° 48′ − 10	=	·1209070	Nat. sec. 40° 48′	=	1·3210126
Log. 792	=	2·8987252			792
Log. 1046·2	=	3·0196322			26420252
					11889134
					9247088
					1046·2419792

Hence the dif. long. = 1046·2 = 17° 26′ E.

∴ long. from 74° 10′ W. − dif. long. 17° 26′ E. gives longitude in 56° 44′ W.

BY INSPECTION.

To 40°, the next less degree to the given latitude, and 198, one-fourth part of the given meridian distance in a latitude column, the distance is 258·5, and to 41° and 198, the distance is 262·4; hence the change of distance answering to a change of one degree of latitude is 3·9. Now, 60′ : 48′ :: 3·9 : 3·1, which being added to 258·5, the sum is 261·6; and this being multiplied by 4, gives 1046·4 miles, the difference of longitude sought.

BY GUNTER'S SCALE.

The extent from 49° 12′, the complement of the latitude, to 90° on the line of sines, being laid in the same direction on the line of numbers, will reach from 792 to 1046, the difference of longitude, on the same line.

Problem XVI.

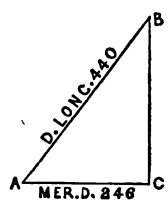
206. Given the number of miles contained in any portion of the equator, and also the number of miles contained in a similar portion of a parallel, to find the latitude of that parallel.

Example.

A ship from a place in longitude 3° 12′ W. sails due east 246 miles, and by observation is found to be in longitude 4° 8′ E.; required the latitude of the parallel on which she sailed.

BY CONSTRUCTION.

Draw the straight line AC, equal to the meridian distance 246; from C draw CB at right angles to AC; then with an extent, 440, equal to the difference of longitude made, describe an arc with the centre A, cutting CB in B, and join AB; and the angle BAC being measured, will be found to be 56° nearly, the latitude sought.



BY CALCULATION.

By Art. (123) or Art. (201) $\frac{\text{mer. dist.}}{\text{dif. long.}} = \text{cos. BAC} = \text{cos. lat.}$

Hence by Logarithms.			By Natural Numbers.
Log. 246 + 10	=	12·3909351	$\frac{246}{440} = \cdot 5590909 = \text{cos. lat.}$
Log. 440	=	2·6434527	
Log. cos. 56° 0′ 25″	=	9·7474824	∴ latitude = 56° 0′ 25″.

BY INSPECTION.

Since the difference of longitude given, 440, exceeds the distances given in the Traverse Tables, take its half and also the half of the meridian distance 246; these are 220 and 123 respectively. Enter the tables with 220 as distance, and 123 as dif. lat., and turning the tables, we find in the page with 56° at the bottom, 220 in the distance column, and 123 in the dif. lat.; hence the latitude sought is 56°.

BY GUNTER'S SCALE.

Extend from 440 back to 246 on the line of numbers; this extent will reach from 90 back to 34 on the line of sines; 34° is therefore the complement of the latitude sought; hence the latitude is 56° very nearly.

Problem XVII.

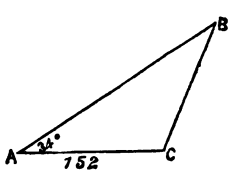
207. Given the number of miles in a portion of a known parallel, to find the number of miles contained in a similar portion of another given parallel.

Example.

A ship in latitude 39° 30′ N. wishes to reach a port on the same meridian in latitude 56° N.; she sails due east 152 miles, and then due north till she is in latitude 56° N.; how far must she now sail direct west to reach the port required?

BY CONSTRUCTION.

Draw the straight line AC = the given distance 152 miles; make the angle BAC 34°, the complement of the latitude 56°, and the angle ACB = 95° 30′, the sum of the two latitudes; then BC will be the distance sought, and being applied to the scale of equal parts, will be found to measure 110 miles.



BY CALCULATION.

By Art. (201) the given mer. dist. = 152 = dif. long. cos. 39° 30′;
and the sought mer. dist. = dif. long. cos. 56°;

$$\therefore \frac{\text{sought mer. dist.}}{\text{given mer. dist.}} = \frac{\text{dif. long. cos. } 56^\circ}{\text{dif. long. cos. } 39^\circ\ 30'} = \frac{\cos. 56^\circ}{\cos. 39^\circ\ 30'} \text{ (202)}.$$

Hence mer. dist. sought = 152 cos. 56° sec. 39° 30′.

Log. 152	=	2·1818436
Log. cos. 56°	=	9·7475617 − 10
Log. sec. 39° 30′	=	10·1125939 − 10
Log. 110·15	=	2·0419992

∴ The mer. distance sought = 110·15 miles.

BY INSPECTION.

In the Traverse Table, with a difference of latitude of 152 in the page headed 39°, we get in the Distance column 195·5, and in the page headed 40°, we get 198·5; half the sum of these two—namely, 197—is the distance, or difference of longitude made at 39½° by a meridian distance of 152 miles; with this distance 197 enter the page with 56° at the bottom, and opposite the distance in the column of Difference of Latitude will be found the meridian distance sought, 110 miles.

BY GUNTER'S SCALE.

Extend from 50° 30′, the complement of 39° 30′, to 34°, the complement of 56°, on the line of sines; this extent being applied to the line of numbers, will reach from 152 back to 110 miles, the distance sought.

Problem XVIII.

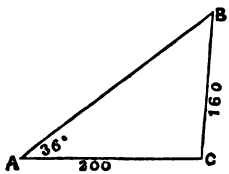
208. Given the length of an arc of a known parallel, and the length of a similar portion of an unknown parallel, to find the latitude of the latter parallel.

Example.

A ship from a port in latitude 54° 0′ N. sailed due west 160 miles, then having run due south an unknown number of miles, sailed due east 200, and by observation found she had arrived at the meridian of the port she sailed from; required the latitude of the place come to, and the distance run in the south direction.

BY CONSTRUCTION.

Make AC = 200, and draw the line AB so as to make the angle BAC = 36°, the complement of the latitude left; then with an extent equal to 160, and one foot of the compasses in C, describe an arc cutting AB in B; join BC, and the angle ABC will measure 47° 17′, the complement of the latitude come to, which is therefore 42° 43′.



BY CALCULATION.

$$\text{Here } \frac{\text{sin. B}}{\text{sin. A}} = \frac{\text{AC}}{\text{BC}} = \frac{200}{160}$$

$$\therefore \text{ sin. B} = \frac{200}{160} \text{ sin. } 36^\circ = \frac{5}{4} \text{ sin. } 36^\circ$$

$$= \frac{10}{8} \times \cdot 5877853$$

$$= \frac{5 \cdot 877853}{8} = \cdot 7347316 = \text{sin. } 47^\circ\ 17'.$$

BY LOGARITHMS.

Ar. co. Log. 160	=	7·7958800
Log. 200	=	2·3010300
Log. cos. 54°	=	9·7692187
Log. cos. 42° 43′	=	9·8661287

54° − (42° 43′) = 11° 17′ = 677 miles, the difference of latitude made, and therefore the distance sailed south.

BY INSPECTION.

In the Traverse Table, with 54° as course, look for 160 in the column Difference of Latitude, opposite it in the Distance column will be found 272·2 nearly, which is the difference of longitude made. With this distance, and 200 of difference of latitude, seek the page where they are found most nearly together; that will be at 42° and 43°, the former giving the dif. lat. 202·4, and the latter 199·2, the difference of which is 3·2, and the difference between the former and the true one 2·4; hence 3·2 : 2·4 :: 60′ : 45′, the correction to be added to 42°; hence the latitude sought is 42° 45′ nearly.

BY GUNTER'S SCALE.

The extent from 160 to 200, on the line of numbers, will reach from 36° 0′, the complement of the given latitude, to 47° 17′, the complement of the required latitude, on the line of sines.

Exercises in Parallel Sailing, 25.

(278.) How many miles in a degree of longitude in the parallel of latitude 51° 17′?

(279.) How many miles in 3° 44′ of longitude in the parallel of latitude 55° 56′ N.?

(280.) How many miles must a ship sail due west on the parallel of latitude 34° 50′ to make a change of longitude of 716 miles?

(281.) A ship from latitude 52° 18′ and longitude 2° 12′ W. sails due east 296 miles; what longitude is she now in?

(282.) A ship from latitude 53° 36′ N. and longitude 10° 18′ E. sails due west 236 miles; required her present longitude.

(283.) How far must a ship sail due east on the parallel of 60° to change her longitude 3°?

(284.) A ship from latitude 57° 58′ N. and longitude 7° 3′ E. sails west 156 miles; required the longitude arrived at.

(285.) A ship in north latitude sails on a parallel 218 miles, and another in south latitude sails 158 miles on a parallel; they have both made the same difference of longitude, 265 miles; required the difference of latitude between the two ships.

(286.) A ship runs 186 miles due west in latitude 51° 16′; required her difference of longitude made good.

(287.) The difference of longitude of two places in the parallel of 54° 20′ is 3° 15′; required their distance as measured along the parallel.

(288.) A ship in latitude 41° 30′ sails due east till her difference of longitude is 396 miles; what distance has she run?

(289.) A ship from latitude 47° 15′ N. and longitude 12° 19′ W. sails due north 220 miles, and then due east 220 miles; required the latitude and longitude now in.

(290.) In what latitude does 423 miles of meridional distance correspond to 540 miles of difference of longitude?

(291.) In what latitude does 100 miles of meridional distance correspond to 300 miles of difference of longitude?

(292.) From two ports in latitude 33° 40′ N., distant 248 miles, two ships sail directly north; how far are they distant from each other when they reach the latitude of 45° 10′ N.?

(293.) From two ports in latitude 45° 12′ S., distant 290 miles, two ships sail directly north for 290 miles each; find the latitude arrived at, and the meridional distance between them.

(294.) If two ships in latitude 50° N., distant from one another 210 miles, should both sail directly south till their meridian distance was 261 miles; what latitude are they come to?

(295.) A ship having run due east for four days at the rate of $4\frac{1}{2}$ knots per hour on an average, finds she has altered her longitude by 9° 52′; what parallel of latitude did she sail on?

(296.) Suppose two ships in the parallel of 45° 47′ N., their difference of longitude being 8° 53′, should both sail directly south 734 miles; how far were they from each other when they set out, and how far are they distant after having run that distance?

(297.) What is the difference of latitude between two parallels on different sides of the equator, when between the same two meridians the meridional distance in the northern parallel is 246, in the southern parallel is 210 miles, and at the equator 300 miles?

MIDDLE-LATITUDE SAILING.

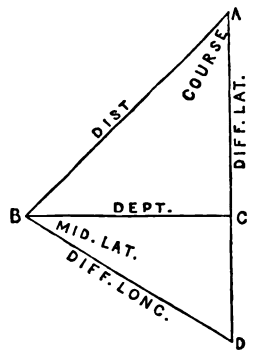
209. This is a method, founded on the principles of Parallel Sailing, by which, when the ship's course cuts the meridians obliquely, and consequently the departure is made on different parallels, *departure* is converted into *longitude*, as if it had all been made on the latitude which is midway between the latitude sailed from and that arrived at; and hence it is called *Middle-latitude Sailing.*

Suppose a ship in sailing from latitude 43° to latitude 50° makes a departure of 120 miles; this departure, if converted into longitude by the principles of parallel sailing, on the supposition that it was all made on the parallel sailed from—namely, 43°—gives the difference of longitude 164·2; and if converted into longitude, on the supposition that it was all made on the parallel arrived at—namely 50°—gives the difference of longitude 186·7. One of these must be too great, and the other too small; and in middle-latitude sailing we assume that the true longitude is obtained by reducing it to longitude on the supposition that it was all made on 46° 30′, which is the latitude midway between that sailed from and that arrived at; by this means we find the true difference of longitude to be 174·3 miles. It will be seen afterwards that this result is not strictly correct, but it is sufficiently near for most practical purposes, especially when the difference between the extreme latitudes is not very great, or does not amount to many degrees.

210. When the latitude sailed from and that arrived at are both north or both south, the *middle latitude* is half their *sum.*

When the latitudes left and that arrived at are of opposite names, the ship is nearly on the equator, and the departure itself is therefore the difference of longitude, for small distances such as are sailed in the course of a day.

211. *To investigate the Rules for Middle-latitude Sailing.*—In the adjoining figure, let AB = the distance sailed, making an angle BAC with the meridian AC; then the angle BAC = *the course*, AC = the difference of latitude made, and BC = the departure. Consider now BC as a meridian distance, as in parallel sailing, make the angle CBD = the middle latitude, and produce AC to meet BD in D; then BCD is a right-angled triangle in which BC = the meridional distance; and since angle CBD is the middle latitude (Art. 201), BD = the difference of longitude made in sailing from A to B on the course BAC, also AC = the difference of latitude made in the same distance and course, and BC = the departure.



Now from the triangle DBC, by Art. (123), we have

$$\frac{\text{dept.}}{\text{dif. long.}} = \text{cos. mid. lat., and therefore}$$

$$\text{dept.} = \text{dif. long.} \times \text{cos. mid. lat.} \qquad \text{(A)}$$

Again, from the triangle ABC, we have, by Art. (122),

$$\frac{\text{dept.}}{\text{dist.}} = \text{sin. course, and therefore}$$

$$\text{dept.} = \text{dist.} \times \text{sin. course.} \qquad \text{(B)}$$

And from the same triangle, by Art. (124), we have

$$\frac{\text{dept.}}{\text{dif. lat.}} = \text{tan. course, and therefore}$$

$$\text{dept.} = \text{dif. lat.} \times \text{tan. course.} \qquad \text{(C)}$$

Having now obtained three values of the departure (A), (B), and (C), these are equal to one another; first (A) = (B); that is,

$$\text{dif. long.} \times \text{cos. mid. lat.} = \text{dist.} \times \text{sin. course,}$$

and dividing by cos. mid. lat., and remembering that to divide by the cosine is the same as to multiply by the secant, we have

$$\text{dif. long} = \text{dist.} \times \text{sin. course} \times \text{sec. mid. lat.} \qquad \text{(D)}$$

Next, equating (A) to (C), we obtain

$$\text{dif. long.} \times \text{cos. mid. lat.} = \text{dif. lat.} \times \text{tan. course},$$

and dividing both sides by cos. mid. lat. as before, we have

$$\text{dif. long.} = \text{dif. lat.} \times \text{tan. course} \times \text{sec. mid. lat.} \qquad \text{(E)}$$

Also, from (A), by dividing both sides by cos. mid. lat., we have

$$\text{dif. long.} = \text{dept.} \times \text{sec. mid. lat} \qquad \text{(F)}$$

Collecting these three results, it may be remarked that they are all contained in (F); for if the two values of the departure which can be obtained from the triangle ABC—namely, dept. = dist. × sin. course, and dept. = dif. lat. × tan. course—be substituted for dept. in (F), (D) and (E) will be obtained; hence we have

$$\begin{aligned} \text{dif. long.} &= \text{dept.} \times \text{sec. mid. lat.} && \text{(F)} \\ &= \text{dist.} \times \text{sin. course} \times \text{sec. mid. lat.} && \text{(D)} \\ &= \text{dif. lat.} \times \text{tan. course} \times \text{sec. mid lat.} && \text{(E)} \end{aligned}$$

212. From these are obtained the three following rules for finding the difference of longitude according to the data, which, being expressed in words, are as follow:

RULE I.—The difference of longitude is equal to the product of the departure, multiplied into the secant of the middle latitude.

RULE II.—The difference of longitude is equal to the continued product of the distance, into the sine of the course, into the secant of the middle latitude.

RULE III.—The difference of longitude is equal to the continued product of the difference of latitude, into the tangent of the course, into the secant of the middle latitude.

213. The same formulæ turned into a logarithmic form, give the following:

Log. dif. long. = log. dept. + log. sec. mid. lat. − 10. (F)

Log. dif. long. = log. dist. + log. sin. course + log. sec. mid. lat. − 20. (D)

Log. dif. long. = log. dif. lat. + log. tan. course + log. sec. mid. lat. − 20. (E)

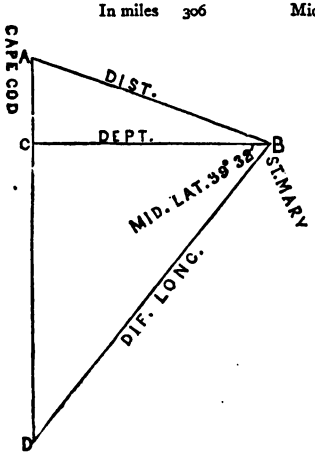
Problem XIX.

214. Given the latitudes and longitudes of two places, to find their bearing and distance.

Example.

Required the bearing and distance between Cape Cod lighthouse, in latitude 42° 5′ N., longitude 70° 4′ W., and the island of St Mary, in the latitude of 36° 59′ N., and longitude 25° 10′ W.

Cape Cod's lat.	42° 5′ N.		42° 5′	Long.	70° 4′ W.
St Mary's lat.	36° 59′ N.		36° 59′	Long.	25° 10′ W.
Dif. lat.	5° 6′	Sum	79° 4′		44° 54′
	60				60
In miles	306	Mid. lat.	39° 32′	Dif. long.	2694 miles.



BY CONSTRUCTION.

Draw the east and west line BC, and at the point B make the angle CBD = 39° 32′, the middle latitude, and draw BD = 2694; from D draw DC perpendicular to BC, and produce it to A till AC = 306, the difference of latitude, and join AB; then A represents Cape Cod, B St Mary's Island, and the angle BAC is the course required, and BC will represent the departure.

These being measured, AB will be found to measure about 2099 miles; BC, the departure, will measure 2078 miles; and the course, CAB, from Cape Cod to St Mary, will be found to be 81° 37′; and therefore the course, S. 81° 37′ E. or E. $\frac{3}{4}$ S. nearly.

BY CALCULATION.

By Art. (211, A) dept. = dif. long. × cos. mid. lat.

By Art. (124) $\frac{\text{dept.}}{\text{dif. lat.}}$ = tan. course.

and Art. (126) dist. = dif. lat. × sec. course.

To find the Course.

Log. dif. long.	=	log. 2694	=	3·4303976
Log. cos. mid. lat.	=	log. cos. 39° 32′	=	9·8871977
∴ Dept. = 2077·7	=	log. 2077·7	=	3·3175953
Log. dept. + 10	=	log. 2077·7 + 10	=	13·3175953
−Log. dif. lat.	=	log. 306	=	2·4857214
		Log. tan. 81° 37′	=	10·8318739

∴ the course = S. 81° 37′ E., or E. $\frac{3}{4}$ S. nearly.

To find the Distance.

Log. dif. lat.	=	log. 306	=	2·4857214
Log. sec. course	=	log. sec. 81° 37′	=	10·8362566
		Log. 2098·8	=	3·3219780

∴ the distance = 2098·8 = 2099 miles nearly.

NOTE.—The course may be found without finding the departure from (E), Art. (211), which gives, by dividing both sides by dif. lat. × sec. mid. lat.,

$$\text{tan. course} = \frac{\text{dif. long.} \times \text{cos. mid. lat.}}{\text{dif. lat.}} = \frac{2694 \times \text{cos. } 39° 32′}{306}$$

Log. 2694	=	3·4303976	
Log. cos. 39° 32′	=	9·8871977	
		13·3175953	
−Log. 306	=	2·4857214	
Tan. 81° 37′	=	10·8318739	the same as above.

BY INSPECTION.

RULE.—Look for the middle latitude, as if it were a course in Plane Sailing, and the difference of longitude in the Distance column, opposite to which, in the column of Difference of Latitude,

will be found the departure; having now the difference of latitude and departure, enter the second Traverse Table, and seek until they are found together; opposite to them will be found, in the Distance column, the distance sought; also, the course will be found at the top of the page, if the difference of latitude is greater than the departure; but when otherwise, the course will be found at the bottom of the page.

BY GUNTER'S SCALE.

1. On the line of sines, extend from 90° to 50° 28′, the complement of the middle latitude; that extent will reach from the difference of longitude, 2694, back to 2078, the departure, on the line of numbers.

2. Extend from the difference of latitude, 306, to the departure, 2078, on the line of numbers; that extent will reach from radius or 45° to the course 81° 37′ on the line of tangents.

3. Extend from 81° 37′, the course, to 90° on the line of sines; that extent will reach from the departure, 2078, to the distance, 2099 miles, on the line of numbers.

Problem XX.

215. Given the latitude sailed from and the latitude arrived at, and the departure from the meridian; to find the course, distance, and difference of longitude.

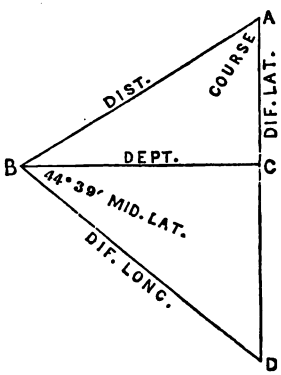
Example.

A ship from latitude 49° 57′ N. and longitude 15° 16′ W., sails south-westerly till her departure is 789 miles, and latitude in 39° 20′ N. Required the course, distance, and longitude in.

Latitude left	=	49° 57′ N.	
Latitude in	=	39° 20′ N.	
Dif. of latitude	=	10° 37′	= 637 miles.
Sum of lats.	=	89° 17′	
Middle latitude	=	44° 39′	

BY CONSTRUCTION.

Draw the meridian ACD, on which lay off AC, equal to the difference of latitude, 637 miles; draw CB perpendicular to AC, and make it equal to the departure, 789 miles; draw BD, making an angle with BC equal to 44° 39′, the middle latitude, and produce AC and BD to meet in D, and join BA. AB will be the distance sailed, and being measured, will be found to be 1014 miles; BD is the difference of longitude, and being measured, will be found to be 1109 miles; and the angle BAC is the course, and will measure 51° 5′.



BY CALCULATION.

To find the Course.

We have, by Art. (124), $\tan \text{course} = \frac{\text{dept.}}{\text{dif. lat.}}$

Hence Log. 789 + 10	=	12·8970770
Log. 637	=	2·8041394
Tan. 51° 5′	=	10·0929376

∴ The course = S. 51° 5′ W.

To find the Distance.

We have, Art. (126), $\frac{\text{dist.}}{\text{dif. lat.}} = \text{sec. course.}$

∴ Dist. = dif. lat. × sec. course.

Hence Log. 637	=	2·8041394
Log. sec. 51° 5′	=	10·2019094
Log. 1014	=	3·0060488

∴ The distance = 1014 miles.

To find the Dif. Long., we have, Art. (211, F), dif. long. = dept. × sec. mid. lat.

Hence Log. 789	=	2·8970770
Log. sec. 44° 39′	=	10·1478782
Log. 1109	=	3·0449552

∴ Dif. long. = 1109 miles	=	18° 29′ W.
Longitude sailed from	=	15° 16′ W.
∴ The longitude in	=	33° 45′ W.

BY INSPECTION.

RULE.—With the difference of latitude and departure, find the course and distance by seeking in the Traverse Table for degrees, until the difference of latitude and departure are found to correspond, against which, in the Distance column, will be found the distance sought, and the course will be found at the top of the page, if the difference of latitude be greater than the departure; otherwise, the course will be found at the bottom of the page.

Then take the middle latitude as a course, and find the departure in the Latitude column, the number corresponding in the Distance column will be the difference of longitude.

In the present example, to bring the distances given within the range of the tables, I take one-tenth of the difference of latitude, and one-tenth of the departure—these are, 63·7 and 78·9; then entering the table with these, the nearest to them found in the table are 63·6 and 78·5, standing together over 51°, and opposite 101 in the Distance column, which, being multiplied by 10, gives the distance 1010 miles; whence, the course by inspection is S. 51° W., and the distance 1010 miles. I then take one-tenth of the departure, 78·9, and seek it in the column of latitude of 45° as course, which is the nearest to 44° 39′, the middle latitude; the nearest that I find to it is 79·2, and the corresponding distance 112, and correcting this for the excess in the latitude, it is reduced to 111·6, which, being multiplied by ten, gives the distance sought, 1116 miles. Again, I seek the difference of latitude, 79·2, in the course of 44°; the nearest found is 79·1, corresponding to the distance 110, consequently, that corresponding to 78·9 being multiplied by ten, gives 1097 miles for the difference of longitude: the difference between these results is 19 miles, and as the differences of the courses is 60′, and that between the first and the true course is 21′ less than 45°, I say 60′ : 21′ :: 19 : 6·7, therefore, subtracting 7 miles from 1116 miles, there remain 1109 miles of difference of longitude—the same as by calculation.

BY GUNTER'S SCALE.

1. The extent from the difference of latitude, 637, to the departure, 789, on the line of numbers, will reach from 45° to the course, 51° 5′, on the line of tangents.

2. The extent from 51° 5′ to 90° on the line of sines, will reach from 789, the departure, to the distance, 1014, on the line of numbers.

3. The extent from the complement of the middle latitude, 45° 21′, to 90° on the line of sines, will reach from 789 to the difference of longitude, 1109, on the line of numbers.

Problem XXI.

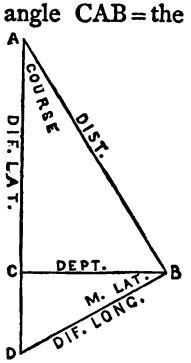
216. Given the course and distance, and one latitude; to find the difference of latitude and difference of longitude.

Example.

A ship from latitude 42° 30′ N., and longitude 58° 51′ W., sails SE. by S. 591 miles; required the latitude and longitude arrived at.

BY CONSTRUCTION.

Draw the meridian ACD, and make the angle CAB = the course 3 points, and make AB = 591, the given distance sailed; from B let fall BC perpendicular to AC, then AC = 491, the difference of latitude made, and BC = 328, the departure made. Hence we obtain the latitude arrived at, and the middle latitude; draw the line BD, making the angle CBD 38° 24′ = the middle latitude, and the distance BD = 419 miles is the difference of longitude made, and hence the longitude in is obtained.



BY CALCULATION.

To find the Dif. Latitude.

By Art. (123) $\frac{\text{dif. lat.}}{\text{dist.}} = \text{cos. course.}$

$\therefore$ Dif. lat. = dist. × cos. course.

Hence Log. 591	=	2·7715875
Log. cos. 3 pts.	=	9·9198464
Log. 491·4	=	2·6914339

$\therefore$ Dif. lat. = 491 miles.

Latitude left	=	42° 30′ N.
Dif. lat. 491	=	8° 11′ S.
Latitude in	=	34° 19′ N.
Sum of lats.	=	76° 49′
Middle lat.	=	38° 25′
Longitude left	=	58° 51′ W.
Dif. long 419	=	6° 59′ E.
Longitude in	=	51° 52′ W.

To find the Departure.

By Art. (122) $\frac{\text{dept.}}{\text{dist.}} = \text{sin. course.}$

$\therefore$ Dept. = dist. × sin. course.

Hence Log. 591	=	2·7715875
Log. sin. 3 pts.	=	9·7447390
Log. 328·34	=	2·5163265

$\therefore$ Departure = 328 miles nearly.

To find the Difference of Longitude.

By (211, F) dif. long. = dept. × sec. course.

$\therefore$ Log. dept.	=	2·5163265
Log. sec. 38° 25′	=	10·1059539
Log. 419	=	2·6222804

Also by Art. (132), taking the whole triangle ABD, we have

$$\frac{\text{dif. long.}}{\text{dist.}} = \frac{\text{sin. course}}{\text{sin. BDC}}$$

$$= \frac{\text{sin. course}}{\text{cos. mid. lat.}}$$

And therefore dif. long. $= \text{dist.} \times \frac{\text{sin. course}}{\text{cos. mid. lat.}}$

= dist. × sin. course × sec. mid. lat.

Hence Log. 591	=	2·7715875
Log. sin. 3 pts.	=	9·7447390
Log. sec. 38° 25′	=	10·1059539
Log. 419	=	2·6222804

BY INSPECTION.

RULE.—With the course, 3 points, enter the first Traverse Table, and dividing the distance 591 into the two parts 300 and 291, we obtain

Dist.	Dif. Lat.	Dept.
300	249·4	166·7
291	242·0	161·7
591	491·4	328·4

The difference of latitude is therefore 491·4, and the departure is 328·4.

Taking now the middle latitude as a course, and the tenth of the departure, 32·8, as difference of latitude, with 38° as a course, we get in the Distance column 41·6; and again, with 39° as course, we get 42·3; the difference is ·7, and $\frac{25}{60}=\frac{5}{12}$ of this is ·3 nearly, which being added to 41·6, gives 41·9, which again being multiplied by 10, gives 419 miles for the difference of longitude—the same as before obtained by calculation.

BY GUNTER'S SCALE.

1. The extent from 8 points to 5 points, the complement of the course, on the line S.R., will reach from the distance 591 back to 491, the difference of latitude on the line of numbers.

2. The extent from 8 points to the course 3 points on the line S.R., will reach back from 591 to 328, the departure on the line of numbers.

3. The extent from the complement of the middle latitude, 51° 35′, to 90° on the line of sines, will reach from the departure, 328, to the difference of longitude, 419, on the line of numbers.

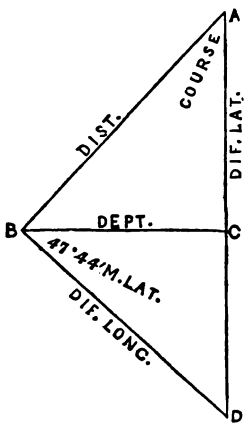
Problem XXII.

217. Given the latitude sailed from and the latitude arrived at, and also the course; to find the distance sailed, the departure, and difference of longitude.

Example.

Suppose a ship sailing from a place in the latitude of 49° 57′ N. and longitude 30° W., makes a course good of S. 39° W., and is then found by observation to be in the latitude of 45° 31′ N.; it is required to find the distance run and the longitude in.

Latitude from	=	49° 57′ N.	
Latitude by observation	=	45° 31′ N.	
Dif. lat.	=	4° 26′	= 266 miles
Sum of latitudes	=	95° 28′	
Middle latitude	=	47° 44′	



BY CONSTRUCTION.

Draw the meridian AD, on which set off AC = 266, the difference of latitude in miles; draw CB perpendicular to AC; draw AB, making an angle of 39° with AC, and meeting BC in B; through B draw BD, making an angle equal to the middle latitude, 47° 44′, with BC, and meeting AC produced in D. Then AB will be the distance = 342·3 miles, BC the departure = 215·4 miles, and BD the difference of longitude = 320 miles.

BY CALCULATION.

To find the Distance.

By Art. (126) $\frac{\text{dist.}}{\text{dif. lat.}}$ = sec. course.

∴ Dist. = dif. lat. × sec. course.

Hence Log. 266	=	2·4248816
Log. sec. 39°	=	10·1094974
Log. 342·3	=	2·5343790

∴ Distance = 342·3 miles.

To find the Departure.

By Art. (124) $\frac{\text{dept.}}{\text{dif. lat.}}$ = tan. course.

∴ Dept. = dif. lat. × tan. course.

Hence Log. 266	=	2·4248816
Log. tan. 39°	=	9·9083692
Log. 215·4	=	2·3332508

∴ Dept. = 215·4 miles.

To find the Difference of Longitude.

1st, by (E), Art. (211).

Log. dif. lat.	=	2·4248816
Log. tan. 39°	=	9·9083692
Log. sec. 47° 44′	=	10·1722547
Log. 320·3	=	2·5055055

2d, by (F), Art. (211).

Log. dept.	=	2·3332508
Log. sec. 47° 44′	=	10·1722547
Log. 320·3	=	2·5055055

∴ The difference of longitude = 320 miles = 5° 20′ W.
Longitude sailed from = 30° 0′ W.
Hence the longitude in = 35° 20′ W.

BY INSPECTION.

RULE.—Look for the course in the first or second Traverse Table according as it is given in points of degrees, and in the column of Dif. Lat. find the given difference of latitude, and in the adjacent columns will be found the distance and departure in their proper columns; then take the middle latitude as a course, and the departure formerly found as a difference of latitude, and in the adjacent Distance column will be found the difference of longitude sought.

Thus, with the course, 39°, and half the difference of latitude, 133, I enter the second Traverse Table; the nearest number in the table to that sought is 132·9, which corresponds to the distance, 171, and to the departure, 107·6; these doubled give the distance, 342, and the departure, 215·2 miles.

Then with the middle latitude, 47° 44′ or 48°, as a course, I enter the same table, and seek for half the departure, 107·6, in the Dif. Lat. column, the nearest number to which is 107·7, which corresponds to the distance, 161; this doubled gives the difference of longitude = 322 miles nearly.

If I enter the table again with 47° as a course and the same as difference of latitude, the nearest I find is 107·8, which gives a distance of 158; the difference between these results is 3, which, being multiplied by $\frac{16}{60}=\frac{4}{15}$, gives ·8; this being subtracted from 161, leaves 160·2, which being doubled, gives the difference of longitude, 320·4 miles, the same as by calculation.

BY GUNTER'S SCALE.

1. The extent from 45° to 39° the course, on the line of tangents, will reach from the difference of latitude, 266, back to the departure, 215·4, on the line of numbers.

2. The extent from the complement of the course, 51°, to 90° on the line of sines, will reach from the difference of latitude, 266, to the distance, 342·3, on the line of numbers.

3. The extent from the complement of the middle latitude, 42° 16′, to 90° on the line of sines, will reach from the departure, 215·4, up to the difference of longitude, 320·3, on the line of numbers.

Problem XXIII.

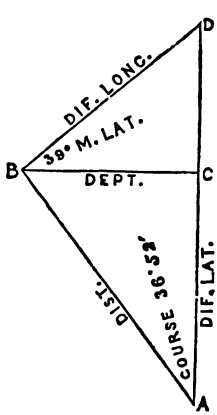
218. Given the latitude sailed from and the latitude arrived at, and also the distance; to find the course, departure, and difference of longitude.

Example.

Suppose a ship sails 300 miles north-westerly from a place in the latitude of 37° N. and the longitude of 32° 16′ W., until she is in the latitude of 41° N.; required her course and longitude in.

Latitude left		37° 0′ N.		37° 0′ N.
Latitude in		41° 0′ N.		41° 0′ N.
		4° 0′	Sum	78° 0′ N.
		60		
Dif. lat.	=	240	Mid. lat.	39° 0′

BY CONSTRUCTION.



Draw the meridian ACD, and make AC = 240, the difference of latitude; draw the line CB perpendicular to AC; take the distance 300 in the compasses, and with one foot in A cut CB in B, and join AB; the angle CAB is the course, and BC is the departure made good. Again, make the angle CBD = 39°, the middle latitude, and produce AC and BD to meet in D, and BD will be the difference of longitude. It will now be found that the angle BAC, the course, = 36° 52′; that BC, the departure, = 180 miles; and that BD, the difference of longitude, = 231·6 miles.

BY CALCULATION.

To find the Course.

By Art. (123) $\cos A = \frac{\text{dif. lat.}}{\text{dist.}} = \frac{240}{300}$

∴ Log. 240 + 10	=	12·3802112
Log. 300	=	2·4771213
Log. cos. 36° 52′	=	9·9030899

To find the Departure.

By Art. (124) $\frac{\text{dept.}}{\text{dif. lat.}} = \text{tan. course.}$

Hence dept. = dif. lat. × tan. course.

∴ Log. 240	=	2·3802112
Log. tan. 36° 52′	=	9·8750102
∴ Dept. = 180	=	2·2552214

To find the Difference of Longitude.

By Art. (211, F)

Dif. long. = dept. × sec. mid. lat.

∴ Log. dept.	=	2·2552214
Log. sec. 39°	=	10·1094974
Log. 231·59	=	2·3647188
Hence dif. long.	=	3° 52′ W.
Long. left	=	32° 16′ W.
∴ Longitude in	=	36° 8′ W.

BY INSPECTION.

RULE.—Seek in the second Traverse Table till the distance, 300, and the difference of latitude, 240, be found in their proper columns, then in the Departure column will be found 180·5, and at the top of the page the course, 37°; this is very nearly correct, since the difference of latitude, 239·6, found in the table, only differs from the true by ·4.

Take now the departure, 180·5, as a difference of latitude, and with the middle latitude as a course, and opposite to 180·3, the nearest found in the table, stands 232, the difference of longitude sought.

BY GUNTER'S SCALE.

1. The extent from 300 to 240, on the line of numbers, will reach from 90° to the complement of the course, 53° 8′, on the line of sines.

2. The extent from 90° to the course, 36° 52′, on the line of sines, will reach from 300, the distance, to 180, the departure, on the line of numbers.

3. The extent from the complement of the middle latitude, 51°, to 90° on the line of sines, will reach from the departure,

180, to the difference of longitude, 231·6, on the line of numbers.

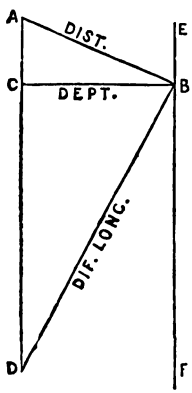
Problem XXIV.

219. Given one latitude, the course, and the departure; to find the difference of latitude, the distance, and the difference of longitude.

Example.

A ship from latitude 50° 10′ S. and longitude 30° E., sails ESE. until her departure is 957 miles; required her distance sailed, and the latitude and longitude in.

BY CONSTRUCTION.



Draw the meridian ACD, and parallel to it, at the distance 957 = the departure, draw the line EF; make the angle CAB = the course, 6 points, and produce the line AB to meet EF in B; draw BC perpendicular to ACD; then AB = the distance sailed is equal to 1036 miles, AC = the difference of latitude made is 396·4 miles. We now obtain the middle latitude, 53° 28′; hence at the point B make the angle CBD = 53° 28′, the middle latitude, and BD will be found to measure 1608, the difference of longitude.

BY CALCULATION.

To find the Difference of Latitude.

By Art. (125) $\frac{\text{dif. lat.}}{\text{dept.}}$ = cot. course.

Hence dif. lat. = dept. × cot. 6 pts.

∴ Log. dept. = 957	=	2·9809119
Log. cot. 6 pts.	=	9·6172243
Log. 396·4	=	2·5981362

To find the Distance.

By Art. (127) $\frac{\text{dist.}}{\text{dept.}}$ = cosec. course.

Hence dist. = dept. × cosec. 6 pts.

∴ Log. 957	=	2·9809119
Log. cosec. 6 pts.	=	10·0343847
Log. 1035·8	=	3·0152966

∴ Dist. = 1036 miles nearly.

∴ Dif. lat. = 396·4	=	6° 36′ S.
Latitude from	=	50° 10′ S.
Latitude in	=	56° 46′ S.
Sum of latitudes	=	106° 56′
Mid. latitude	=	53° 28′

To find the Difference of Longitude.

By Art. (211, F)

Dif. long. = dept. × sec. mid. lat.

∴ Log. 957	=	2·9809119
Log. sec. 53° 28′	=	·2252712
Log. 1607·6	=	3·2061831

∴ Dif. longitude = 1608	=	26° 48′ E.
Longitude left	=	30° 0′ E.
Longitude in	=	56° 48′ E.

BY INSPECTION.

RULE.—Find the course among the points or degrees at the top or bottom of the first or second Traverse Table, as the case may be, and the departure in its column, corresponding to which, in the columns of Distance and Difference of Latitude will be found the distance and difference of latitude respectively; then with the middle latitude as a course, seek the departure in the Difference of Latitude column, corresponding to which, in the Distance column, will be found the difference of longitude sought.

Thus I enter the first Traverse Table with 6 points as a course and one-tenth of the departure, 95·7, which is nearly midway between 95·2 and 96·1, and taking half the sum of the corresponding distances and difference of latitudes, gives 103·5 and 39·6; these being multiplied by 10, give the distance = 1035 miles, and the difference of latitude 396. Again, the middle latitude being nearly 53½°, I enter the second Traverse Table first with 53° as a course, and second with 54° as a course, and in both cases with 95·7, the tenth of the departure, as a difference of latitude; I find in the Distance column first 159, and second 163; half the sum of these is 161, which being multiplied by 10, gives 1610 miles for the difference of longitude.

BY GUNTER'S SCALE.

1. The extent from the course, 6 points, to its complement, 2 points, on the line of sines, will reach from the departure, 957, back to 396·4 on the line of numbers.

2. The extent from 6 points to 8 points on the line marked S.R., will reach from the departure, 957, up to the distance, 1036, on the line of numbers.

3. The extent from the complement of the middle latitude, 36° 32′, to 90° on the line of sines, will reach from the departure, 957, to the difference of longitude, 1608, on the line of numbers.

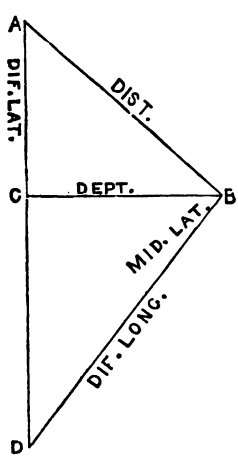
Problem XXV.

220. Given one latitude, distance sailed, and departure from the meridian; to find the course, the difference of latitude, and the difference of longitude.

Example.

A ship from latitude 49° 30′ N. and longitude 25° 0′ W., sails south-easterly 645 miles, and her departure from the meridian is then 500 miles; required the course steered, and the latitude and longitude the ship is in.

BY CONSTRUCTION.



Draw the horizontal line CB = 500, the departure; and perpendicular thereto, through C, draw the meridian line ACD; take an extent equal to the distance, 645 miles, in your compasses, and with one foot in B as a centre, describe an arc cutting AC in A, join AB; then will AC be the difference of latitude, 407·5 miles, and BAC the course, S. 50° 49′ E., hence we have the latitude in, and hence the middle latitude; make the angle CBD equal to the middle latitude, and produce BD to meet ACD in D; then BD will be the difference of longitude, 721·1 miles.

BY CALCULATION.

To find the Course.

By Art. (122) sin. course $= \frac{\text{dept.}}{\text{dist.}}$

Hence log. 500 + 10	=	12·6989700
− Log. 645	=	2·8095597
Log. sin. 50° 49′	=	9·8894103

∴ The course is S. 50° 49′ E.

To find the Difference of Latitude.

By Art. (123) $\frac{\text{dif. lat.}}{\text{dist.}} = \text{cos. course.}$

∴ Dif. lat. = dist. × cos. course

= 645 × cos. 50° 49′.

Hence log. 645	=	2·8095597
Log. cos. 50° 49′	=	9·8005823
Log. 407·52	=	2·6101420

Latitude left	=	49° 30′ N.
Dif. lat. 408	=	6° 48′ S.
Latitude in	=	42° 42′ N.
Sum of latitudes	=	92° 12′
Middle latitude	=	46° 6′

To find the Difference of Longitude.

By (211, F)

Dif. long. = dept. × sec. mid. lat.

= 500 × sec. 46° 6′.

But log. 500	=	2·6989700
Log. sec. 46° 6′	=	10·1590150
Log. 721·1	=	2·8579850
Longitude left	=	25° 0′ W.
Dif. of long. 721	=	12° 1′ E.
Longitude in	=	12° 59′ W.

BY INSPECTION.

RULE.—Find the course by seeking in the second Traverse Table, till against the distance, in its column, is found the given departure in one of the following columns adjoining to which in the other column will be found the difference of latitude, which, if greater than the departure, the course will be found at the top, but if less, the course will be found at the bottom.

Then take the middle latitude as a course, and find the departure in the column of Dif. Lat., against which, in the *Distance* column, will be found the difference of longitude.

Thus, one-third of the distance, 215, and one-third of the departure, 166·7, are found nearly to correspond to a course of 51°, and a difference of latitude of 135·3, which, multiplied by 3, gives the difference of latitude 406 nearly; then with the middle latitude, 46°, as a course, I enter the table and seek for one-fifth of the departure = 100, in the Difference of Latitude column, the distance corresponding, 144, being multiplied by 5, gives 720, the difference of longitude nearly.

BY GUNTER'S SCALE.

1. The extent from the distance, 645, to the departure, 500, on the line of numbers, will reach from 90° to the course 50° 49′, on the line of sines.

2. The extent from 90° to 39° 11′, the complement of the course, on the line of sines, will reach from the distance, 645, to the difference of latitude, 407·5, on the line of numbers.

3. The extent from the complement of the middle latitude, 43° 54′, to 90° on the line of sines, will reach from the departure, 500, up to the difference of longitude, 721·1, on the line of numbers.

MERCATOR'S SAILING.

221. The calculations by Middle-latitude Sailing, though sufficiently exact for a short run, are liable to great errors in calculating the situations of places differing greatly in latitude and longitude, particularly in high latitudes; to remedy this inconveniency, a chart was published in 1566, by Gerard Mercator, a Flemish geographer, in which all the meridians are parallel to each other, but proportionally lengthened so as to conform to the spherical figure of the earth. The principles on which this chart is constructed were first explained in the year 1599, by Edward Wright, an Englishman, and are as follows:

222. In Art. (201, B) it was shewn that dif. long. = mer. dist. × sec. lat.; hence if the meridians are supposed to be parallel to one another, the meridian distances will be all equal, and the dif. long. becomes Mercator's meridional parts: if therefore the first meridional distance be taken 1′, its corresponding meridional parts will be the natural secant of 1′; in the same way the meridional parts to be added for the second mile will be the natural secant of 2′, that for the third mile will be the natural secant of 3′, &c. Therefore the length of the expanded arc of

the meridian may be found by a continued addition of natural secants to every minute of the quadrant; nearly in this way has been calculated the Table of Meridional Parts (page 298), by means of which Mercator's charts may be constructed, and all the cases of Mercator's sailing calculated.

223. In using the Table of Meridional Parts, the degrees are to be found at the top of the page, and the minutes or miles at either side; under the degrees and opposite the minutes will be found the expanded length of the meridian, measured from the equator, usually called *Meridional Parts.*

To find the expanded arc intercepted between two given parallels, or, as it is generally called, the *meridional difference of latitude.* Take out the meridional parts opposite to each in the table, then, if the places be both on the *same side* of the equator, their *difference* is the *meridional difference of latitude;* but, if the places be on *opposite sides* of the equator, their sum is the *meridional difference of latitude* required.

Example I.

Required the meridional parts corresponding to the latitude 47° 18′. Look at the top of the page till you find 47°, marked $\left\{\begin{matrix}47\\ \text{Deg.}\end{matrix}\right.$ and opposite to 18′, and under 47°, will be found 3229·2, the meridional parts required.

Example II.

Find the meridional difference of latitude between Cape Cod, in latitude of 42° 5′ N., and the island of St Mary, in the latitude of 36° 59′ N.

Cape Cod's lat.	=	42° 5′ N.	Mer. parts	=	2788·5
St Mary's lat.	=	36° 59′ N.	Mer. parts	=	2391·4
		Meridional difference of latitude		=	397·1 miles.

Example III.

Required the meridional difference of latitude between a place in latitude 16° 37′ N. and another in latitude 24° 17′ S.

First latitude	=	16° 37′ N.	Mer. parts	=	1011·3
Second latitude	=	24° 17′ S.	Mer. parts	=	1502·7
		Meridional difference of latitude		=	2514·0 miles.

224. From the above it is evident that the difference of longitude has the same ratio to the meridional difference of latitude that the departure has to the real difference of latitude; that is,

$$\frac{\text{dif. long.}}{\text{mer. dif. lat.}} = \frac{\text{dept.}}{\text{dif. lat.}}$$

Therefore dif. long. $= \text{dept.} \times \frac{\text{mer. dif. lat.}}{\text{dif. lat.}} = \text{dept.} \times \text{sec. mid. lat.}$ (Art. 211, F). Wherefore sec. mid. lat. $= \frac{\text{mer. dif. lat.}}{\text{dif. lat.}}$; the middle latitude calculated from this formula is that which would give correct results.

To illustrate this, let us suppose that the latitude sailed from was 42° 16′ N., and that arrived at 53° 44′ N., the latitude midway between them, which, in Middle-latitude Sailing, we took for the middle latitude, half their sum = 48°. Calculating now the middle latitude which would give correct results, and which we shall therefore call the *true* middle latitude, by the above formula,

Latitude from	=	42° 16′ N.	Mer. parts	=	2803·3
Latitude in	=	53° 44′ N.	Mer. parts	=	3837·5
Dif. lat. 688	=	11° 28′ N.	Mer. dif. lat.	=	1034·2

$$\therefore \text{sec. (true mid. lat.)} = \frac{1034{\cdot}2}{688} = 1{\cdot}5031977 = \text{sec. } 48° 18′$$

∴ The true middle latitude is 48° 18′ instead of 48°; hence the results found by the former Rule are too small.

225. By the above formula has been calculated the following table for correcting the middle latitude so as to obtain correct results by middle-latitude sailing. The quantities in the table are always to be added to the middle latitude, as found by the rule, to obtain the *true* middle latitude. The readings along the top are the differences between the latitude sailed from and that arrived at; and those down the side of the table are the half sums of the two latitudes.

WORKMAN'S TABLE.

A Table of Corrections to be added to the Mean Middle Latitude to find the True Middle Latitude.

Mid. Lat. Deg.	DIFFERENCE OF LATITUDE.*																		
	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	
15	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 19	0 23	0 27	0 31	0 35	0 40	0 45	0 51	0 58	1 06	1 14	15
16	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 18	0 22	0 26	0 30	0 34	0 38	0 43	0 49	0 56	1 03	1 11	16
17	0 02	0 03	0 04	0 06	0 08	0 11	0 14	0 17	0 21	0 25	0 28	0 32	0 37	0 42	0 48	0 54	1 01	1 08	17
18	0 02	0 03	0 04	0 06	0 08	0 11	0 14	0 17	0 20	0 24	0 27	0 31	0 36	0 41	0 46	0 52	0 58	1 06	18
19	0 02	0 03	0 04	0 06	0 07	0 10	0 13	0 16	0 19	0 23	0 26	0 30	0 34	0 40	0 45	0 50	0 56	1 03	19
20	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 18	0 22	0 25	0 29	0 33	0 38	0 43	0 48	0 54	1 00	20
21	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 18	0 21	0 25	0 29	0 33	0 37	0 42	0 47	0 53	0 58	21
22	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 17	0 20	0 24	0 28	0 32	0 36	0 41	0 46	0 51	0 56	22
23	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 17	0 20	0 24	0 28	0 32	0 36	0 40	0 45	0 50	0 55	23
24	0 02	0 03	0 04	0 06	0 07	0 09	0 11	0 14	0 16	0 19	0 23	0 27	0 31	0 35	0 39	0 44	0 48	0 53	24
25	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 19	0 23	0 27	0 31	0 35	0 39	0 43	0 47	0 52	25
26	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 19	0 22	0 26	0 30	0 34	0 38	0 42	0 47	0 52	26
27	0 02	0 03	0 04	0 05	0 07	0 08	0 11	0 14	0 16	0 19	0 22	0 26	0 30	0 33	0 38	0 42	0 46	0 51	27
28	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 25	0 29	0 33	0 37	0 41	0 46	0 51	28
29	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 25	0 29	0 32	0 36	0 41	0 45	0 50	29
30	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 25	0 28	0 32	0 36	0 41	0 45	0 50	30
31	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 25	0 28	0 32	0 36	0 41	0 45	0 50	31
32	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 25	0 28	0 31	0 36	0 41	0 45	0 50	32
33	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49	33
34	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49	34
35	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49	35
36	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49	36
37	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49	37
38	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49	38
39	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 21	0 25	0 28	0 32	0 36	0 41	0 45	0 50	39
40	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 22	0 25	0 28	0 32	0 36	0 41	0 45	0 50	40
41	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 22	0 25	0 28	0 32	0 37	0 41	0 45	0 50	41
42	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15	0 18	0 22	0 26	0 29	0 33	0 37	0 42	0 46	0 51	42
43	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 19	0 23	0 26	0 30	0 34	0 38	0 42	0 46	0 51	43
44	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 19	0 23	0 27	0 30	0 34	0 38	0 43	0 47	0 52	44
45	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 19	0 23	0 27	0 31	0 35	0 39	0 43	0 47	0 52	45
46	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 19	0 23	0 27	0 31	0 35	0 39	0 44	0 48	0 53	46
47	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 20	0 23	0 27	0 31	0 35	0 40	0 44	0 49	0 54	47
48	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16	0 20	0 23	0 27	0 31	0 35	0 40	0 45	0 50	0 55	48
49	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 17	0 21	0 24	0 28	0 32	0 36	0 41	0 45	0 51	0 57	49
50	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 17	0 21	0 24	0 28	0 32	0 36	0 41	0 46	0 52	0 58	50
51	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 17	0 21	0 24	0 28	0 32	0 37	0 42	0 47	0 53	0 59	51
52	0 02	0 03	0 04	0 05	0 07	0 09	0 12	0 15	0 18	0 22	0 25	0 29	0 33	0 37	0 42	0 48	0 54	1 00	52
53	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 18	0 22	0 25	0 29	0 33	0 38	0 43	0 49	0 55	1 01	53
54	0 02	0 03	0 04	0 06	0 08	0 10	0 13	0 16	0 19	0 23	0 26	0 30	0 34	0 39	0 44	0 50	0 56	1 02	54
55	0 02	0 03	0 04	0 06	0 08	0 10	0 13	0 16	0 19	0 23	0 26	0 30	0 35	0 40	0 45	0 51	0 57	1 03	55
56	0 02	0 03	0 04	0 06	0 08	0 10	0 13	0 16	0 20	0 24	0 27	0 31	0 36	0 41	0 46	0 52	0 58	1 04	56
57	0 02	0 03	0 04	0 06	0 08	0 11	0 14	0 17	0 20	0 24	0 28	0 32	0 37	0 42	0 48	0 54	1 00	1 06	57
58	0 02	0 03	0 04	0 06	0 09	0 11	0 14	0 17	0 21	0 25	0 29	0 33	0 38	0 44	0 50	0 55	1 02	1 08	58
59	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 18	0 22	0 26	0 30	0 34	0 39	0 45	0 51	0 57	1 04	1 10	59
60	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 19	0 23	0 27	0 31	0 35	0 40	0 46	0 52	0 59	1 06	1 13	60
61	0 02	0 03	0 05	0 07	0 09	0 12	0 15	0 19	0 23	0 27	0 31	0 36	0 41	0 47	0 54	1 01	1 08	1 15	61
62	0 02	0 03	0 05	0 07	0 09	0 12	0 16	0 20	0 24	0 28	0 32	0 37	0 42	0 49	0 56	1 03	1 10	1 18	62
63	0 02	0 04	0 05	0 07	0 09	0 13	0 16	0 20	0 24	0 29	0 33	0 39	0 44	0 51	0 58	1 05	1 12	1 21	63
64	0 02	0 04	0 06	0 08	0 09	0 13	0 17	0 21	0 25	0 29	0 34	0 40	0 46	0 53	1 00	1 07	1 14	1 24	64

* When the difference *of latitude* is under 2°, the correction may be neglected; when it is 2° *and under* 3°, *add a correction of* 0° 01′.

Problem XXVI.

226. Given the latitudes and longitudes of two places; to find the direct course and distance between them.

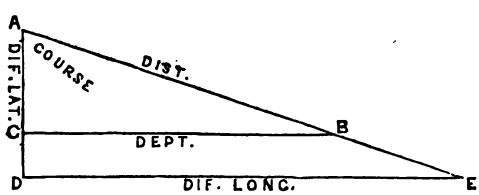
Example.

Required the bearing and distance from Cape Cod Light-house, in latitude 42° 5′ N. and longitude 70° 4′ W., to the island of St Mary, one of the Western Islands, in latitude 36° 59′ N., and longitude 25° 10′ W.

Cape Cod's latitude = 42° 5′ N.	Mer. parts. = 2788·5	Long. = 70° 4′ W.
St Mary's latitude = 36° 59′ N.	Mer. parts. = 2391·4	Long. = 25° 10′ W.
5° 6′	Mer. dif. lat. = 397·1	44° 54′
60		60
Difference of latitude = 306		Dif. long. = 2694

BY CONSTRUCTION.

Draw the meridian AD = 397·1, the mer. dif. lat., and set off also AC = 306, the real dif. lat.; draw DE perpendicular to AD, and make DE = 2694, the difference of longitude, join AE, and through C draw CB parallel to DE; then will CAB be the course = S. 81° 37′ E., and AB the distance, 2099 miles.



BY CALCULATION.

To find the Course.

$$\frac{DE}{AD} = \frac{\text{dif. long.}}{\text{mer. dif. lat.}} = \text{tan. course.}$$

∴ Log. 2694 + 10	=	13·4303976
Log. 397·1	=	2·5988999
Log. tan. 81° 37′	=	10·8314977

∴ Course = S. 81° 37′ E.

To find the Distance.

$$\frac{\text{dist.}}{\text{dif. lat.}} = \text{sec. course.}$$

∴ Dist. = 306 × sec. 81° 37′.

Log. 306	=	2·4857214
Log. sec. 81° 37′	=	10·8362566
Log. 2098·8	=	3·3219780

∴ The distance = 2099 miles nearly.

BY INSPECTION.

RULE.—With the meridional difference of latitude used as a dif. lat., and the difference of longitude used as a departure, enter the Traverse Tables until these numbers, or corresponding parts of them, are found together in their proper columns, the course will then be found at the top or bottom of the page; with this course and the true difference of latitude, find the corresponding distance.

Thus one-tenth of the mer. dif. lat. and dif. long. are found to agree nearly to a course of $7\frac{1}{4}$ points; this course and one-tenth of the proper difference of latitude, 30·6, is found to correspond to the distance 209; this being multiplied by 10, gives the distance 2090, differing a little from that by logarithms, owing to the neglect of about 3′ in the course.

BY GUNTER'S SCALE.

1. Extend from the mer. dif. lat. = 397 to the dif. long. = 2694 on the line of numbers; that extent will reach from radius or 45° to the course, 81° 37′, on the line of tangents.

2. Extend from the complement of the course, 8° 23′, to radius 90° on the line of sines, that extent will reach from the proper difference of latitude, 306, to the distance, 2099 miles, on the line of numbers.

Problem XXVII.

227. Given both latitudes and the departure; to find the course, distance, and difference of longitude.

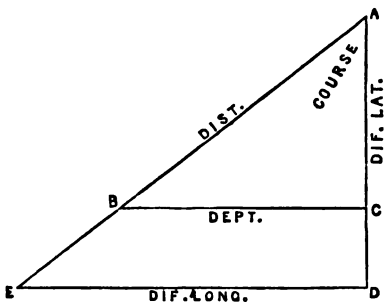
Example.

A ship from latitude 49° 57′ N. and longitude 15° 16′ W. sails south-westerly until her departure is 789 miles, and then by observation she has arrived in latitude 39° 20′ N.; required her course sailed, distance and longitude in.

Lat. left	=	49° 57′ N.	Mer. parts	=	3469·8
Lat. in	=	39° 20′ N.	Mer. parts	=	2570·7
		10° 37′ = 637 m.	Mer. dif. lat.	=	899·1

BY CONSTRUCTION.

Draw the meridian line ACD, make AD = 899·1, the mer. dif. lat., and AC = 637, the proper difference of latitude; from C draw BC perpendicular to AC, and make it = 789, the departure; join AB, and through D draw DE parallel to BC, and produce AB to meet it in E; then the angle BAC will be the course, S. 51° 5′ W., AB will be the distance sailed, 1014 miles, and DE will be the difference of longitude = 1114 miles.



BY CALCULATION.

To find the Course.

By Art. (124) tan. course $= \frac{BC}{AC}$

∴ Log. (BC = 789) + 10	=	12·8970770
Log. (AC = 637)	=	2·8041394
Log. tan. 51° 5′	=	10·0929376

∴ The course is S. 51° 5′ W.

To find the Dif. Long.

By Art. (124) $\frac{DE}{AD}$ = tan. course.

Hence DE = 899·1 × tan. 51° 5′.

∴ Log. 899·1	=	2·9538080
Log. tan. 51° 5′	=	10·0929376
Log. 1113·7	=	3·0467456

∴ Dif. long. = 1114 miles nearly.

To find the Distance.

By Art. (126) $\frac{AB}{AC}$ = sec. course.

Hence AB = 637 × sec. 51° 5′.

∴ Log. 637	=	2·8041394
Log. sec. 51° 5′	=	10·2019094
Log. 1014	=	3·0060488

∴ The distance = 1014 miles.

Longitude left	=	15° 16′ W.
Dif. long. 1114	=	18° 34′ W.
Longitude in	=	33° 50′ W.

The dif. long. in this case may be found without either finding the course or the distance, for it is plain that

AC : CB :: AD : DE

$$\therefore DE = \frac{\text{dept.} \times \text{mer. dif. lat.}}{\text{dif. lat.}} = \frac{789 \times 899{\cdot}1}{637}$$

BY INSPECTION.

RULE.—Seek in the Traverse Tables with the proper difference of latitude and departure till they are found to agree in their respective columns, corresponding to which will be the distance in its column, and the course at the top or bottom of the page, according as the difference of latitude is greater or less than the departure; with the same course, and the meridian difference of latitude as a dif. lat., will be found in the Departure column the difference of longitude.

Thus, with one-tenth of the true difference of latitude and departure, 63·7 and 78·9, I find the course, 51°, and the distance, 101, which, multiplied by 10, gives 1010, the true distance nearly; in the same table, opposite to one-tenth of the meridian difference of latitude, 89·9, I find in the departure 111·1, which, being multiplied by 10, gives 1111 miles for the difference of longitude.

BY GUNTER'S SCALE.

1. The extent from the dif. lat., 637, to the departure, 789, on the line of numbers, will reach from radius 45° to the course 51° 5′, on the line of tangents.

2. The extent from 51° 5′ to 90°, on the line of sines, will reach from the departure, 789, to the distance, 1014, on the line of numbers.

3. The extent from 45° to the course, 51° 5′, on the line of tangents, will reach from the mer. dif. lat., 899, to the difference of longitude, 1114, on the line of numbers.

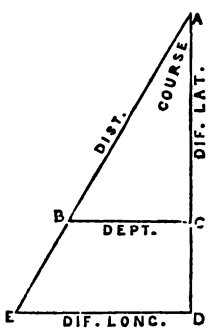
Problem XXVIII.

228. Given one latitude, the course, and distance; to find the difference of latitude and the difference of longitude.

Example.

A ship from latitude 42° 30′ N. and longitude 58° 51′ W. sails SW. by S. 591 miles; required the latitude and longitude in.

BY CONSTRUCTION.



Draw the meridian ACD and ABE, making an angle with it equal to the course, 3 points; make AB = 591 miles, the distance sailed, and from B let fall BC perpendicular to ACD; then will BC be the departure, and AC will be the difference of latitude, 491 miles. Hence we have both latitudes, and can thence find the mer. dif. lat., equal to which make AD, and through D draw DE parallel to CB, meeting AB produced in E, and DE will be the difference of longitude, 419·6 miles.

BY CALCULATION.

To find the Dif. of Latitude.

By Art. (123) dif. lat. = dist. × cos. 3 pts.

Log. 591	=	2·7715875
Log. cos. 3 pts.	=	9·9198464
Log. 491·4	=	2·6914339

Lat. left = 42° 30′ N. Mer. pts. = 2822·3
Dif. lat. 491 = 8° 11′ S.
Lat. in = 34° 19′ N. Mer. pts. = 2194·5
Mer. dif. lat. = 627·8

To find the Dif. of Longitude.

By Art. (124) $\frac{ED}{AD}$ = tan. 3 pts.

Hence ED = 627·8 × tan. 3 pts.

Log. 627·8	=	2·7978213
Log. tan. 3 pts.	=	9·8248926
Log. 419·5	=	2·6227139
Long. left	=	58° 51′ W.
Dif. long. 419·5	=	7° 0′ W.
Long. in	=	65° 51′ W.

BY INSPECTION.

RULE.—Find the course at the top or bottom of the page,

either among the points or degrees, and in that page opposite the distance will be found, in their proper columns, the difference of latitude and departure. Then in the same page find the meridional difference of latitude in the Dif. Lat. column, and against it, in the Departure column, will be found the difference of longitude.

Thus, under the course SW. by S., or 3 points, and opposite one-third of the distance, 197, stands 163·8 in the Latitude column, which, multiplied by 3, gives the difference of latitude, 491·4 miles; then find one-fourth of the meridional difference of latitude, 157 miles, in the Latitude column, against which stands 105 in the Departure column, which, multiplied by 4, gives 420, the difference of longitude.

BY GUNTER'S SCALE.

1. The extent from 8 points to the complement of the course, 5 points, on the line marked S.R., will reach from the distance, 591, to the difference of latitude, 491·4, on the line of numbers.

2. The extent from the radius, 4 points, to the course, 3 points, on the line marked T.R., will reach from the meridional difference of latitude, 628, to the difference of longitude, 419·6, on the line of numbers.

Problem XXIX.

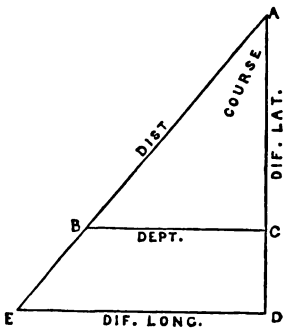
229. Given both latitudes and the course; to find the distance and difference of longitude.

Example.

A ship from latitude 49° 57′ N. and longitude 30° W. sails S. 39° W. till she arrives in latitude 45° 31′ N.; required the distance run and the longitude in.

Lat. left	=	49° 57′ N.	Mer. parts	=	3469·8
Lat. in	=	45° 31′ N.	Mer. parts	=	3074·0
Dif. lat	=	4° 26′ = 266 m.	Mer. dif. lat.	=	395·8

BY CONSTRUCTION.



Draw the meridian ACD, on which take AC, equal the proper difference of latitude, 266 miles, and AD, equal the meridional difference of latitude, 396 miles; make the angle BAC = the course 39°, and draw CB and BE perpendicular to AD, and meeting ABE in B and E; then will AB be the distance, 342 miles, and DE the difference of longitude, 321 miles.

BY CALCULATION.

To find the Distance.			To find the Dif. Longitude.		
Log. AC = 266	=	2·42488816	Log. AD = 395·8	=	2·5974758
Log. sec. 39°	=	10·1094974	Log. tan. 39°	=	9·9083692
Log. 342·3	=	2·5343790	Log. 320·51	=	2·5058450

Longitude left	=	30° 0′ W.
Dif. long. 321	=	5° 21′ W.
Longitude in	=	35° 21′ W.

BY INSPECTION.

RULE.—Find the course at the top or bottom of the Traverse Table, and the proper difference of latitude in its proper column, adjoining to which will be found, in their proper columns, the distance and departure; then in the same page find the mer. dif. lat. in the Latitude column, adjoining to which, in the Departure column, will be found the difference of longitude.

Thus, under the course 39°, and opposite the half dif. lat., 133 (the nearest to which is 132·9), stand 171 and 107·6, these doubled give the distance, 342, and departure, 215·2 miles; and in the same page, opposite the half mer. dif. lat., 198, found in the Latitude column, stands 160·5, in the Dept. column, which being doubled, gives the difference of longitude, 321 miles.

BY GUNTER'S SCALE.

1. The extent from the complement of the course, 51°, to the radius, 90°, on the line of sines, will reach from the proper difference of latitude, 266, to the distance, 342·3, on the line of numbers.

2. The extent from radius 45° to the course, 39°, on the line of tangents, will reach from the meridional difference of latitude, 396, back to the difference of longitude, 321, on the line of numbers.

Problem XXX.

230. Given both latitudes and the distance; to find the course and the difference of longitude.

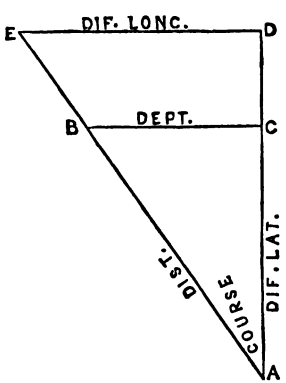
Example.

A ship from latitude 37° N. and longitude 32° 16′ W. sails 300 miles north-westerly, until she is in latitude 41° N.; required the course steered and the longitude in.

Lat. left	=	37° N.	Mer. parts	=	2392·7
Lat. in	=	41° N.	Mer. parts	=	2701·6
Dif. lat.	=	4° = 240 m.	Mer. dif. lat.	=	308·9

BY CONSTRUCTION.

Draw the meridian ACD; make AC equal to the proper difference of latitude, 240 miles, and AD equal to the meridional difference of latitude, 309 miles; draw CB and DE perpendicular to ACD, with an extent equal to 300, the distance, and one foot in A, cut with the other CB in B; join AB, and produce it to meet DE in E, and it is done; for the angle BAC is equal to the course, 36° 52′, BC is the departure, and DE is the difference of longitude, 231·7 miles.



BY CALCULATION.

To find the Course.		To find the Dif. of Longitude.	
Log. (AC = 240) + 10 = 12·3802112		Log. (mer. dif. = 308·9) =	2·4898179
Log. (AB = 300) = 2·4771213		Log. tan. 36° 52′ =	9·8750102
Log. cos. 36° 52′ = 9·9030899		Log. 231·65 =	2·3648281
Longitude left =	32° 16′ W.		
Dif. of long. 232 =	3° 52′ W.		
Longitude in =	36° 8′ W.		

BY INSPECTION.

RULE.—Seek in the Tables till against the distance taken in its column is found the given difference of latitude in one of the following columns; adjoining to it will be found the departure, which if less than the difference of latitude, the course will be found at the top; but if greater, the course will be found at the bottom. In the same page, find the meridional difference of latitude in the Latitude column, adjoining to which, in the Departure column, will be found the difference of longitude.

Thus, the distance, 300, and the difference of latitude, 240, are found to correspond nearly to a course of 37°, and a departure of 180·5; and in the Latitude column, opposite half the meridional difference of latitude, 154·5 (the nearest to which is 154·1), stands 116·2, in the Departure column, which doubled, gives the difference of longitude, 232·4.

BY GUNTER'S SCALE.

1. The extent from the distance, = 300, to the proper dif. lat., = 240, on the line of numbers, will reach from radius or 90° to 53° 8′, the complement of the course, on the line of sines.

2. The extent from radius 45° to the course, 36° 52′, on the line of tangents, will reach from the meridional difference of latitude, = 309, to the difference of longitude, 231·7, on the line of numbers.

Problem XXXI.

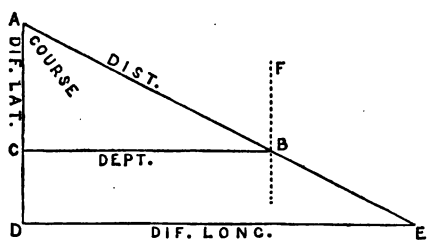
231. Given one latitude, the course, and the departure; to find the distance, difference of latitude, and difference of longitude.

Example.

A ship from latitude 50° 10′ S. and longitude 30° E. sails ESE. until her departure is 957 miles; required the distance sailed, and the latitude and longitude in.

BY CONSTRUCTION.

Draw the meridian ACD, and at a distance from it equal to the departure, 957 miles, draw the line FB parallel to ACD; make the angle CAB = 6 points, the course; draw AB to cut BF in B, draw BC perpendicular to ACD; then will AB be the distance, 1036 miles; AC, the difference of latitude, 396 miles; hence we have both latitudes, and thence the meridional difference of latitude, 667 miles; make the line AD = 396, and through D draw DE perpendicular to AD, meeting AB produced in E; then will DE be the difference of longitude, 1610 miles.



BY CALCULATION.

To find the Distance.

By Art. (127) $\frac{AB}{BC}$ = cosec. course.

∴ AB = 957 × cosec. 6 pts.

Log. 957	=	2·9809119
Log. cosec. 6 pts.	=	10·0343847
Log. 1036	=	3·0152966

∴ The distance = 1036 miles.

To find the Dif. of Longitude.

By Art. (124) dif. long. = mer. dif. lat. × tan. course.

∴ Log. 666·9	=	2·8240607
Log. tan. 6 pts.	=	10·3827757
Log. 1610·04	=	3·2068364

∴ Dif. long. = 1610 = 26° 50′ E.

Longitude left = 30° 0′ E.

Longitude in = 56° 50′ E.

To find the Dif. of Latitude.

By Art. (125) $\frac{AC}{BC}$ = cot. course.

∴ AC = 957 × cot. 6 pts.

Log. 957	=	2·9809119
Log. cot. 6 pts.	=	9·6172243
Log. 396·4	=	2·5981362

Lat. left = 50° 10′ S. Mer. pts. = 3490·1

Dif. lat. 396 = 6° 36′ S.

Lat. in = 56° 46′ S. Mer. pts. = 4157·0

Mer. dif. lat. = 666·9

BY INSPECTION.

RULE.—Find the course and the departure in either of the Traverse Tables, and adjacent to the departure, in their proper columns, will be found the distance and difference of latitude. The two extreme latitudes being now known, the meridional difference of latitude may be found; with this meridional difference of latitude as a dif. of lat. enter the same page, and adjacent to it, in the Departure column, will be found the difference of longitude.

Thus, with the course ESE. or 6 points, and one-tenth of the departure, 95·7, enter the Table, and the nearest to this departure is 95·2 and 96·1; as 95·7 is nearly half their sum, we get the distance, 103·5, and the difference of latitude, 39·6; these being multiplied by 10, give the distance, 1035 miles, and the difference of latitude, 396 miles; then in the Latitude column find a tenth of the meridional difference of latitude, 66·7, the nearest to which in the Table is 66·6, which corresponds to 160·8; and as this is more than double of 66·6, we may conclude that 66·7 corresponds to 161·0, which, multiplied by 10, gives the difference of longitude, 1610 miles.

BY GUNTER'S SCALE.

1. The extent from the course, 6 points, to radius, 8 points, on the line marked S.R., will reach from 957, the departure, to 1036, the distance, on the line of numbers.

2. The extent from radius, 4 points, to the complement of the course, 2 points, on the line marked T.R., will reach from the departure, 957, to the difference of latitude, 396, on the line of numbers.

3. The extent from the radius, 4 points, to the course, 6 points, on the line marked T.R., will reach from the meridional difference of latitude, 66·7, to the difference of longitude, 1610, on the line of numbers.

Problem XXXII.

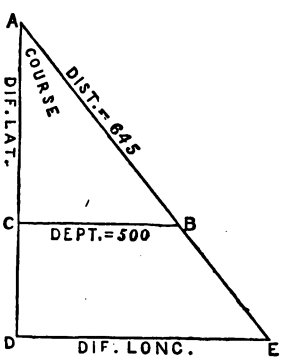
232. Given one latitude and longitude, the distance sailed, and the departure made; to find the course, the difference of latitude, and the difference of longitude.

Example.

A ship from latitude 49° 30′ N. and longitude 25° W., sails south-easterly 645 miles, making 500 miles of departure; required the course steered, and the latitude and longitude in.

BY CONSTRUCTION.

Draw the meridian ACD, and from any point C in it draw CB = 500, and perpendicular to AC; then with an extent in the compasses = 645, the distance, put one foot in B, and with the other cut AC in A, join AB; then will AC be the proper difference of latitude, 407·5 miles, and the angle BAC will be the course, 50° 49′; hence we have the other latitude, and the meridional difference of latitude, to which make AD equal; and draw DE parallel to CB, meeting AB produced in E; then will DE be the difference of longitude, 722·6 miles.



BY CALCULATION.

To find the Course.

By Art. (122) sin. course = $\frac{\text{dept.}}{\text{dist.}}$

∴ Log. 500 + 10	=	12·6989700
Log. 645	=	2·8095597
Log. sin. 50° 49′	=	9·8894103

∴ The course = S. 50° 49′ E.

To find the Dif. of Latitude.

By Art. (123) $\frac{\text{dif. lat.}}{\text{dist.}}$ = cos. course.

∴ Dif. lat. = dist. × cos. course.

Log. 645	=	2·8095597
Log. cos. 50° 49′	=	9·8008523
Log. 407·8	=	2·6104120

Dif. lat. 408 =	6° 48′ S.		
Lat. left	= 49° 30′ N.	Mer. pts. =	3428·1
Lat. in	= 42° 42′ N.	Mer. pts. =	2838·6
			589·5

To find the Dif. Longitude.

Log. 589·5	=	2·7704838
Log. tan. 50° 49′	=	10·0887913
Log. 723·22	=	2·8592751

∴ Dif. long. = 723	=	12° 3′ E.
Longitude left	=	25° 0′ W.
Hence longitude in	=	12° 57′ W.

BY INSPECTION.

RULE.—Seek in the Traverse Tables till the distance and departure are found to correspond in their respective columns, adjoining to which, in the column of Latitude, will be found the true difference of latitude; which, if greater than the departure, the *course* will be found at the top; but if less, the course will be found at the bottom. With this course seek the meridional difference of latitude in the Latitude column, adjoining to which, in the Departure column, will be found the difference of longitude.

Thus, one-third of the distance, 215, and one-third of the departure, 166·7, are found to correspond nearly to a course of 51°, and a difference of latitude of 135·3, which, multiplied by 3, gives the true difference of latitude, 406 miles nearly. Then one-fourth of the meridional difference of latitude, 147, in the Latitude column, is found nearly to correspond to the departure, 181·5; this multiplied by 4, gives 726, the difference of longitude nearly.

BY GUNTER'S SCALE.

1. The extent from the distance, 645, to the departure, 500, on the line of numbers, will reach from the radius, 90°, to the course, 50° 49′, on the line of sines.

2. The extent from the radius, 90°, to the complement of the course, 39° 11′, on the line of sines, will reach from the distance, 645, to the difference of latitude, 407·8, on the line of numbers.

3. The extent from the radius, 45°, to the course, 50° 49′, on the line of tangents, will reach from the meridional difference of

latitude, 589, to the difference of longitude, 722·6, on the line of numbers. Or the extent from the proper difference of latitude, 407·8, to the departure, 500, will reach from the meridional difference of latitude, 589, to the difference of longitude, 722·7, on the line of numbers.

Problem XXXIII.

233. Given the latitude and longitude sailed from, and the courses and distances since sailed; to find the latitude and longitude in, and the true course and distance to a third place, whose latitude and longitude are known.

Example.

Suppose a ship sails from latitude 53° 11′ N. and longitude 7° 15′ W., the following courses: W. by S. 34 miles; SSE. 49 miles; SW. by S. 58 miles; SSW. ½ W. 46 miles; W. by N. ½ N. 53 miles; SW. by S. 62 miles; required the latitude and longitude in, and the course and distance to a place in latitude 42° 47′ N. and longitude 18° 43′ W.

Latitude left	=	53° 11′ N.	Mer. parts	=	3782·1
Dif. lat. 177	=	2° 57′ S.			
Lat. in	=	50° 14′ N.	Mer. parts	=	3496·3
Sum of latitudes	=	103° 25′			
Middle latitude	=	51° 43′			
			Meridional dif. lat.	=	285·8

To find the Difference of Longitude made by Middle-latitude Sailing.

Dif. long. = dept. × sec. mid. lat. Art. (211, F).

Log. 153·5	=	2·1861084
Log. sec. 51° 43′	=	10·2079231
Log. 247·8	=	2·3940315

Hence by middle-latitude sailing the difference of longitude made is 248 miles = 4° 8′ W.

Courses.	Dist.	Dif. Lat. N.	Dif. Lat. S.	Departure. E.	Departure. W.
W. by S.	34	..	6·6	..	33·3
SSE.	49	..	45·3	18·8	..
SW. by S.	58	..	48·2	..	32·2
SSW. ½ W.	46	..	40·6	..	21·7
W. by N. ½ N.	53	15·4	..	..	50·7
SW. by S.	62	..	51·5	..	34·4
	..	..	192·2	18·8	172·3
	..	..	15·4	..	18·8
	..	..	176·8	..	153·5

By inspection, it is found that a difference of latitude 176·6 corresponds to a departure of 153·5, with the course 41°, and distance run 234 miles; again, with the course, 41°, and the meridional difference of latitude, 285·8, as a latitude, the half of which, 142·9, corresponds to a departure of 124·2, which doubled, gives 248·4, the difference of longitude sought.

The ship is therefore in latitude 50° 14′ N., having run a distance of 234 miles.

Longitude left	=	7° 15′ W.				
Dif. long. 248	=	4° 8′ W.				
Longitude in	=	11° 23′ W.				
Longitude to	=	18° 43′ W.				
Dif. of longitude	=	7° 20′ W.	=	440 miles.		
Latitude in	=	50° 14′ N.	Mer. parts	=	3496·3	
Latitude to	=	42° 47′ N.	Mer. parts	=	2845·4	
Dif. of lat.	=	7° 27′ = 447 m.	Mer. dif. lat.	=	650·9	
Sum of the lats.	=	93° 1′				
Middle latitude	=	46° 31′				
Correction (p. 153)	=	+ 7′				
True mid. lat.	=	46° 38′				

To find the Course by Middle-latitude Sailing.

$$\text{tan. course} = \frac{\text{dept.}}{\text{dif. lat.}} = \frac{\text{dif. long.} \times \text{cos. mid. lat.}}{\text{dif. lat.}}$$

Hence log. 440	=	2·6434527
Log. cos. 46° 38′	=	9·8367447
		12·4801974
Log. 447	=	2·6503075
Log. tan. 34° 3	=	9·8298899

To find the Course by Mercator's Sailing.

$$\text{Tan. course} = \frac{\text{dif. long.}}{\text{mer. dif. lat.}} = \frac{440}{650{\cdot}9}$$

Log. 440 + 10	=	12·6434527
Log. 650·9	=	2·8135143
Tan. 34° 3′	=	9·8299384

To find the Distance.

Dist. = dif. lat. × sec. course, Art. (126).

Log. 447	=	2·6503075
Log. sec. 34° 3′	=	10·0816817
Log. 539·5	=	2·7319892

∴ The distance = 539·5 miles.

The latitude and longitude in are therefore lat. 50° 14′ N. and long. 11° 23′ W., and the course to the place sought S. 34° 3′ W., and the distance to it is 539·5 miles.

In the same manner may the longitudes arrived at be

calculated in all the questions given in Traverse Sailing where the longitude sailed from is given. By correcting the middle latitude by Workman's Table we have obtained the same course by both methods; had this correction not been applied, the resulting courses, and consequently also the distances, would have been different; in this case the course would be found 34° 7′, and the distance 540.

Exercises in Middle-latitude and Mercator's Sailing, 26.

(298.) Required the bearing and distance from Cape Clear, in latitude 51° 25′ N. and longitude 9° 29′ W., to Funchal in Madeira, in latitude 32° 37′ N. and longitude 16° 58′ W.

(299.) Required the bearing and distance from Funchal, in latitude 32° 37′ N. and longitude 16° 58′ W., to Geer Cape, NW. coast of Africa, in latitude 30° 38′ N. and longitude 9° 52′ W.

(300.) Required the bearing and distance from Geer Cape, in latitude 30° 38′ N. and longitude 9° 52′ W., to a place in latitude 22° 15′ N. and longitude 20° 17′ W.

(301.) Required the bearing and distance from a place in latitude 22° 15′ N. and longitude 20° 17′ W., to Cape Formosa, W. coast of Africa, in latitude 4° 25′ N. and longitude 5° 59′ E.

(302.) A ship from latitude 51° 18′ N. and longitude 15° 12′ W. sails south-easterly till her departure is 473 miles, and is then found to be in latitude 45° 19′ N.; required her course, distance, and longitude in.

(303.) A ship from latitude 55° 58′ N. and longitude 3° 11′ W. sails north-easterly till her departure is 178 miles, and is then found by observation to be in latitude 57° 5′ N.; required her course, distance, and longitude in.

(304.) A ship from latitude 34° 11′ S. and longitude 5° 12′ E. sails between the north and west till her departure is 620 miles,

and is found by observation to be in latitude 30° 1′ S.; required her course, distance, and longitude in.

(305.) A ship from latitude 0° 12′ N. and longitude 15° 17′ W. sails north-westerly till her departure is 598 miles, and her latitude by observation 8° 19′ N.; required her course, distance, and longitude in.

(306.) A ship from Cape Formosa, W. coast of Africa, in latitude 4° 25′ N. and longitude 5° 59′ E., sails SW. ½ W. 495 miles; required the latitude and longitude in.

(307.) A ship from latitude 35° 17′ S. and longitude 31° 12′ E. sails NNW. 708 miles; required the latitude and longitude then in.

(308.) A ship from latitude 1° 12′ S. and longitude 15° 11′ W. sails SW. ½ S. 518 miles; required the latitude and longitude arrived at.

(309.) A ship from Cape Horn, in latitude 55° 58′ S. and longitude 67° 21′ W., sails NE. by N. 642 miles; required the latitude and longitude arrived at.

(310.) A ship from latitude 51° 8′ N. and longitude 9° 57′ W. sails N. 58° W. till she arrives in the latitude of 56° 3′ N.; required the distance run, the departure made, and the longitude in.

(311.) A ship from latitude 18° 57′ N. and longitude 5° 51′ E. sails N. 35° E. till she arrives in the latitude of 12° 14′ N.; required the distance run, the departure made, and the longitude in.

(312.) A ship from latitude 57° 4′ N. and longitude 7° 12′ W. sails SW. ½ S. till she arrives in latitude 54° 10′ N.; required the distance run, the departure made, and the longitude in.

(313.) A ship from latitude 40° 18′ S. and longitude 175° E. sails N. 56° W. till she arrives in latitude 32° 55′ S; required the distance run, the departure made, and the longitude in.

(314.) A ship from latitude 43° 47′ S. and longitude 15° 25′ E.

sails 420 miles north-easterly till she is found by observation to be in latitude 38° 18′ S.; required the course steered and the longitude in.

(315.) A ship from latitude 40° 43′ N. and 12° 17′ W. sails 509 miles between the north and west till she arrives in latitude 43° 40′ N.; required the course steered and the longitude arrived at.

(316.) A ship from latitude 50° 18′ N. and longitude 59° 23′ W. sails 365 miles between the south and east till she is found by observation to be in latitude 47° 13′ N.; required the course steered and the longitude arrived at.

(317.) A ship from New York, in latitude 40° 42′ N. and longitude 73° 59′ W., sails north-easterly 518 miles, and is then by observation in latitude 45° 7′ N.; required the course steered and the longitude arrived at.

(318.) A ship from Valparaiso, in latitude 33° 0′ S. and longitude 71° 38′ W., sails WNW. ½ N. until her departure is 460 miles; required the distance sailed and the latitude and longitude in.

(319.) A ship from Cape Verd, in latitude 14° 44′ N. and longitude 17° 30′ W., sails NW. by N. until her departure is 370 miles; required the distance sailed and the latitude and longitude in.

(320.) A ship from Cape Ortegal, in latitude 43° 47′ N. and longitude 7° 49′ W., sails N. 28° W. till her departure is 195 miles; required the distance run and the latitude and longitude in.

(321.) A ship from Funchal, Madeira, in latitude 32° 37′ N. and longitude 16° 58′ W., sails SSE. till her departure is 120 miles; required the distance run and the latitude and longitude in.

(322.) A ship from latitude 37° 29′ N. and longitude 64° 15′ W. sails north-easterly 480 miles, and makes a departure of 434 miles; required the course steered and the latitude and longitude in.

(323.) A ship from Cape Roca, in latitude 38° 46′ N. and longitude 9° 30′ W., sails north-westerly 524 miles, and makes a departure of 308 miles; required the course steered and the latitude and longitude in.

(324.) A ship from Cape of Good Hope, in latitude 34° 29′ S. and longitude 18° 23′ E., sails south-easterly 600 miles, and makes a departure of 318 miles; required the course steered and the latitude and longitude in.

(325.) A ship from Cape St Sebastian, in latitude 22° 5′ S. and longitude 35° 58′ E., sails between the north and east 564 miles, having made 462 miles of departure. Required the course steered, and the latitude and longitude in.

(326.) A ship bound for New York, in latitude 40° 42′ N. and longitude 73° 59′ W., sails from Cape Finisterre, in latitude 42° 53′ N. and longitude 9° 15′ W., the following courses—*1st*, W. by S. 64 miles; *2d*, WNW. ½ W. 84 miles; *3d*, W. ½ S. 50 miles; *4th*, SW. ½ S. 73 miles; *5th*, NW. by W. 85 miles; *6th*, W. ½ S. 65 miles; and *7th*, WNW. ½ W. 78 miles. Required the latitude and longitude in, and the direct course and distance to New York.

(327.) A ship from Cape Blanco, in latitude 47° 12′ S. and longitude 65° 44′ W., sails the following courses—*1st*, NE. by N. 60 miles; *2d*, ENE. 71 miles; *3d*, E. by N. 53 miles; *4th*, NE. by N. 85 miles; *5th*, N. by E. 84 miles; *6th*, NNW. 64 miles; *7th*, NNE. 103 miles; *8th*, NE. ¾ E. 96 miles. Required the latitude and longitude in, and the course to Cape Clear, in latitude 51° 26′ N. and longitude 9° 29′ W.

(328.) A ship from Cape of Good Hope, in latitude 34° 29′ S. and longitude 18° 23′ E., sails the following courses—*1st*, SW. by W. 47 miles; *2d*, SW. ½ S. 80 miles; *3d*, W. ½ S. 87 miles; *4th*, SW. by S. 140 miles; *5th*, SSW. ¾ W. 118 miles; *6th*, SE. 124 miles; *7th*, W. by N. 130 miles; *8th*, WSW. 140 miles. Required the latitude and longitude in, and the course and distance to Cape Horn, in latitude 55° 58′ S. and longitude 67° 21′ W.

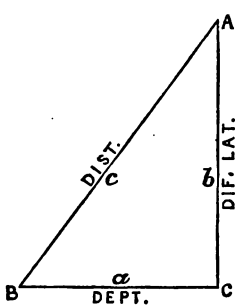
(329.) A ship from Sydney in Australia, latitude 33° 52′ S. and longitude 151° 14′ E., sails the following courses—*1st*, ESE. 72 miles; *2d*, E. by S. ½ S. 59 miles; *3d*, ESE. ¼ S. 64 miles; *4th*, SE. ¾ S. 80 miles; *5th*, E. by N. 43 miles; and *6th*, SE. by S. 71 miles. Required the latitude and longitude in, and the course and distance to Auckland in New Zealand, latitude 36° 50′ S. and longitude 174° 49′ E.

CHAPTER VII.

ELEMENTS OF TRIGONOMETRY CONTINUED.

234. In Articles (112–136) we investigated as many of the elementary principles of trigonometry as have been sufficient to solve all the problems in Plane, Parallel, Middle-latitude, and Mercator's Sailing; but there are other problems in trigonometry which it will be necessary to solve, in order to render the solutions of some problems that follow intelligible.

Resuming the definitions, and observing that the angle A in navigation = the course, a = the side opposite the angle A = the departure, b = the side adjacent to the angle A = the difference of latitude, and c = the hypotenuse = the distance sailed; also the definitions of the trigonometrical ratios contain each only two of the three letters, a, b, and c; if we first divide both numerator and denominator of the ratio by the letter which it does not contain, and then both numerator and denominator by the letter in its numerator, we will thus obtain two values of each of the trigonometrical ratios; thus,



(A) (B)

(122.) $$\text{Sin. A} = \frac{\text{side opposite } \angle \text{A}}{\text{hypotenuse}} = \frac{a}{c} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{\text{tan. A}}{\text{sec. A}} = \frac{1}{\frac{c}{a}} = \frac{1}{\text{cosec. A.}}$$

(123.) $$\text{Cos. A} = \frac{\text{side adjacent } \angle \text{A}}{\text{hypotenuse}} = \frac{b}{c} = \frac{\frac{b}{a}}{\frac{c}{a}} = \frac{\text{cot. A}}{\text{cosec. A}} = \frac{1}{\frac{c}{b}} = \frac{1}{\text{sec. A.}}$$

(124.) $$\text{Tan. A} = \frac{\text{side opposite } \angle \text{A}}{\text{side adjacent } \angle \text{A}} = \frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{\text{sin. A}}{\text{cos. A}} = \frac{1}{\frac{b}{a}} = \frac{1}{\text{cot. A.}}$$

(125.) $$\text{Cot. A} = \frac{\text{side adjacent } \angle \text{A}}{\text{side opposite } \angle \text{A}} = \frac{b}{a} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{\text{cos. A}}{\text{sin. A}} = \frac{1}{\frac{a}{b}} = \frac{1}{\text{tan. A.}}$$

(126.) $$\text{Sec. A} = \frac{\text{hypotenuse}}{\text{side adjacent } \angle \text{A}} = \frac{c}{b} = \frac{\frac{c}{a}}{\frac{b}{a}} = \frac{\text{cosec. A}}{\text{cot. A.}} = \frac{1}{\frac{b}{c}} = \frac{1}{\text{cos. A.}}$$

(127.) $$\text{Cosec. A} = \frac{\text{hypotenuse}}{\text{side opposite } \angle \text{A}} = \frac{c}{a} = \frac{\frac{c}{b}}{\frac{a}{b}} = \frac{\text{sec. A}}{\text{tan. A}} = \frac{1}{\frac{a}{c}} = \frac{1}{\text{sin. A.}}$$

We have thus in columns (A) and (B) obtained two values of each of the trigonometrical ratios.

235. Since by Euclid I. 47, $a^2 + b^2 = c^2$, dividing both sides by c^2, gives

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1, \text{ or } \text{sin.}^2 \text{A} + \text{cos.}^2 \text{A} = 1 \qquad (a)$$

hence also $\text{sin. A} = \sqrt{1 - \text{cos.}^2 \text{A}}$, and $\text{cos. A} = \sqrt{1 - \text{sin.}^2 \text{A}}$ (*b*)

Again, dividing both sides by b^2, gives

$$\frac{a^2}{b^2} + 1 = \frac{c^2}{b^2} \therefore 1 + \tan.^2 A = \sec.^2 A \qquad (c)$$

hence sec. $A = \sqrt{1 + \tan.^2 A}$, and tan. $A = \sqrt{\sec.^2 A - 1}$

Lastly, dividing both sides by a^2, gives

$$1 + \frac{b^2}{a^2} = \frac{c^2}{a^2} \therefore 1 + \cot.^2 A = \operatorname{cosec}.^2 A \qquad (d)$$

hence cosec. $A = \sqrt{1 + \cot.^2 A}$, and cot. $A = \sqrt{\operatorname{cosec}.^2 A - 1}$

236. From the value of sin. A in the last column of (122) above, namely—

$$\text{sin. } A = \frac{1}{\text{cosec. } A}, \text{ multiplying by cosec. A, we have}$$

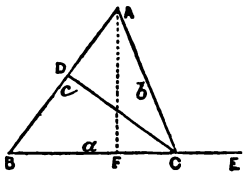
$$\text{sin. } A \times \text{cosec. } A = 1.$$

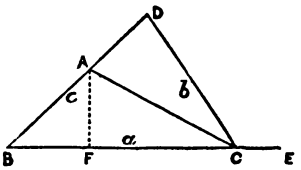
In the same manner we obtain from (123), cos. A × sec. A = 1, and from (124), tan. A × cot. A = 1; also from (235) (*a*), (*c*), and (*d*), we have $\sin.^2 A + \cos.^2 A = 1$ $\sec.^2 A - \tan.^2 A = 1$, and $\operatorname{cosec}.^2 A - \cot.^2 A = 1$. We thus obtain six values of 1, all of which are equal to one another, and by equating each of them to all the remaining ones fifteen different equations may be obtained; these values of one are as follow:

1 = sin. A cosec. A = cos. A sec. A = tan. A cot. A = $\sin.^2 A + \cos.^2 A = \sec.^2 A - \tan.^2 A = \operatorname{cosec}.^2 A - \cot.^2 A$; any one of these six may be substituted for 1, or may be written as a denominator to an integer without altering the value.

237. Having now found the principal relations of the trigonometrical functions of a single arc or angle, we proceed to investigate the relations of the sine and cosine of the sum and difference of two arcs or angles. For this purpose we take the triangles (1) and (2), and in (1) let the ∠ A be acute and produce the side BC opposite the angle A to E, then (Euc. I. 32) ∠ ACE = ∠ (A + B), and the sine of ACB = the sine of ACE =

sin. (A + B); also cos. ACB = − cos. ACE = − cos. (A + B). Draw CD perpendicular to AB in (1), and to BA produced in





(2), and call the lengths of the sides opposite the angles A, B, C, a, b, and c; then in (1)

$$c = \text{BD} + \text{DA} = a \cos. \text{B} + b \cos. \text{A} \qquad (e)$$

$$\frac{\text{sin. C}}{\text{sin. A}} = \frac{\text{sin. (A + B)}}{\text{sin. A}} = \frac{a \cos. \text{B} + b \cos. \text{A}}{a}$$

$$= \cos. \text{B} + \frac{\text{sin. B cos. A}}{\text{sin. A}}; \text{ since } \frac{b}{a} = \frac{\text{sin. B}}{\text{sin. A}}.$$

Multiplying by sin. A, gives

$$\text{sin. (A + B)} = \text{sin. A cos. B} + \text{cos. A sin. B} \qquad (f)$$

Again, $\cos. \text{C} = \frac{\text{CF}}{b} = \frac{a - \text{BF}}{b} = \frac{a - c \cos. \text{B}}{b}$ which by (e)

$$= \frac{a - (a \cos. \text{B} + b \cos. \text{A}) \cos. \text{B}}{b}$$

$$= \frac{a - a \cos.^2 \text{B} - b \cos. \text{A} \cos. \text{B}}{b}$$

$$= \frac{a (1 - \cos.^2 \text{B}) - b \cos. \text{A} \cos. \text{B}}{b}$$

$$= \frac{a \sin.^2 \text{B} - b \cos. \text{A} \cos. \text{B}}{b};$$

$$\therefore - \cos. (\text{A} + \text{B}) = \frac{\text{sin. A sin.}^2 \text{B}}{\text{sin. B}} - \cos. \text{A} \cos. \text{B}.$$

Hence, changing the signs,

$$\cos (\text{A} + \text{B}) = \cos. \text{A} \cos. \text{B} - \sin. \text{A} \sin. \text{B}. \qquad (h)$$

Next, in the triangle (2), let the exterior angle

$$\text{DAC} = \angle \text{A}, \text{ then } \angle \text{ACB} = \angle (\text{A} - \text{B}).$$

And $$\frac{\text{sin. C}}{\text{sin. A}} = \frac{\text{sin. (A} - \text{B)}}{\text{sin. A}} = \frac{c}{a} = \frac{a \text{ cos. B} - b \text{ cos. A}}{a}$$

$$= \text{cos. B} - \frac{\text{sin. B. cos. A}}{\text{sin. A}}.$$

Multiplying by sin. A, gives

$$\text{sin. (A} - \text{B)} = \text{sin. A cos. B} - \text{sin. B cos. A.} \qquad (g)$$

Again, cos. C =

$$\text{cos. (A} - \text{B)} = \frac{\text{CF}}{b} = \frac{a - \text{BF}}{b} = \frac{a - (a \text{ cos. B} - b \text{ cos. A}) \text{ cos. B}}{b}$$

$$= \frac{a(1 - \text{cos.}^2 \text{ B}) + b \text{ cos. A cos. B}}{b}$$

$$= \frac{a \text{ sin.}^2 \text{ B} + b \text{ cos. A cos. B}}{b}$$

$$\text{cos. (A} - \text{B)} = \text{cos. A cos. B} + \text{sin. A sin. B.} \qquad (k)$$

Collecting the above results, marked (f), (g), (h), and (k), we have the following very important results:

$$\text{sin. (A} + \text{B)} = \text{sin. A cos. B} + \text{cos. A sin. B,} \qquad (f)$$
$$\text{sin. (A} - \text{B)} = \text{sin. A cos. B} - \text{cos. A sin. B,} \qquad (g)$$
$$\text{cos. (A} + \text{B)} = \text{cos. A cos. B} - \text{sin. A sin. B,} \qquad (h)$$
$$\text{cos. (A} - \text{B)} = \text{cos. A cos. B} + \text{sin. A sin. B.} \qquad (k)$$

238. To find the sine and cosine of 2A, in terms of the sine and cosine of A.

In (f) and (h) above, let B = A, then (f) becomes

$$\text{sin. 2A} = \text{sin. A cos. A} + \text{cos. A sin. A} = 2 \text{ sin. A cos. A.}$$

Similarly, $$\text{sin. A} = 2 \text{ sin. } \tfrac{1}{2} \text{ A cos. } \tfrac{1}{2} \text{ A.} \qquad (l)$$

In (h) let B = A, then $\text{cos. 2A} = \text{cos.}^2 \text{ A} - \text{sin.}^2 \text{ A.}$ (m)

Similarly, $$\text{cos. A} = \text{cos.}^2 \tfrac{1}{2} \text{ A} - \text{sin.}^2 \tfrac{1}{2} \text{ A.}$$

From (g) and (k), by making B = A, we obtain

$$\text{sin. o} = \text{o and cos. o} = 1. \qquad (n)$$

By (235, *a*) $1 = \cos.^2 A + \sin.^2 A,$

and (238, *m*) $\cos. 2A = \cos.^2 A - \sin.^2 A;$

adding these two gives

$$1 + \cos. 2A = 2 \cos.^2 A.$$

Subtracting, gives $1 - \cos. 2A = 2 \sin.^2 A;$

and taking the halves of the arcs, we have

$$1 + \cos. A = 2 \cos.^2 \tfrac{1}{2} A, \qquad (p)$$

and

$$1 - \cos. A = 2 \sin.^2 \tfrac{1}{2} A. \qquad (q)$$

239. Adding and subtracting (*f*) and (*g*), and also (*k*) and (*h*), gives the four following important formulæ:

$$\sin.(A+B) + \sin.(A-B) = 2 \sin. A \cos. B, \qquad (r)$$
$$\sin.(A+B) - \sin.(A-B) = 2 \cos. A \sin. B, \qquad (s)$$
$$\cos.(A-B) + \cos.(A+B) = 2 \cos. A \cos. B, \qquad (t)$$
$$\cos.(A-B) - \cos.(A+B) = 2 \sin. A \sin. B. \qquad (u)$$

240. Let now $A+B=S$, and $A-B=D$; then $A=\frac{1}{2}(S+D)$, and $B=\frac{1}{2}(S-D)$; and substituting these values in (*r*), (*s*), (*t*), and (*u*), they are changed into—

$$\sin. S + \sin. D = 2 \sin. \tfrac{1}{2}(S+D) \cos. \tfrac{1}{2}(S-D), \qquad (r)$$
$$\sin. S - \sin. D = 2 \cos. \tfrac{1}{2}(S+D) \sin. \tfrac{1}{2}(S-D), \qquad (s)$$
$$\cos. D + \cos. S = 2 \cos. \tfrac{1}{2}(S+D) \cos. \tfrac{1}{2}(S-D), \qquad (t)$$
$$\cos. D - \cos. S = 2 \sin. \tfrac{1}{2}(S+D) \sin. \tfrac{1}{2}(S-D). \qquad (u)$$

241. Again, since S and D may be any arcs or angles, for S and D let us write A and B, and we have finally,

$$\sin. A + \sin. B = 2 \sin. \tfrac{1}{2}(A+B) \cos. \tfrac{1}{2}(A-B), \qquad (r)$$
$$\sin. A - \sin. B = 2 \cos. \tfrac{1}{2}(A+B) \sin. \tfrac{1}{2}(A-B), \qquad (s)$$
$$\cos. B + \cos. A = 2 \cos. \tfrac{1}{2}(A+B) \cos. \tfrac{1}{2}(A-B), \qquad (t)$$
$$\cos. B - \cos. A = 2 \sin. \tfrac{1}{2}(A+B) \sin. \tfrac{1}{2}(A-B). \qquad (u)$$

242. Next divide (*r*) by (*s*), and (*t*) by (*u*), and reduce the second sides to their simplest forms; we obtain

$$\frac{\sin. A + \sin. B}{\sin. A - \sin. B} = \frac{2 \sin. \frac{1}{2}(A+B) \cos. \frac{1}{2}(A-B)}{2 \cos. \frac{1}{2}(A+B) \sin. \frac{1}{2}(A-B)} = \frac{\tan. \frac{1}{2}(A+B)}{\tan. \frac{1}{2}(A-B)}. \qquad (v)$$

$$\frac{\cos. B+\cos. A}{\cos. B-\cos. A}=\frac{2\cos.\frac{1}{2}(A+B)\cos.\frac{1}{2}(A-B)}{2\sin.\frac{1}{2}(A+B)\sin.\frac{1}{2}(A-B)}=\frac{\cot.\frac{1}{2}(A-B)}{\tan.\frac{1}{2}(A+B)}. \quad (w)$$

But (Art. 132) $\frac{\sin. A}{\sin. B}=\frac{a}{b}\therefore\frac{\sin. A+\sin. B}{\sin. A-\sin. B}=\frac{a+b}{a-b}$, and equating this value of $\frac{\sin. A+\sin. B}{\sin. A-\sin. B}$ with that obtained in (v), we have

$$\frac{a+b}{a-b}=\frac{\tan.\frac{1}{2}(A+B)}{\tan.\frac{1}{2}(A-B)}, \text{ or}$$

$$\frac{\tan.\frac{1}{2}(A-B)}{\tan.\frac{1}{2}(A+B)}=\frac{a-b}{a+b}, \text{ and } \therefore \tan.\tfrac{1}{2}(A-B)=\frac{a-b}{a+b}\tan.\tfrac{1}{2}(A+B). \quad (x)$$

This result shews, that if in a triangle the sides a and b, and the contained angle C, be given, the other angles can be found, and then by the former case, the third side; for $\frac{1}{2}(A+B)=\frac{1}{2}(180°-C)$, which is known when $\angle C$ is known, and then by (x) $\frac{1}{2}(A-B)$ is known; and then $\frac{1}{2}(A+B)+\frac{1}{2}(A-B)=A$, and $\frac{1}{2}(A+B)-\frac{1}{2}(A-B)=B$.

243. From (x) and the following remark, we have this important case: Given two sides and the contained angle in a triangle, to find the remaining angles.

RULE.—*Subtract the given angle from* 180°, *and take half of the remainder, this is* $\frac{1}{2}(A+B)$; *add the sides* a *and* b *and also subtract them, this will give* a+b *and* a−b; *then the sum of the sides is to their difference as the tangent of half the sum of the remaining angles to the tangent of half their difference.* The half sum and half difference being added, gives the greater angle, which is always opposite the greater side; and being subtracted, gives the less. Or in symbols, the Rule is as follows:

$$a+b:a-b::\tan.\tfrac{1}{2}(A+B):\tan.\tfrac{1}{2}(A-B)=\frac{a-b}{a+b}\tan.\tfrac{1}{2}(A+B),$$

as (x) above.

244. Again, by Art. (132), we have

$$\frac{\sin. A}{\sin. C}=\frac{a}{c} \text{ and } \frac{\sin. B}{\sin. C}=\frac{b}{c};$$

taking the sum and difference of these equals, we obtain

$$\frac{\text{sin. A} + \text{sin. B}}{\text{sin. C}} = \frac{a+b}{c} \text{ and } \frac{\text{sin. A} - \text{sin. B}}{\text{sin. C}} = \frac{a-b}{c}.$$

Now, $\text{sin. A} + \text{sin. B} = 2 \text{ sin. } \frac{1}{2}(\text{A} + \text{B}) \text{ cos. } \frac{1}{2}(\text{A} - \text{B})$, by 241 (*r*),

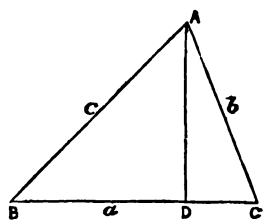
$\text{sin. A} - \text{sin. B} = 2 \text{ cos. } \frac{1}{2}(\text{A} + \text{B}) \text{ sin. } \frac{1}{2}(\text{A} - \text{B})$, by 241 (*s*),

and $\text{sin. C} = \text{sin.}(\text{A} + \text{B}) = 2 \text{ sin. } \frac{1}{2}(\text{A} + \text{B}) \text{ cos. } \frac{1}{2}(\text{A} + \text{B})$, by 238 (*l*).

Substituting these values, and dividing the common factor out of the numerators and denominators, they become

$$\frac{\text{cos. } \frac{1}{2}(\text{A} - \text{B})}{\text{cos. } \frac{1}{2}(\text{A} + \text{B})} = \frac{a+b}{c} \ (y), \text{ and } \frac{\text{sin. } \frac{1}{2}(\text{A} - \text{B})}{\text{sin. } \frac{1}{2}(\text{A} + \text{B})} = \frac{a-b}{c}. \qquad (z)$$

245. Let ABC be a triangle whose sides opposite the angles A, B, and C are represented by a, b, and c; it is required to find an expression for the cosine of any angle in terms of the three sides.



Draw AD perpendicular to BC, then (Art. 123) $\text{BD} = c \text{ cos. B}$, and $\text{CD} = b \text{ cos. C}$; therefore $\text{BD} + \text{DC} = c \text{ cos. B} + b \text{ cos. C}$; but $\text{BD} + \text{DC} = a$,

and $$\therefore a = b \text{ cos. C} + c \text{ cos. B},$$

and similarly it can be proved that

$$b = c \text{ cos. A} + a \text{ cos. C},$$

and $$c = b \text{ cos. A} + a \text{ cos. B}.$$

Multiplying the first by a, the second by b, and the third by c, gives

$$a^2 = ab \text{ cos. C} + ac \text{ cos. B},$$

$$b^2 = bc \text{ cos. A} + ab \text{ cos. C},$$

and $$c^2 = bc \text{ cos. A} + ac \text{ cos. B}.$$

Adding the second and third of these expressions, and from their sum subtracting the first, we obtain

$$b^2 + c^2 - a^2 = 2bc \text{ cos. A};$$

whence we have $\cos. A = \frac{b^2+c^2-a^2}{2bc}$.

Also adding the first and third, and subtracting the second,

$$\cos. B = \frac{a^2+c^2-b^2}{2ac};$$

and similarly, $\cos. C = \frac{a^2+b^2-c^2}{2ab}$.

To both sides of $\cos. A = \frac{b^2+c^2-a^2}{2bc}$ add 1, and it becomes

$$1 + \cos. A = 1 + \frac{b^2+c^2-a^2}{2bc} = \frac{2bc+b^2+c^2-a^2}{2bc}.$$

Hence, $1 + \cos. A = \frac{(b+c)^2-a^2}{2bc} = \frac{(a+b+c)(b+c-a)}{2bc}$;

but (238, *p*) $1 + \cos. A = 2 \cos.^2 \frac{1}{2}A$;

$$\therefore 2 \cos.^2 \tfrac{1}{2}A = \frac{(a+b+c)(b+c-a)}{2bc},$$

and dividing by 2,

$$\cos.^2 \tfrac{1}{2}A = \frac{\frac{1}{2}(a+b+c) \times \frac{1}{2}(b+c-a)}{bc}.$$

246. Let now $\frac{1}{2}(a+b+c) = s$, then will $\frac{1}{2}(b+c-a) = s-a$, and $\frac{1}{2}(a+c-b) = s-b$, and also $\frac{1}{2}(b+a-c) = s-c$; whence we have

$$\cos.^2 \tfrac{1}{2}A = \frac{s(s-a)}{bc},$$

and extracting the square root,

$$\cos. \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}.$$

Similarly, $\cos. \frac{1}{2}B = \sqrt{\frac{s(s-b)}{ac.}}$,

and $\cos. \frac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}}$.

247. In the same manner, if each side of the equation, $\cos. A = \frac{b^2+c^2-a^2}{2bc}$, be subtracted from 1, we obtain

$$1 - \cos. A = 1 - \frac{b^2+c^2-a^2}{2bc} = \frac{a^2-(b-c)^2}{2bc}$$

$$= \frac{(a+b-c)(a+c-b)}{2bc}.$$

Hence, (238, q) and (246), $\sin^2 \frac{1}{2}A = \frac{(s-b)(s-c)}{bc}$, and extracting the square root;

$$\therefore \sin. \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Similarly, $$\sin. \tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}},$$

and $$\sin. \tfrac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{ab}}.$$

248. Dividing now the values of the sine found in (247) by the values of the cosine found in (246), we obtain the values of the tangent—namely,

$$\tan. \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc} \times \frac{bc}{s(s-a)}}$$

$$= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

Multiply numerator and denominator by the square root of $(s-a)$, and we have

$$\tan. \tfrac{1}{2}A = \frac{1}{s-a}\sqrt{\frac{1}{s}(s-a)(s-b)(s-c)},$$

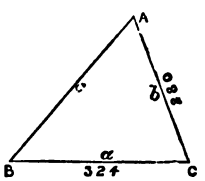
and similarly, $$\tan. \tfrac{1}{2}B = \frac{1}{s-b}\sqrt{\frac{1}{s}(s-a)(s-b)(s-c)},$$

and $$\tan. \tfrac{1}{2}C = \frac{1}{s-c}\sqrt{\frac{1}{s}(s-a)(s-b)(s-c)}.$$

249. When the three sides of a triangle are given, the angles can be calculated either from the expressions for the cosine of half the angle given in (246), the sine of half the angle in (247), or that for the tangent of half the angle as given in (248). When all the three angles are required, the expression for the tangent is the best, and for large or small angles gives more accurate results than either of the other two. The expressions for the tangents of the half-angles require only the use of four quantities to calculate all the three angles; whereas, if the three angles be calculated either from the expressions for the cosine or sine of their halves, seven quantities are required.

Problem XXXIV.

250. Given two sides and the contained angle of a triangle, to find the remaining angles and the third side. The Rules for solving this case will be found in Arts. (243) and (244), which see.



Example.

Given AC = 280, BC = 324, and the contained angle ACB = 72° 30′; to find the remaining angles and the third side.

Here $a = 324$
and $b = 280$
Hence $a + b = 604$
and $a - b = 44$

$\angle A + \angle B = 180° - 72° 30'$
$= 107° 30'$
$\therefore \frac{1}{2}(A + B) = 53° 45'$

Whence $\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B) = \frac{44}{604} \times \tan 53° 45'$

Log. 44	=	1·6434527
Log. tan. 53° 45′	=	10·1347596
		11·7782123
Log. 604	=	2·7810369
Log. tan. 5° 40′ 25″·7 =		8·9971754

Hence $\frac{1}{2}(A + B) = 53° 45'$
and $\frac{1}{2}(A - B) = 5° 40' 25''·7$
$\therefore \angle A = 59° 25' 25''·7$
and $\angle B = 48° 4' 34''·3$

To find the Third Side, AB = c.

This may be done three ways—*1st*, by the former case, which gives $c = \frac{\text{sin. C} \times a}{\text{sin. A}}$; or *2d*, by (244, *y*), which gives $c = \frac{(a + b) \text{ cos. } \frac{1}{2}(\text{A} + \text{B})}{\text{cos. } \frac{1}{2}(\text{A} - \text{B})}$; or *3d*, by (244, *z*), which gives $c = \frac{(a - b) \text{ sin. } \frac{1}{2}(\text{A} + \text{B})}{\text{sin. } \frac{1}{2}(\text{A} - \text{B})}$.

1st.	*2d.*	*3d.*
2·5105450	2·7810369	1·6434527
9·9794195	9·7718150	9·9065745
12·4899645	12·5528519	11·5500272
9·9349796	9·9978671	8·9950417
2·5549849 = log. 358·909	2·5549848	2·5549855

Hence AB = 358·909, by either of the three methods.

Exercises 27.

(330.) Given AC = 400, BC = 530, and the contained angle ACB = 48° 42′; to find the angles at A and B, and the side AB.

(331.) Given AC = 490, BC = 608, and the contained angle ACB = 70°; to find the angles at A and B, and the side AB.

(332.) Given AC = 78·42, BC = 98·64, and the contained angle ACB = 104° 18′; to find the angles at A and B, and the side AB.

(333.) Given the sides AC and BC respectively 280 and 345, and the angle ACB = 94° 30′; to find the angles at A and B, and the side AB.

(334.) Given AC = 620, BC = 804, and the angle ACB = 71° 50′; to find the angles at A and B, and the side AB.

(335.) Given AC = 57·3, BC = 80·5, the angle at C = 65° 30′; to find the angles at A and B, and the side AB.

(336.) Given AC = 900, BC = 1200, and the angle at C = 86° 14′; to find the angles at A and B, and the side AB.

(337.) Given AC = 486, BC = 596, and the angle at C = 47° 30′; to find the angles at A and B, and the side AB.

Problem XXXV.

251. Given the three sides of a triangle, to find the angles.

RULE.—Add the three sides together, and take half the sum; from the half sum subtract each side separately, which gives s, $s-a$, $s-b$, and $s-c$, where a, b, and c are the lengths of the three sides of the triangle, and s half their sum; then subtract the logarithm of s from 20; to the remainder add the logarithms of $s-a$, $s-b$, and $s-c$, and take half the sum; from this half sum subtract successively log. $(s-a)$, log. $(s-b)$, log. $(s-c)$, and the three remainders will be the log. tan. $\frac{1}{2}$A, log. tan. $\frac{1}{2}$B, and log. tan. $\frac{1}{2}$C; doubling the angles thus obtained, we get the three angles of the triangle, whose sum is always 180°.

For the proof of the Rule, see Art. (248).

Example.

In the triangle ABC there are given BC $= a = 480$, CA $= b =$ 464, and AB $= c = 422$; to find the three angles.

	a	= 480				
	b	= 464				
	c	= 422				
Sum	$= 2s$	= 1366				
Half sum	$= s$	= 683	20 − log. s	=	17·1655793	
	$s-a$	= 203	log. $(s-a)$	=	2·3074960	
	$s-b$	= 219	log. $(s-b)$	=	2·3404441	
	$s-c$	= 261	log. $(s-c)$	=	2·4166405	
					2)24·2301599	
					12·1150799·5	= Const.
	Log. tan. $\frac{1}{2}$A	= 32° 42′ 12″·2		=	9·8075839·5	= C − $(s-a)$.
	Log. tan. $\frac{1}{2}$B	= 30° 45′ 34″·4		=	9·7746358·5	= C − $(s-b)$.
	Log. tan. $\frac{1}{2}$C	= 26° 32′ 13″·4		=	9·6984394·5	= C − $(s-c)$.

Hence, doubling these, we have

$$\angle A = 65° 24′ 24″·4$$
$$\angle B = 61° 31′ 8″·8$$
and $$\angle C = 53° 4′ 26″·8$$

Adding these three, we have

$$\angle A + \angle B + \angle C = 180° 0′ 0″$$

Exercises 28.

In the following Exercises, it is required to find the three angles:

		BC	CA	AB
(338.)	Given	498	654	872
(339.)	"	562	781	426
(340.)	"	218·5	317·4	300·1
(341.)	"	400	500	600
(342.)	"	1000	1100	1250
(343.)	"	285	345	360
(344.)	"	291	311	331
(345.)	"	785	555	1200

CHAPTER VIII.

CURRENT SAILING.

252. As yet we have proceeded on the supposition that the surface of the ocean has no motion. This may answer tolerably well in those places where the tides are regular; as the effect of the *flood* will nearly counterbalance that of the *ebb*. But in places where there is a constant current, or setting of the sea toward the same point, an allowance must be made for the change of the ship's place, arising from this cause. And the method of resolving problems in sailing, in which the effect of a current is taken into account, is called *Current Sailing*.

253. That point of the compass towards which the current runs is called its *setting;* and its rate per hour is called its *drift*.

254. If a ship sail in the same direction as the *setting* of the current, her distance made good will be her distance sailed increased by the drift, multiplied by the number of hours she has sailed in the current; but if she sail directly against the current, her distance made good will be the difference of the same quantities. If the ship's course be oblique to the direction of the current, her true course and distance will be compounded of the course and distance given by the log, and of the setting and drift of the current; and the distance made good in a given time will be represented by the third side of a triangle, of which

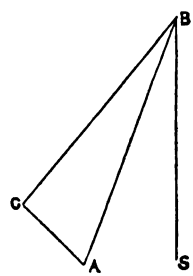
the distance given by the log, and the drift of the current in the same, are the other two sides.

Example I.

A ship sailed SW. by S. at the rate of 7 miles an hour, in a current setting SE. $1\frac{1}{2}$ miles an hour; required the course and distance made good in 24 hours.

BY CONSTRUCTION.

Draw BS to represent the meridian; and draw BC, making an angle SBC = 3 points (SW. by S.), and make it equal to $7 \times 24 = 168$ miles; then draw CA SE., and make it equal to $1\frac{1}{2} \times 24 = 36$ miles, and join AB, which will be the distance made good; and the angle SBA will be the course made good. Now, the distance will measure about 179 miles, and the course SSW.



BY CALCULATION.

In the triangle ABC are given the distances BC, CA equal to 168 and 36 miles respectively, and the contained angle BCA equal 9 points; to find the other angles and distance AB.

To find the Angles.

Here, by Art. (243), $168 + 36 : 168 - 36 :: \tan. \frac{1}{2}(A + B) : \tan. \frac{1}{2}(A - B)$

Or, $204 : 132 :: \tan. 39^\circ 22\frac{1}{2}' : \tan. \frac{1}{2}(A - B)$

Ar. co. log. 204	=	7·6903698
Log. 132	=	2·1205739
Log. tan. 39° 22′ 30″	=	9·9141732
Log. tan. 27° 58′ 10″	=	9·7251169
∴ ∠ CAB = 67° 20′ 40″		
∠ CBA = 11° 24′ 20″		
But ∠ CBS = 33° 45		

∴ The ∠ ABS = S. 22° 21′ W. = the course, or SSW. nearly.

To find the Distance.

$$\frac{AB}{AC} = \frac{\text{sin. ACB}}{\text{sin. ABC}}; \text{ or } \frac{AB}{36} = \frac{\text{sin. 9 pts.}}{\text{sin. } 11^\circ 4\frac{1}{2}'}$$

Log. 36	=	1·5563025
Log. sin. 7 points	=	9·9915739
Log. cosec. 11° 24½′	=	10·7038783
∴ The distance = 178·6	=	2·2517547

255. This problem can be more readily solved by a traverse

for the result is the same as if the ship had first sailed SW. by S. 168 miles, and then SE. 36 miles; thus,

Courses.	Dist.	Dif. Lat.		Departure.	
		N.	S.	E.	W.
SW. by S...	168	..	139·7	..	93·3
SE.	36	..	25·5	25·5	..
	..	..	165·2	25·5	93·3
	..	..	..	..	25·5
	..	..	..	..	67·8

From this it is plain that the dif. lat. made is 165·2 S., and the departure 67·8 W.; and by inspection of the Table it is found that this corresponds to a course of 2 points, or SSW. nearly, and a distance of 179 miles.

Example II.

A ship sails NNE. 40 miles in 5 hours, then E. by S. 60 miles in 12 hours, and then SE. 70 miles in 7 hours, while all the time a current was setting SW. 2 miles an hour; required the direct course and distance the ship has made good.

Solution.

256. Here the ship sails $(5 + 12 + 7) = 24$ hours in the current; wherefore, the setting and drift of the current for all the time is SW. 48 miles, which is to be taken in, as another course and distance, with those actually sailed.

For these courses and distances, the Traverse Table will be as follows:

Courses.	Dist.	Dif. Lat.		Departure.	
		N.	S.	E.	W.
NNE.	40	37·0	..	15·3	..
E. by S. ...	60	..	11·7	58·8	..
SE.	70	..	49·5	49·5	..
SW.	48	..	33·9	..	33·9
	..	..	95·1	123·6	..
	..	..	37·0	33·9	..
	..	..	58·1	89·7	..

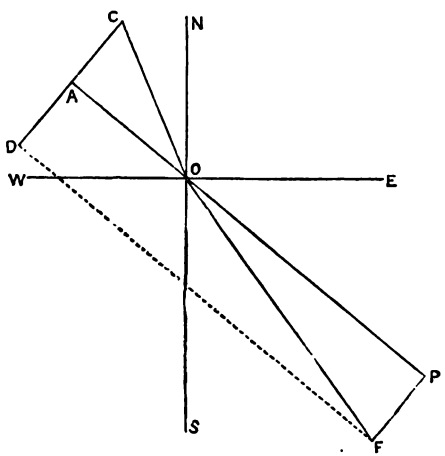
Here the difference of latitude made good is 58·1 S., and the departure 89·7 E.; to which the corresponding course and distance is found to be S. 57° 4′ E. 106·9 miles, or SE. by E. 106·9 miles nearly.

Example III.

257. A ship has to sail to a port bearing from her present position SE., in a current setting N. by W. $1\frac{1}{2}$ miles an hour; find the courses on which she must sail so as to run direct to the port, when her speed is 6, 7, 8, and 9 miles an hour; and the rate per hour at which she approaches the port with each rate of sailing.

Solution.

Let O be the place of the ship, and P the port to be sailed to, bearing SE.; also, let OC, N. by W., represent the setting of the current; produce PO to A, then the angle COA = 3 points; and if OC represent the drift of the current, then CA is the distance the ship is carried to the left of OP by the current, and OA is the distance the ship is carried directly away from the port. Produce CA to D, making AD = CA; then the course must be such that in the same time in which the current carries the ship from O to C, the ship must make a departure from the line OP, on the other side, equal to AD; hence, through D draw DF parallel to OP; and from the centre O, with a radius equal to the rate of sailing per hour, cut the line DF in F, and draw FP perpendicular to OP; then FP = AD = AC = OC sin. COA; now ∠ POF is plainly the angle which the ship's course must make with OP, and the course SOF = SOP − FOP. The course is thus found. OP = OF cos. FOP = the distance the ship advances towards P;



but she is carried by the current from O to A = drift cos. AOC; hence, OP − OA = the ship's approach to P.

1. To find the course and rate of approach when the rate of sailing is 6 miles an hour.

$$\text{Sin. POF} = \frac{\text{PF}}{\text{OF}} = \frac{\text{CO} \times \text{sin. COA}}{\text{OF}} = \frac{1\frac{1}{2} \times \cdot 5555702}{6} = \cdot 1388925 = \text{sin. } 7^\circ\ 59'$$

OP	= cos. 7° 59′ × 6	=	·9903085 × 6	=	5·9418510
OA	= cos. 3 pts. × 1½	=	·8314696 × 1½	=	1·2472044
OP − OA = the rate of approach				=	4·6946466

2. To find the course and rate of approach when the rate of sailing is 7 miles an hour.

$$\text{Sin. FOP} = \frac{\text{CO} \times \text{sin. COA}}{\text{OF}} = \frac{1\frac{1}{2} \times \cdot 5555702}{7} = \cdot 1190508 = \text{sin. } 6^\circ\ 50'$$

OP	= cos. 6° 50′ × 7	=	·9928965 × 7	=	6·9502755
OA	= cos. 3 pts. × 1½			=	1·2472044
OP − OA = the rate of approach				=	5·7030711

3. To find the course and rate of approach when the rate of sailing is 8 miles an hour.

$$\text{Sin. FOP} = \frac{\text{CO} \times \text{sin. COA}}{\text{OF}} = \frac{1\frac{1}{2} \times \cdot 5555702}{8} = \cdot 1041694 = \text{sin. } 5^\circ\ 58'$$

OP	= cos. 5° 58′ × 8	=	·9945825 × 8	=	7·9566600
OA	= cos. 3 pts. × 1½			=	1·2472044
OP − OA = the rate of approach				=	6·7094556

4. To find the course and rate of approach when the rate of sailing is 9 miles an hour.

$$\text{Sin. FOP} = \frac{\text{CO} \times \text{sin. COA}}{\text{OF}} = \frac{1\frac{1}{2} \times \cdot 5555702}{9} = \cdot 0925950 = \text{sin. } 5^\circ\ 19'$$

OP	= cos. 5° 19′ × 9	=	·9956978 × 9	=	8·9612802
OA	= cos. 3 pts. × 1½			=	1·2472044
OP − OA = the rate of approach				=	7·7140758

258. The angle FOP, which we have found above, is the angle between the SE. and the ship's course, so that subtracting its respective values from 45°, leaves the true courses—namely, S. 37° 1′ E., S. 38° 10′ E., S. 39° 2′ E., and S. 39° 41′ E.

If the setting of the current had been E. by S., its direction

with the bearing of the port would still have been 3 points; but then the current would have advanced the ship towards the port instead of carrying her from it, as in the above example; the courses to be steered would still have been those given above; but the rate of approach to the port would then have been OP + OA instead of OP − OA; these rates would therefore have been 7·1890554 miles, 8·1974799 miles, 9·2038644 miles, and 10·2084846 miles.

259. This problem may be very expeditiously solved, with sufficient accuracy for all ordinary purposes, by inspection; thus, enter the Table with a course of 3 points and a distance of 150 = 100 times the drift, opposite which will be found in the Departure column, 83·3, and in the Difference of Latitude, 124·7; the one-hundredth part of which gives for the departure from the direction of the port, ·833, and the hourly motion along the direction, 1·247. When the ship sails at the rate of 6 miles an hour, enter the Traverse Table with 10 times the run = 60, and 10 times the departure, 8·33; the nearest course is 8°, and the tenth part of the difference of latitude, = 5·94, the approach per hour towards the port; this diminished by the distance by which she is carried from it by the current, 1·247, gives the approach made good, 4·693. Proceeding in the same manner with the other rates of sailing, 7, 8, and 9 miles an hour, we get the rates of approach, 5·703, 6·713, and 7·723 miles per hour, none of which differ from the former by so much as one-hundredth part of a mile per hour.

Exercises 29.

(346.) A ship sails NW. by W. 8 miles an hour in a current setting SE. by E. $2\frac{1}{2}$ miles an hour; required her true course and rate of sailing per hour.

(347.) A ship sails E. by S. 9 miles an hour in a current setting NE. 2 miles an hour; required her true course and rate of sailing per hour.

(348.) A ship sailing S. at the rate of $8\frac{1}{2}$ miles per hour in a

current, finds that at the end of 24 hours she has made a change of latitude of 3 degrees, and a departure of 25 miles to the west; required the setting and drift of the current.

(349.) A ship has made by the reckoning S. ½ E. 20 miles; but by observation it is found that, owing to a current, she has actually gone SSW. 28 miles; required the setting and drift of the current in the time the ship has been running.

(350.) A ship's course to her port is NW. by W., and she is running by the log 9 miles an hour, but meeting with a current setting W. ½ S. 3 miles an hour; what course must she steer in the current that her true course may be NW. by W., and at what rate will she approach the port?

(351.) In a tide running WNW. 3 miles an hour, I wish to weather a cape bearing NE. by N. 21 miles; what course must I steer so as to clear the point, the ship going 8 miles an hour by the log, and in what time will I reach the cape?

(352.) If a ship sail due W. 9 miles an hour by the log in a current setting NW. by W. 3½ miles an hour; what is her true course and hourly rate of sailing?

(353.) A ship at sea in the night has sight of Scilly light bearing NW. by N. distant 15 miles, it being then flood-tide setting ENE. 2 miles an hour, and the ship running at the rate of 5 miles an hour; required what course and distance she must sail to reach the Lizard, which bears from Scilly E. ½ S. distant 51 miles.

(354.) In a river two miles broad, in which the ebb tide is running E. by S. at the rate of 3 miles an hour, a waterman who can row at the rate of 5 miles an hour, intends to cross the river to a point on the opposite side, SSW. from his present position. In what direction must he pull that he may reach the intended place in the shortest time possible, and in what time will he reach it?

(355.) The conditions being all the same as in the last question, except that the point on the opposite side bears direct south; required the course and the time required to cross.

CHAPTER IX.

ON PLYING TO WINDWARD.

260. *Windward Sailing* is the method of gaining an intended port by the shortest and most direct method possible, when the wind is in a direction unfavourable to the course the ship ought to steer, so as to reach the port directly.

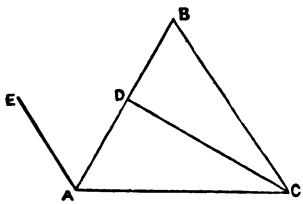
261. In order to attain this important point, it is evident the ship must sail on different tacks; the object, therefore, of this sailing is to find the proper courses to be steered, and the distance to be sailed on each board, that the vessel may arrive at the intended port the soonest possible. The following examples will shew the rules to be observed in this sailing.

Example I.

A ship is bound to a port 34 miles direct to windward, the wind being NNE.; she can lie within 6 points of the wind on each tack; required how far she must sail on each tack so as to reach the port in two boards.

Solution.

Draw AB, NNE. = 34 miles, and through A draw AC and AE, each making an angle of 6 points with AB, the direction of the port; through B draw BC parallel to AE, and produce AC and BC till they meet in C; draw CD perpendicular to AB, and AD will evidently be the half of AB, and the angles CAB and CBA being equal, the side AC = BC (Euc. I. 6). Again, in the triangle ADC, right-angled at D (Art. 126),



$$AC = AD \times \text{sec. } DAC = 17 \times \text{sec. } 6 \text{ points.}$$

$$\therefore AC = 17 \times 2{\cdot}6131259$$

$$= 44{\cdot}4231403 \text{ miles, the distance to be sailed on}$$

each board; hence,

262. RULE I.—When the ship has to sail directly against the wind. Multiply half the distance into the secant of the angle within which the ship can sail of the wind, and the product will be the distance to be sailed on each tack.

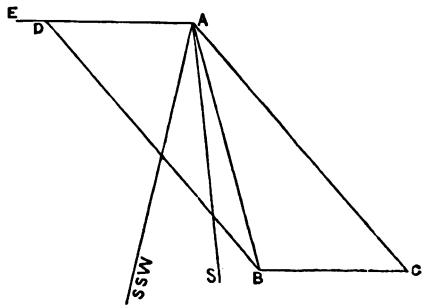
263. RULE II.—By inspection; with the angle within which the ship can sail of the wind as a course, and half the distance of the port as a difference of latitude, enter the Traverse Table, and opposite to half the distance of the port, in the Dif. Lat. column, will be found, in the Distance column, the distance to be sailed on each tack.

Example II.

The wind being SSW., a ship is bound to a port bearing S. by E., distant 54 miles, which it is proposed to reach in two boards, each being within $5\frac{1}{2}$ points of the wind, the first being in the SE. quarter; required the distance to be run on each board.

Solution.

Draw AS, a meridian, and A, SSW., to represent the direction of the wind, and AB, S. by E., to represent the direction of the port; and make AB = 54 miles, the distance of the port; draw AC and AE, making angles of $5\frac{1}{2}$ points with the direction of the wind; and through B draw BC parallel to AE, and meeting AC in C;



then AC = the distance to be sailed on the starboard tack, and CB = the distance to be sailed on the port tack. Angle CAB = $5\frac{1}{2} - 3 = 2\frac{1}{2}$ points, and angle ABC = EAB = $5\frac{1}{2} + 3 = 8\frac{1}{2}$ points; hence also, ACB = 5 points.

To find AC.

$$\frac{AC}{AB} = \frac{\text{sin. } 8\frac{1}{2} \text{ pts.}}{\text{sin. } 5 \text{ pts.}} \text{ (Art. 132)}$$

$\therefore$ AC = 54 × sin. $8\frac{1}{2}$ pts. cosec. 5 pts.

Log. 54	=	1·7323938
Log. sin. $7\frac{1}{2}$ pts.	=	9·9979037
Log. cosec. 5 pts.	=	10·0801536
AC = 64·632	=	1·8104511

To find BC.

$$\frac{BC}{AB} = \frac{\text{sin. } 2\frac{1}{2} \text{ pts.}}{\text{sin. } 5 \text{ pts.}} \text{ (Art. 132)}$$

$\therefore$ BC = 54 × sin. $2\frac{1}{2}$ pts. cosec. 5 pts.

Log. 54	=	1·7323938
Log. sin. $2\frac{1}{2}$ pts.	=	9·6733866
Log. cosec. 5 pts.	=	10·0801536
BC = 30·615	=	1·4859340

264. If BD be drawn through B parallel to AC, to meet AE in D, it is plain that the ship may reach the port either by first sailing on the *starboard tack* from A to C, and then on the port tack from C to B; or, by first sailing on the *port tack* from A to D, and then on the *starboard tack* from D to B.

265. If it were required to reach the port B in three boards, there are two ways in which this might be done, either by first sailing any portion of the line AC, and then sailing across parallel to AD or CB till the ship reach the line DB, and

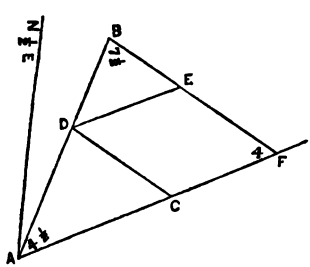
then sailing the remaining part of the starboard tack on DB; or, by first sailing any portion of AD, and then along on the *starboard tack* parallel to AC or DB, till she reach the line CB, and then sailing the remaining part of the *port tack* along CB. In all the four cases above mentioned, the distance or distances sailed on the *starboard tack* is equal to AC = 64·632 miles, and the distance or distances sailed on the *port tack* is equal to AD or CB = 30·615 miles.

Example III.

The wind being at N. $\frac{1}{2}$ E., a ship is bound to a port bearing NNE., distant 68 miles, which she intends to make at four boards. The coast which is to westward also tends NNE., so that the ship must go about as soon as she reaches the straight line joining the two ports; required the course and distance on each board, the ship making her way good within 6 points of the wind.

Solution.

266. Draw AB, NNE. = 68 miles; then draw AF, making an angle of 6 points with the direction of the wind; and hence an angle of $4\frac{1}{2}$ points with AB; draw also BF, making an angle of 6 points with the direction of the wind, and therefore an angle of $7\frac{1}{2}$ points with AB; produce these lines to meet in F; then the angle at F = 16 points − $(7\frac{1}{2} + 4\frac{1}{2}) = 4$ points. Take any point D in AB, and through it draw DE parallel to AF, and DC parallel to BF; then the four boards are evidently AC, CD, DE, and EB; but they are together equal to AF + FB, the two distances on the port tack—namely, AC and DE—being together equal to AE, and the



two, CD and EB, on the starboard tack, together equal to BF. Now, in the triangle ABF we have

To find AF.			To find FB.		
$\frac{AF}{AB} = \frac{\text{sin. } 7\frac{1}{2} \text{ pts.}}{\text{sin. } 4 \text{ pts.}}$ (Art. 132).			$\frac{BF}{AB} = \frac{\text{sin. } 4\frac{1}{2} \text{ pts.}}{\text{sin. } 4 \text{ pts.}}$ (Art. 132).		
$\therefore$ AF = 68 × sin. $7\frac{1}{2}$ pts. × cosec. 4 pts.			$\therefore$ BF = 68 × sin. $4\frac{1}{2}$ pts. × cosec. 4 pts.		
$\therefore$ Log. 68	=	1·8325089	Log. 68	=	1·8325089
Log. sin. $7\frac{1}{2}$ pts.	=	9·9979037	Log. sin. $4\frac{1}{2}$ pts.	=	9·8881854
Log. cosec. 4 pts.	=	10·1505150	Log. cosec. 4 pts.	=	10·1505150
AF = 95·703	=	1·9809276	BF = 74·337	=	1·8712093

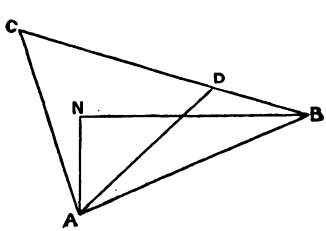
267. Hence the whole distance to be sailed on the port tack is 95·703 miles, and the whole distance to be sailed on the starboard tack is 74·337 miles; if now the two boards on each tack be made equal, AC and DE, on the port tack, will each be 47·85 miles, and CD and EB will each equal 37·17 miles; but if they be unequal, they must be proportional to these distances.

Example IV.

A ship that makes her way good within $5\frac{3}{4}$ points of the wind, made her port in two boards—the first in the NW. quarter on the starboard tack 68 miles, and the other on the port tack 96 miles; and by observation the port come to is 36 miles to the northward of that sailed from. Required the bearing and distance of the port, and the direction of the wind.

Solution.

268. With the given distances 68 and 96 miles, and the included angle equal to $16 - 5\frac{3}{4} \times 2 = 4\frac{1}{2}$ points, construct the triangle ACB; from A, with a radius equal to 36 miles, describe an arc through N; and from B draw BN, touching the arc in N, and join AN, which will be the meridian. Now the distance AB applied



to the scale will measure $74\frac{1}{2}$ miles, and the angle NAB 61° 7′, or NE. by E. $\frac{1}{2}$ E., the bearing of the port; also, make CD = AC, and join AD; then the angle NAD will be 41° 16′ 25″, the direction of the wind.

BY CALCULATION.

In the triangle ACB are given the two sides AC and CB, equal respectively to 68 and 96 miles, and the included angle ACB = $4\frac{1}{2}$ points; to find the angles and the distance AB. By Arts. (243, 244), we have

$a = 96$
$b = 68$
$a + b = 164$ ar. co. log. = 7·7851562
$a - b = 28$ log. = 1·4471580
Log. tan. 64° 41′ 15″ = 10·3251712
Log. tan. 19° 50′ 55″ = 9·5574854
$\therefore \angle A = 84° 32' 10''$
$\angle B = 44° 50' 20''$
and AB = 74·548 miles.

$\frac{1}{2}(A + B) = \frac{1}{2}(16 - 4\frac{1}{2}) = 5\frac{3}{4}$ pts. = 64° 41′ 15″

To find AB.

1·4471580
sin. = 9·9561633
cosec. = 10·4691141
$\therefore$ AB = 74·548 = 1·8724354

Now, in the right-angled triangle ANB, there are given AN = 36 miles, and AB = 74·548 miles; to find the angle NAB, the bearing of the port B from A.

By Art. (123) $\cos. NAB = \frac{36}{74·548}$

Hence log. cos. NAB = log. 36 + 10 − log. 74·548

Log. 36 + 10 = 11·5563025
Log. 74·548 = 1·8724360
Log. cos. 61° 7′ 20″ = 9·6838665

Hence ∠ CAB = 84° 32′ 10″
∠ NAB = 61° 7′ 20″
$\therefore$ ∠ CAN = 23° 24′ 50″
But ∠ CAD = 64° 41′ 15″
Hence ∠ NAD = N. 41° 16′ 25″ E., the direction of the wind.

Exercises 30.

(356.) The wind is NNE., and I am bound to a port directly to windward, and distant 28 miles; the ship can lie within 6

points of the wind on each tack. What must be my course and distance on each tack so as to reach the port in two boards?

(357.) A ship that is bound to a port 80 miles directly to windward, the wind being NW. by W. $\frac{1}{2}$ W., can sail within 6 points of the wind on each tack; how far must she sail on each tack to reach the port in two boards, and what time will she be in reaching the port sailing on each tack at $6\frac{1}{2}$ miles an hour?

(358.) A ship which can lie within $5\frac{1}{2}$ points of the wind is bound to a port bearing S. $\frac{1}{2}$ E. 28 miles, the wind being at SE.; required her course and distance on each tack, to reach her port, close hauled, in two boards.

(359.) Wanting in two boards to reach the mouth of a river which bore NE. 15 miles, I found my ship could lie within $5\frac{1}{2}$ points of the wind (then at E. by N.) on each tack; but that on the port tack she made $1\frac{1}{2}$ points of leeway, and on the starboard tack she made $\frac{1}{2}$ point of leeway; required the course and distance on each tack.

(360.) Wishing to go round a point which bore NE. by E. 17 miles, the wind being NNE., I was obliged to ply to windward; I found my ship could lie within 6 points of the wind; required the course and distance on each tack.

(361.) The wind being SW., a ship plied to windward 320 miles on the port tack, and then 320 miles on the starboard tack, and finds she has made 155·52 miles direct to windward, on each tack she made $1\frac{1}{4}$ points of leeway; find her apparent and real courses.

(362.) If a ship can lie within 6 points of the wind on the port tack, but within $5\frac{1}{2}$ points on the starboard tack; required her course and distance on each tack to reach a port lying SSE. 32 miles, the wind being at SSW.

(363.) From a ship, A, the mouth of a river bore NNE., and from another, B, it bore N. by W., distant from each 20 miles; if each ship can lie within 5 points of the wind, and

sail with the given wind, close hauled, 4 miles an hour, which will reach the river sooner, and how much, the wind being at N.?

(364.) Two ships sail from the same port, the wind being direct ahead; the one sails within 6 points of the wind, including leeway, and runs $3\frac{1}{2}$ miles an hour; the other sails within $6\frac{1}{2}$ points of the wind, including leeway, and runs 5 miles an hour; find the time each of the ships will take to advance 54 miles directly to windward.

(365.) The wind is at S. by E.; a ship sails within 5 points of the wind on each tack, and runs $4\frac{1}{2}$ miles an hour, but on each tack she makes $1\frac{3}{4}$ points of leeway; in what time will she advance 43 miles direct to windward?

(366.) I wish to reach a port SE. by E. distant 64 miles, the wind being S. by E.; my ship will lie within 6 points of the wind on either tack, and sail 5 miles an hour; to reach the port in *three* boards, two on the port tack and one on the starboard tack, what must be the distance sailed on each board, the two port tacks being equal in length, and in what time will I reach the port?

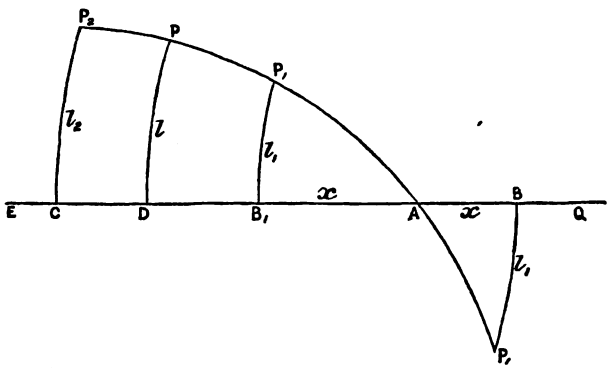
GREAT-CIRCLE SAILING.

269. When a ship sails on a great circle she is constantly changing her course, unless that great circle be a meridian or the equator; in the former case, she sails direct north or south, and in the latter, direct east or west. Now a ship cannot be continually changing her course, therefore a ship cannot be made to sail on a great circle; but she may be made to sail very nearly on a great circle by changing her course at short intervals, such as at every 5° of change of longitude made, and by this means all the advantage is gained which would be by sailing on a great circle.

270. The arc of a great circle passing through two places on the surface of the earth is the shortest distance between

these places; and hence, the nearer a ship can sail to this great circle, the shorter will her passage be from the one place to the other. In sailing between two places in high latitudes, whose difference of longitude is large, the distance in sailing between them is much shortened by keeping on or near the great circle joining them. The following rules and formulæ, deduced from the principles of Spherical Trigonometry, can easily be applied to the complete solution of the problem.

Let P_1, P_2 be two places on the earth, and let them be joined by a great circle produced, if necessary, to meet the equator EQ in A; and let P_1 be the place whose latitude is least, P_2 that whose latitude is greatest, and P any other place on the great circle.



Take l_1 = the latitude of the place P_1 nearest to the equator,
l_2 = the latitude of the other place P_2,
l = the latitude of any other place on the great circle joining P_1 and P_2,
A = the point where the great circle cuts the equator,
x = the arc of the equator intercepted between A and the meridian of P_1,
d = the difference of longitude of the two places,
L = the arc of the equator measured from A to the meridian of any place P on the great circle; then,

271. cot. $x = \frac{1}{\text{sin. } d}\left\{\frac{\text{tan. } l_2}{\text{tan. } l_1} \mp \text{cos. } d\right\}$, where the upper sign is to be taken when the two places are on the same side of the equator, and the under when they are on opposite sides; but observing that if d is greater than 90°, cos. d is then negative, and consequently its signs in the formula must be changed.

272. To find the angle A at which the great circle cuts the equator, $\text{tan. A} = \text{tan. } l_1 \text{ cosec. } x$.

273. To find the course at any point on the great circle,

$\text{cos. course} = \text{sin. A cos. L}$, or $\text{sin. course} = \text{cos. A sec. } l$.

274. The courses thus found for two adjacent values of L or l, should be changed from the one into the other midway between these values; by this means the ship will be on the great circle at the point for which the calculation is made; and the tracks which the ship makes will be tangents to the great circle.

275. To find the distance between two places on the arc of a great circle joining them.

Using the same notation as in Article (270), we have

$\text{cos. AP}_1 = \text{cos. } x \text{ cos. } l_1$ and $\text{cos. AP}_2 = \text{cos. } (x + d) \text{ cos. } l_2$;

then $\text{AP}_2 - \text{AP}_1 =$ the distance between the places measured on the great circle joining them.

Example I.

Find the various courses for each 5° of longitude in sailing on a great circle of the sphere from New York, in latitude 40° 42′ N. longitude 73° 59′ W., to Cape Clear, in latitude 51° 25′ N. longitude 9° 29′ W.

Here l_1	=	40° 42′ N.	Long. New York	=	73° 59′ W.
l_2	=	51° 25′ N.	Long. Cape Clear	=	9° 29′ W.
			$\therefore d$	=	64° 30′ W.

To find x by Art. (271) we have

$$\text{cot. } x = \frac{1}{\text{sin. } d}\left\{\frac{\text{tan. } l_2}{\text{tan. } l_1} - \text{cos. } d\right\} = \text{cosec. } d\left\{\frac{\text{tan. } l_2}{\text{tan. } l_1} - \text{cos. } d\right\}$$

First calculate $\frac{\text{tan. } l_2}{\text{tan. } l_1} - \text{cos. } d$ as follows:

Log. tan. 51° 25′	=	10·0980987			
Log. tan. 40° 42′	=	9·9345670			
Difference	=	·1635317	=	Log. 1·4572424	
Log. cosec. d =	10·0445118	cos. (d = 64° 30′)	=	·4305111	
	·0114568		=	Log. 1·0267313	
Log. cot. x =	10·0559686	∴ x =	41° 19′ 8″		

Having now found x or AB_1 = 41° 19′, we can now find angle A by Art. (272),

Log. tan. (l_1 = 40° 42′) + 10	=	19·9345670
Log. sin. (x = 41° 19)	=	9·8196888
Log. tan. 52° 29′	=	10·1148782

∴ A = 52° 29′

which is also the highest latitude reached in sailing between the two places on a great circle.

Next, to find the *course* to be sailed at every 5° of longitude, we have, by Art. (273), cos. course = sin. A cos. L; and in starting from New York, L = x = 41° 19′; and at Cape Clear L = d + x = 64° 30′ + 41° 19′ = 105° 49′. We shall here calculate the course for L = 41° 19′, then for L = 45° 49′, 50° 49′, 55° 49′, 60° 49′, 65° 49′, 70° 49′, 75° 49′, 80° 49′, 85° 49′, 90° 49′, 95° 49′, 100° 49′, and for L = 105° 49′ in succession; thus,

(1.) Log. sin. A	=	9·8993697	
Log. cos. 41° 19′	=	9·8756816	
Log. cos. 53° 26′	=	9·7750513	∴ Course = N. 53° 26′ E. from 41° 19′ to 43° 34′.
(2.) Log. sin. A	=	9·8993697	
Log. cos. 45° 49′	=	9·8432057	
Log. cos. 56° 26′	=	9·7425754	∴ Course = N. 56° 26′ E. from 43° 34′ to 48° 19′.
(3.) Log. sin. A	=	9·8993697	
Log. cos. 50° 49′	=	9·8005823	
Log. cos. 59° 55′	=	9·6999520	∴ Course = N. 59° 55′ E. from 48° 19′ to 53° 19′.
(4.) Log. sin. A	=	9·8993697	
Log. cos. 55° 49′	=	9·7496148	
Log. cos. 63° 32′	=	9·6489845	∴ Course = N. 63° 32′ E. from 53° 19′ to 58° 19′.
(5.) Log. sin. A	=	9·8993697	
Log. cos. 60° 49′	=	9·6880688	
Log. cos. 67° 15′	=	9·5874385	∴ Course = N. 67° 15′ E. from 58° 19′ to 63° 19′.
(6.) Log. sin. A	=	9·8993697	
Log. cos. 65° 49′	=	9·6124211	
Log. cos. 71° 2′	=	9·5117908	∴ Course = N. 71° 2′ E. from 63° 19′ to 68° 19′.

(7.) Log. sin. A = 9·8993697
Log. cos. 70° 49′ = 9·5166569
Log. cos. 74° 54′ = 9·4160266 ∴ Course = N. 74° 54′ E. from 68° 19′ to 73° 19′.

(8.) Log. sin. A = 9·8993697
Log. cos. 75° 49′ = 9·3892111
Log. cos. 78° 48′ = 9·2885808 ∴ Course = N. 78° 48′ E. from 73° 19′ to 78° 19′

(9.) Log. sin. A = 9·8993697
Log. cos. 80° 49′ = 9·2030167
Log. cos. 82° 44′ = 9·1023864 ∴ Course = N. 82° 44′ E. from 78° 19′ to 83° 19′.

(10.) Log. sin. A = 9·8993697
Log. cos. 85° 49′ = 8·8630139
Log. cos. 86° 41′ = 8·7623836 ∴ Course = N. 86° 41′ E. from 83° 19′ to 88° 19′.

(11.) Log. sin. A = 9·8993697
Log. cos. 90° 49′ = 8·1539075
Log. cos. 89° 20′ = 8·0532772 ∴ Course = S. 89° 20′ E. from 88° 19′ to 93° 19′.

(12.) Log. sin. A = 9·8993697
Log. cos. 95° 49′ = 9·0058053
Log. cos. 85° 23′ = 8·9051750 ∴ Course = S. 85° 23′ E. from 93° 19′ to 98° 19′.

(13.) Log. sin. A = 9·8993697
Log. cos. 100° 49′ = 9·2733880
Log. cos. 81° 26′ = 9·1727577 ∴ Course = S. 81° 26′ E. from 98° 19′ to 103° 19′.

(14.) Log. sin. A = 9·8993697
Log. cos. 105° 49′ = 9·4354623
Log. cos. 77° 31′ = 9·3348320 ∴ Course = S. 77° 31′ E. from 103° 19′ to 105° 49′.

276. It should be observed that in every case, when the arc AP passes 90°, the *course* changes from *North* to *South* or from *South* to *North;* and that when the arc AP = 90°, the course is always directly east or west.

The above also contains the courses to be sailed from Cape Clear to New York, by beginning at the end and reading forwards, changing *from* into *to*, and *to* into *from; North* into *South*, and *East* into *West;* thus,

From AP_2 = 105° 49′ to AP = 103° 19′ the course is N. 77° 31′ W.
" AP = 103° 19′ " AP = 98° 19′ " " N. 81° 26′ W.

From AP	=	98° 19′	to AP	=	93° 19′	the course is	N. 85° 23′	W.
" AP	=	93° 19′	" AP	=	88° 19′	" "	N. 89° 20′	W.
" AP	=	88° 19′	" AP	=	83° 19′	" "	S. 86° 41′	W.
" AP	=	83° 19′	" AP	=	78° 19′	" "	S. 82° 44′	W.
" AP	=	78° 19′	" AP	=	73° 19′	" "	S. 78° 48′	W.
" AP	=	73° 19′	" AP	=	68° 19′	" "	S. 74° 54′	W.
" AP	=	68° 19′	" AP	=	63° 19′	" "	S. 71° 2′	W.
" AP	=	63° 19′	" AP	=	58° 19′	" "	S. 67° 15′	W.
" AP	=	58° 19′	" AP	=	53° 19′	" "	S. 63° 32′	W.
" AP	=	53° 19′	" AP	=	48° 19′	" "	S. 59° 55′	W.
" AP	=	48° 19′	" AP	=	43° 34′	" "	S. 56° 26′	W.
" AP	=	43° 34′	" AP_1	=	41° 19′	" "	S. 53° 26′	W.

To find the distance between New York and Cape Clear on a *great circle.*

By the formulæ in Art. (275), taking the values in this case—namely, $l_1 = 40° 42'$, $l_2 = 51° 25'$, $x = 41° 19'$, and $(x + d) = 105° 49'$—we have

Log. cos. 41° 19′	=	9·8756816	Log. cos. 105° 49′	=	9·4354623
Log. cos. 40° 42′	=	9·8797462	Log. cos. 51° 25′	=	9·7949425
Log. cos. 55° 18′	=	9·7554278	Log. cos. 99° 47′	=	9·2304048

$\therefore AP_2 = 99° 47'$

$AP_1 = 55° 18'$

$AP_2 - AP_1 = 44° 29' = 2669$ miles, the distance between New York and Cape Clear by Great-circle Sailing.

To find the course and distance by Mercator's Sailing.

Latitude of New York	=	40° 42′ N.	Mer. pts.	=	2677·8	Long.	73° 59′ W.
Latitude of Cape Clear	=	51° 25′ N.	Mer. pts.	=	3608·7	Long.	9° 29′ W.
		10° 43′	Mer. dif.	=	930·9		64° 30′
		60					60
		643					3870

To find the Course.

Log. 3870 + 10	=	13·5877110
Log. 930·9	=	2·9689030
Log. tan. 76° 28′	=	10·6188080

To find the Distance.

Log. sec. 76° 28′	=	10·6307637
Log. 643	=	2·8082110
Log. 2747·7	=	3·4389747

Hence the distance by Mercator's Sailing = 2747·7

And the distance by Great-circle Sailing = 2669

Difference of the distances is = 78·7 miles.

277. In calculating the distance by Great-circle Sailing, as above, it is necessary to observe that if $x + d$ be greater than 90°, then the arc AP is also greater than 90°; hence the reason of taking the arc AP_2 in the above = 99° 47′.

278. Another method of finding the courses in Great-circle Sailing is to find the latitudes for two points on the great circle sailed on, and from their difference of latitude and the distance between them measured on that circle, find the course from the one to the other, the same as in Plane Sailing; the results so obtained will be more nearly correct than the course can be measured by the compass.

In this method, the latitudes of the two points on the course are found by the formula

$$\text{sin. latitude} = \text{sin. A sin. AP};$$

having found two latitudes by this formula, their difference divided by the arc intercepted between them is the cosine of the course from the one to the other.

279. To find the various courses from New York to Cape Clear by this method, observe that the arc $AP_1 = 55° 18'$, and $AP_2 = 99° 47'$, were already found in the former calculation; and that the calculation may be made for every 5° of the arc AP, take the second, AP = 59° 47′, the third, AP = 64° 47′, and so on, adding 5° to AP at each step; thus,

Sin. latitude = sin. (A = 52° 29′) × sin. 59° 47′

Sin. A	= 9·8993697		Lat. New York	=	40° 42′
Sin. 59° 47′	= 9·9365783		Lat. to	=	43° 16′
Sin. 43° 16′	= 9·8359480		Dif lat. = 154	=	2° 34′

Distance to sail = 59° 47′ − 55° 18′ = 4° 29′ = 269′

Cos. course = $\frac{154}{269}$ (Art. 123).

∴ Log. cos. course = log. 154 + 10 − log. 269.

Log. 154 + 10	= 12·1875207	
Log. 269	= 2·4297523	
Log. cos. 55° 5′	= 9·7577684	∴ Course = N. 55° 5′ E.

To find the second latitude, the latitude in being 43° 16′; and at the point to be sailed to AP = 64° 47′.

Log. sin. A	=	9·8993697	Lat. from	=	43° 16′	
Log. sin. 64° 47′	=	9·9565061	Lat. to	=	45° 51′	
Sin. 45° 51′	=	9·8558758	Dif. lat.	=	2° 35′	= 155 miles.

To find the second course, the distance being 300 miles.

Log. 155 + 10	=	12·1903317	
Log. 300	=	2·4771213	
Cos. 58° 53′	=	9·7132104	∴ Course = N. 58° 53′ E.

To find the third latitude, the latitude in being 45° 51′, and at the point to be sailed to AP = 69° 47′.

Log. sin. A	=	9·8993697	Lat. from	=	45° 51′	
Log. sin. 69° 47′	=	9·9723845	Lat. to	=	48° 6′	
Sin. 48° 6′	=	9·8717542	Dif. lat.	=	2° 15′	= 135 miles.

To find the third course, the distance being 300 miles.

Log. 135 + 10	=	12·1303338	
Log. 300	=	2·4771213	
Cos. 63° 15′	=	9·6532125	∴ Course = N. 63° 15′ E.

To find the fourth latitude, the latitude in being 48° 6′, and at the point to be sailed to AP = 74° 47′.

Log. sin. A	=	9·8993697	Lat. from	=	48° 6′	
Log. sin. 74° 47′	=	9·9845004	Lat. to	=	49° 56′	
Sin. 49° 56′	=	9·8838701	Dif. lat.	=	1° 50′	= 110 miles.

To find the fourth course, the distance being 300 miles.

Log. 110 + 10	=	12·0413927	
Log. 300	=	2·4771213	
Cos. 68° 29′	=	9·5642714	∴ Course = N. 68° 29′ E.

To find the fifth latitude, the latitude in being 49° 56′, and at the point to be sailed to AP = 79° 47′.

Log. sin. A	=	9·8993697	Lat. from	=	49° 56′	
Log. sin. 79° 47′	=	9·9930587	Lat. to	=	51° 19′	
Sin. 51° 19′	=	9·8924284	Dif. lat.	=	1° 23′	= 83 miles.

To find the fifth course, the distance being 300 miles.

Log. 83 + 10	=	11·9190781	
Log. 300	=	2·4771213	
Cos. 73° 56′	=	9·4419568	∴ Course = N. 73° 56′ E.

To find the sixth latitude, the latitude in being 51° 19′, and at the point to be sailed to AP = 84° 47′.

Log. sin. A	=	9·8993697	Lat. from	=	51° 19′
Log. sin. 84° 47′	=	9·9981974	Lat. to	=	52° 11′
Sin. 52° 11′	=	9·8975671	Dif. lat.	=	52 miles.

To find the sixth course, the distance being 300 miles.

Log. 52 + 10	=	11·7160033	
Log. 300	=	2·4771213	
Cos. 80° 1′	=	9·2388820	∴ Course = N. 80° 1′ E.

To find the seventh latitude, the latitude in being 52° 11′, and at the point to be sailed to AP = 89° 47′.

Log. sin. A	=	9·8993697	Lat. from	=	52° 11′
Log. sin. 89° 47′	=	9·9999969	Lat. to	=	52° 29′
Sin. 52° 29′	=	9·8993666	Dif. lat.	=	18 miles.

To find the seventh course, the distance being 300 miles.

Log. 18 + 10	=	11·2552725	
Log. 300	=	2·4771213	
Cos. 86° 34′	=	8·7781512	∴ Course = N. 86° 34′ E.

To find the eighth latitude, the latitude in being 52° 29′, and at the point to be sailed to AP = 94° 47′.

Log. sin. A	=	9·8993697	Lat. from	=	52° 29′
Log. sin. 94° 47′	=	9·9984848	Lat. to	=	52° 13′
Log. sin. 52° 13′	=	9·8978545	Dif. lat.	=	16 miles.

To find the eighth course, the distance being 300 miles.

Log. 16 + 10	=	11·2041200	
Log. 300	=	2·4771213	
Log. cos. 86° 57′	=	8·7269987	∴ Course = S. 86° 57′ E.

The ninth latitude, being that of Cape Clear, is 51° 25′ N.
Hence dif. lat. = 52° 13′ − (51° 25′) = 48 miles, and therefore

To find the ninth course, the distance being 300 miles.

Log. 48 + 10	=	11·6812412	
Log. 300	=	2·4771213	
Log. cos. 80° 47′	=	9·2041199	∴ Course = S. 80° 47′ E.

Example II.

Find the course at every 5° of difference of longitude in sailing on a great circle from Cape St Roque, in latitude 5° 10′ S., longitude 35° 40′ W., to Cape Clear, in latitude 51° 25′ N., longitude 9° 29′ W.

In this example the latitudes are of opposite names, or the places are on opposite sides of the equator; hence the formula for finding x is

$$\text{cot. } x = \text{cosec. } d \left\{ \frac{\text{tan. } l_2}{\text{tan. } l_1} + \text{cos. } d \right\};$$

where $l_1 = 5° 10'$, $l_2 = 51° 25'$, and $d = 35° 40' - (9° 29') = 26° 11'$.

Log. tan. ($l_2 = 51° 25'$)	=	10·0980987	
Log. tan. ($l_1 = 5° 10'$)	=	8·9562672	
		1·1418315	= Log. 13·86218
Nat. cos. 26° 11′	=	·8973868	
		14·7595668	= tan. l_2 / tan. l_1 + cos. d
Log. 14·7595668	=	1·1690735	
Log. cosec. ($d = 26° 11'$)	=	10·3553204	
Log. cot. 1° 43′	=	11·5243939	∴ $x = 1° 43'$

and hence $d - x = 26° 11' - (1° 43') = 24° 28'$

To find A, we have (Art. 272) tan. A = tan. l_1 cosec. x.

Log. tan. ($l_1 = 5° 10'$)	=	8·9562672	
Log. cosec. ($x = 1° 43'$)	=	11·5235016	
Log. tan. 71° 40′	=	10·4797688	∴ A = 71° 40′.

To find the course at St Roque, where L = 1° 43′.

Log. sin. (A = 71° 40′)	=	9·9773772	
Log. cos. 1° 43′	=	9·9998050	
Log. cos. 18° 25′	=	9·9771822	∴ Course = N. 18° 25′ E.

At A the course is 90° − (71° 40′) = 18° 20′.

To find the course at 5° of longitude from A, we have

Log. sin. A	=	9·9773772	
Log. cos. 5°	=	9·9983442	
Log. cos. 19°	=	9·9757214	∴ Course = N. 19° E.

To find the course at 10° of longitude from A.

Log. sin. A	=	9·9773772	
Log. cos. 10°	=	9·9933515	
Log. cos. 20° 48′	=	9·9707287	∴ Course = N. 20° 48′ E.

To find the course at 15° of longitude from A.

Log. sin. A	=	9·9773772	
Log. cos. 15°	=	9·9849438	
Log. cos. 23° 31′	=	9·9623210	∴ Course = N. 23° 31′ E.

To find the course at 20° of longitude from A.

Log. sin. A	=	9·9773772	
Log. cos. 20°	=	9·9729858	
Log. cos. 26° 52′	=	9·9503630	∴ Course = N. 26° 52′ E.

To find the course at Cape Clear, longitude 24° 28′ from A.

Log. sin. A	=	9·9773772	
Log. cos. 24° 28′	=	9·9591380	
Log. cos. 30° 14′	=	9·9365152	∴ Course = N. 30° 14′ E.

To find the distance between Cape St Roque and Cape Clear on a great circle.

By the formula in Art. (275), taking the values in this case, $l_1 = 5° 10'$, $l_2 = 51° 25'$, $x = 1° 43'$, and $d - x = 24° 28'$, we have

Log. cos. ($l_1 = 5° 10'$)	=	9·9982318	Log. cos. ($l_2 = 51° 25'$)	=	9·7949425
Log. cos. ($x = 1° 43'$)	=	9·9998050	Log. cos. ($d - x = 24° 28'$)	=	9·9591380
Log. cos. 5° 27′	=	9·9980368	Log. cos. 55° 25′	=	9·7540805

Whole distance = 55° 25′ + 5° 27′ = 60° 52′ = 3652 miles.

To find the distance by Mercator's Sailing.

Lat. Cape St Roque =	5° 10′ S.	Mer. parts =	310·4	Long. =	35° 40′ W.
Lat. of Cape Clear =	51° 25′ N.	Mer. parts =	3608·7	Long. =	9° 29′ W.
	56° 35′	Mer. dif. lat. =	3919·1	Dif. long. =	26° 11′
	60				60
	3395				1571

To find the Course from Cape Clear to St Roque.			To find the Distance.		
Log. 1571 + 10	=	13·1961762	Log. sec. 21° 51′	=	10·0323765
Log. 3919·1	=	3·5931863	Log. 3395	=	3·5308398
Log. tan. 21° 51′	=	9·6029899	Log. 3657·7	=	3·5632163

Hence the distance by Mercator's Sailing = 3657·7
And the distance by Great-circle Sailing = 3652
Difference of distances = 5·7 miles.

280. It is necessary to remark that the courses found above should be changed in every case midway between the places for which they are calculated, except in the method given in Arts. (278, 279), where the change of course takes place at the extreme points of the arc. By changing the course midway between the points, the ship's course will touch the great circle at each of these points with the course calculated for that point.

Example III.

To find the course by Great-circle Sailing for every 5° of longitude in sailing from Cape of Good Hope, in latitude 34° 29′ S., longitude 18° 23′ E., to Jaffa Cape, in Australia, in latitude 36° 57′ S., longitude 139° 41′ E.

Here l_2	=	36° 57′ S.	Mer. pts.	=	2388·9	Long.$_2$	= 139° 41′ E.
l_1	=	34° 29′ S.	Mer. pts.	=	2206·6	Long.$_1$	= 18° 23′ E.
Dif. lat.	=	2° 28′ = 148 miles.			182·3	Dif. long.	= 121° 18′ = 7278 miles.

To find the arc x or AP_1.

Log. tan. (l_2 = 36° 57′)	=	9·8763257
Log. tan. (l_1 = 34° 29′)	=	9·8368636
	=	0·0394621 = Log. 1·0951215
+ Nat. cos. 121° 18′	=	·5195191
	=	Log. 1·6146406 = 0·2080758
Log. cosec. (d = 121° 18′)	=	10·0683089
Log. cot. (x = 27° 53′)	=	10·2763847

$\therefore x = 27° 53′$, and hence $AP_2 = 121° 18′ + 27° 53′ = 149° 11′$

To find A (Art. 272) tan. A = tan. l_1 cosec. x.

Log. tan. (l_1 = 34° 29′)	=	9·8368636
Log. cosec. (x = 27° 53′)	=	·3300580
Log. tan. 55° 45′	=	10·1669216 $\therefore$ A = 55° 45′.

To find the course at Cape of Good Hope, L = 27° 53′.

Log. sin. (A = 55° 45′)	=	9·9172900	
Log. cos. 27° 53′	=	9·9464040	
Log. cos. 43° 4′	=	9·8636940	∴ Course = S. 43° 4′ E.

To find the course at L = 32° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 32° 53′	=	9·9241644	
Log. cos. 46° 2′	=	9·8414544	∴ Course = S. 46° 2′ E.

To find the course for the point at which L = 37° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 37° 53′	=	9·8972216	
Log. cos. 49° 17′	=	9·8145116	∴ Course = S. 49° 17′ E.

To find the course for the point where L = 42° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 42° 53′	=	9·8649504	
Log. cos. 52° 43′	=	9·7822404	∴ Course = S. 52° 43′ E.

To find the course for the point where L = 47° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 47° 53′	=	9·8264910	
Log. cos. 56° 20′	=	9·7437810	∴ Course = S. 56° 20′ E.

To find the course for the point where L = 52° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 52° 53′	=	9·7806341	
Log. cos. 60° 5′	=	9·6979241	∴ Course = S. 60° 5′ E.

To find the course for the point where L = 57° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 57° 53′	=	9·7256217	
Log. cos. 63° 55′	=	9·6429117	∴ Course = S. 63° 55′ E.

To find the course for the point where L = 62° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 62° 53′	=	9·6587780	
Log. cos. 67° 52′	=	9·5760680	∴ Course = S. 67° 52′ E.

To find the course where L = 67° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 67° 53′	=	9·5757578	
Log. cos. 71° 52′	=	9·4930478	∴ Course = S. 71° 52′ E.

To find the course where L = 72° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 72° 53′	=	9·4688173	
Log. cos. 75° 55′	=	9·3861073	∴ Course = S. 75° 55′ E.

To find the course where L = 77° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 77° 53′	=	9·3220186	
Log. cos. 80° 1′	=	9·2393086	∴ Course = S. 80° 1′ E.

To find the course where L = 82° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 82° 53′	=	9·0930367	
Log. cos. 84° 7′	=	9·0103267	∴ Course = S. 84° 7′ E.

To find the course where L = 87° 53′.

Log. sin. A	=	9·9172900	
Log. cos. 87° 53′	=	8·5674310	
Log. cos. 88° 15′	=	8·4847210	∴ Course = S. 88° 15′ E.

281. Since by Art. (276) the course is always directly east or west when L = 90°, and the course is the same for an arc greater than 90° as for one as much less; we may now take in the next 90°, which is greater than 87° 53′ by 2° 7′; hence the course for 92° 7′ will be the same as for 87° 53′—namely, N. 88° 15′ E.

For L = 97° 7′, the same as for 82° 53′ = N. 84° 7′ E.
" L = 102° 7′, " " 77° 53′ = N. 80° 1′ E.
" L = 107° 7′, " " 72° 53′ = N. 75° 55′ E.
" L = 112° 7′, " " 67° 53′ = N. 71° 52′ E.
" L = 117° 7′, " " 62° 53′ = N. 67° 52′ E.
" L = 122° 7′, " " 57° 53′ = N. 63° 55′ E.

For L = 127° 7′, the same as for 52° 53′ = N. 60° 5′ E.
" L = 132° 7′, " " 47° 53′ = N. 56° 20′ E.
" L = 137° 7′, " " 42° 53′ = N. 52° 43′ E.
" L = 142° 7′, " " 37° 53′ = N. 49° 17′ E.
" L = 147° 7′, " " 32° 53′ = N. 46° 2′ E.

Lastly, to find the course for L = 149° 11′, Jaffa Cape.

Log. sin. A	=	9·9172900	
Log. cos. 149° 11′	=	9·9338976	
Log. cos. 44° 47′	=	9·8511876	∴ Course = N. 44° 47′ E.

282. In finding the courses to be sailed from Cape Jaffa to Cape of Good Hope, taking the same values of L as before, we have only to change N. into S., S. into N., and E. into W., and the courses are all known without any more calculation. As formerly remarked, the courses should be changed midway between the adjacent values of L.

To find the distance on a great circle between the Cape of Good Hope and Jaffa Cape.

Cos. $AP_1 = \cos. x \cos. l_1$ and Cos. $AP_2 = \cos. (x + d) \cos. l_2$.

Log. cos. 27° 53′	=	9·9464040	Log. cos. 149° 11′	=	9·9338976
Log. cos. 34° 29′	=	9·9160805	Log. cos. 36° 57′	=	9·9026339
Log. cos. 43° 14′	=	9·8624845	Log. cos. 133° 20′	=	9·8365315

$AP_2 - AP_1 = 133° 20′ - (43° 14′) = 90° 6′ = 5406$ miles = distance.

By Mercator's Sailing.

To find the Course and Distance.			To find the Distance.		
Log. 7278 + 10	=	13·8620121	Log. sec. 88° 34′	=	11·6018207
Log. 182·3	=	2·2607867	Log. dif. lat. = 148	=	2·1702617
Log. tan. 88° 34′	=	11·6012254	Log. 5916·7	=	3·7720824

Hence the distance by Great-circle Sailing is less than the distance by Mercator's Sailing by 5916·7 − 5406 = 510·7 miles.

Exercises 31.

(367.) In sailing from the Lizard, in latitude 49° 58′ N., longitude 5° 12′ W., to latitude 17° 50′ N., longitude 76° 42′ W.; it is required to find x and A, and the courses at the extreme points.

(368.) In sailing from the Lizard, in latitude 49° 58′ N., longitude 5° 12′ W., to Cape Hatteras, in latitude 35° 14′ N. longitude 75° 34′ W.; it is required to find x and A, and the courses at the extreme points.

(369.) Find the distance on a great circle from Cape Horn, in latitude 55° 58′ S., longitude 67° 21′ W., to Cape of Good Hope, in latitude 34° 29′ S., longitude 18° 23′ E.; find also x and A, and the direction of the great circle at each of these places.

(370.) Find the courses to be sailed for each 5° of longitude on the great circle joining St Helena, in latitude 15° 55′ S., longitude 5° 43′ W., and Cape Horn, in latitude 55° 58′ S., longitude 67° 21′ W.; and first find x and A.

DEMONSTRATION OF THE RULES FOR GREAT-CIRCLE SAILING.

In the spherical triangle, AB_1P_1 (page 203), we have by Art. (321), Rule II., $\sin. x = \cot. A . \tan. l_1$ (1)

and from the triangle, ACP_2,

$$\sin. (d . \mp x) = \cot. A . \tan. l_2 \qquad (2)$$

Dividing (2) by (1),

$$\frac{\sin. (d \mp x)}{\sin. x} = \frac{\cot. A . \tan. l_2}{\cot. A . \tan. l_1} = \frac{\tan. l_2}{\tan. l_1}.$$

Whence (Art. 237) $$\frac{\sin. d \cos. x \mp \cos. d \sin. x}{\sin. x} = \frac{\tan. l_2}{\tan. l_1}.$$

or, $$\sin. d . \cot. x \mp \cos. d = \frac{\tan. l_2}{\tan. l_1};$$

transposing and dividing, we have

$$\cot. x = \frac{1}{\sin. d}\left\{\frac{\tan. l_2}{\tan. l_1} \pm \cos. d\right\} = \operatorname{cosec}. d\left\{\frac{\tan. l_2}{\tan. l_1} \pm \cos. d\right\}$$

Again, from (1) we have $\tan. A = \tan. l_1 \operatorname{cosec}. x$.

And from the spherical triangle ADP, by Art. (321),

$$\cos. APD = \cos. \text{course} = \cos. AD . \sin. A = \cos. L . \sin. A.$$

$$\therefore \cos. \text{course} = \cos. L . \sin. A.$$

CHAPTER X.

NAUTICAL ASTRONOMY.

DEFINITIONS AND EXPLANATION OF TERMS.

283. AXIS.—The axis of the heavens is the diameter of the celestial sphere, about which the diurnal rotation of the celestial sphere takes place, and which is due to the real rotation of the earth. The axis of the heavens is only the axis of the earth prolonged; the imaginary extremities of this axis are the poles of the heavens.

284. EQUINOCTIAL.—The celestial great circle, to the plane of which the axis of the heavens is perpendicular, is called the *equinoctial*, or the celestial equator. It is traced out by extending the plane of the terrestrial equator to the heavens.

285. MERIDIANS.—The celestial meridians are in like manner marked out by extending the planes of the terrestrial meridians; or they are semicircles terminating in the poles of the heavens, and perpendicular to the equinoctial.

286. The *Altitude* of a celestial body is its distance above the horizon, measured on the vertical circle passing through the body. The complement of the altitude is called the *zenith distance*. In the case of sun and moon, the *true* altitude is measured from the rational horizon, and is a little greater than

the altitude measured from the sensible horizon. In the case of the stars, on account of their great distance, the difference of altitude is insensible, whichever horizon be referred to.

287. The *Azimuth* of a celestial body is the arc of the horizon intercepted between the meridian of the observer and the vertical circle passing through the object. The arc of the horizon here spoken of is the measure of the angle at the zenith between the meridian and the vertical circle passing through the object. Vertical circles are sometimes called azimuth circles.

288. AMPLITUDE is also an arc of the horizon. It is the arc comprised between the east point of the horizon and the point where the body rises, or between the west point and where it sets; the former arc is called the rising amplitude of the body, and the latter its setting amplitude. Azimuth is measured either from the north or south points of the horizon; amplitude either from the east or west. When we speak of the azimuth of a body, we refer merely to the azimuth of the vertical circle on which the body is, whatever its altitude on that circle may be; but when we speak of its amplitude, we refer exclusively to its position with respect to the east or west point of the horizon at rising or setting.

289. DECLINATION.—The declination of a celestial object is its distance from the equinoctial, measured on the celestial meridian passing through it, and is either north or south. What is latitude as respects a body on the earth, is declination in reference to a point in the heavens. Celestial meridians are thus sometimes called circles of declination, and what are parallels of latitude on the earth become parallels of declination on the celestial sphere.

290. POLAR DISTANCE is the distance of a celestial object from the elevated pole. It is the complement of the declination when the elevated pole and the object are both on the same side of the equinoctial; but when the pole and object are on opposite sides of the equinoctial, it is 90° increased by the declination.

291. THE ECLIPTIC.—The ecliptic is the great circle on the celestial sphere described by the sun in his apparent annual motion about the earth; in reality, it is the path of the earth about the sun in the contrary direction. The ecliptic crosses the equinoctial at an angle subject to continuous but very small variation, determinable by observation. It is always given with the utmost attainable accuracy in the *Nautical Almanac*, and is at present (April 5, 1866) 23° 27′ 15·43″.

The two points where the ecliptic crosses the equinoctial are called the *equinoctial points*. The sun, in his apparent annual course, passes through these points about the 20th of March and 22d of September—the former being the time of the vernal equinox, and the latter of the autumnal equinox.

292. CELESTIAL LONGITUDE is measured along the ecliptic, from the point of the vernal equinox up to 360°.

293. CELESTIAL LATITUDE is measured from the ecliptic on the great circles that pass through the poles of the ecliptic, and hence cut the ecliptic at right angles; it is measured up to 90°, which is the pole of the ecliptic.

294. RIGHT ASCENSION.—The right ascension of a celestial object is the arc of the equinoctial intercepted between the *vernal equinoctial* point and the declination circle or meridian passing through the object. It is evident from the above that *declination* in a celestial object corresponds to *latitude* in a terrestrial; and that *right ascension* in a celestial object corresponds to *longitude* in a terrestrial, only the right ascension is measured from the vernal equinoctial point, from west to east, up to 360° or 24 hours of time.

295. TIME.—A *sidereal day* is the interval between two successive transits of a fixed star over any meridian; and a *solar* or an *apparent day* is the interval between two successive transits of the sun. The sidereal day commences when the first point of Aries is on the meridian, and the solar or astronomical day when the sun is on the meridian. The *sidereal time* of day is measured by the arc of the equator, or the angle at the pole between the first point of Aries and the meridian of the observer,

estimated in the order of the signs; and the solar or *apparent time* of day is measured by the arc of the equator, or the angle at the pole intercepted between the meridian on which the sun is and the meridian of the observer, estimated also in the order of the signs.

296. The solar time added to the sun's right ascension is the sidereal time, or the sun's right ascension taken from the sidereal time, leaves the solar or apparent time.

297. APPARENT TIME.—When the sun is on the meridian of any place, it is *apparent noon* at that place; when it is in any other position, the angle between the meridian of the place and that on which the sun is, is called the *hour angle* from noon at that place and instant; this angle, converted into time, at the rate of 15° to an hour, is the *apparent* time from noon at the place.

298. MEAN TIME is that which would be shewn by the sun if he revolved in the plane of the equator with the *mean* angular velocity with which he revolves in the ecliptic; the difference between mean and apparent time is called the *Equation of time.* It is given for every day of the year in the *Nautical Almanac,* page 1, for the meridian of Greenwich; that is to say, when it is apparent noon at Greenwich, on any day of the year, the almanac shews the time to be added or subtracted to obtain the corresponding mean time at that meridian.

PREPARATORY PROBLEMS.

Problem I.

299. To convert longitude into time.

RULE.—Multiply the longitude by 4, divide the product of the degrees by 60; the quotient will be the hours, and the remainder the minutes, and the product of the minutes in the angle will be seconds of time, &c. Or reduce by Table, page 313.

Example.

Find the time corresponding to 86° 15′ 30″ of longitude.

86° 15′ 30″
4

5 h. 45 m. 2 s. 0 the time sought.

Exercises 32.

Convert into time the following longitudes:

(371.) 20° 17′ 45″; 29° 18′ 16″; 42° 11′ 15″; and 51° 19′ 45″.
(372.) 17° 18′ 30″; 49° 54′ 11″; 89° 16′; and 75° 12′ 30″.
(373.) 58° 11′ 15″; 65° 13′ 45″; 69° 29′; and 93° 17′ 45″.
(374.) 110° 12′ 30″; 127° 5′ 17″; 130° 19′; and 112° 58′ 45″.
(375.) 149° 11′ 15″; 178° 2′ 30″; 170° 16′; and 156° 46′ 30″.

Problem II.

300. To reduce time into longitude.

RULE.—Reduce the hours and minutes into minutes, after which place the seconds, then divide by 4, and the quotient will be the degrees, minutes, and seconds of longitude required. Or reduce by the Table on page 313.

Example.

Reduce 3 h. 17 m. 14 s. into longitude.

3 h. 17 m. 14 s.
60

4)197 m. 14 s.

49° 18′ 30″ = longitude sought.

Exercises 33.

Reduce the following times into corresponding longitudes:

(376.) 3 h. 12 m. 24 s.; 5 h. 16 m. 19 s.; 7 h. 20 m. 15 s.; and 29 m. 48 s.

(377.) 4 h. 29 m. 5 s.; 5 h. 59 m. 4 s.; 6 h. 3 m. 18 s.; and 50 m. 3 s.

(378.) 8 h. 13 m. 56 s.; 9 h. 28 m. 12 s.; 10 h. 20 m. 6 s.; and 59 m. 16 s.

(379.) 11 h. 15 m. 43 s.; 10 h. 58 m. 14 s.; 9 h. 43 m. 8 s.; and 57 m. 56 s.

(380.) 10 h. 12 m. 40 s.; 11 h. 39 m. 8 s.; 11 h. 59 m. 20 s.; and 50 m. 50 s.

Problem III.

301. To express civil time astronomically and conversely.

The civil day begins at midnight, and the astronomical day at the following noon, so that astronomical time is 12 hours slower than civil time; but astronomical time is estimated from one noon to the next up to 24 hours, while the civil day is divided into two periods of 12 hours each; the first 12 hours, from midnight to noon, is called A.M.; and the second, from noon to midnight, is called P.M.: hence the following:

RULE I.—If the civil time be P.M., the astronomical time is the same; but if it be A.M., add 12 hours to the civil time, and deduct one day for the corresponding astronomical time.

RULE II.—If the hour of the astronomical time be less than 12, write P.M. after it, and it will be the required civil time; but if the astronomical time be greater than 12 hours, add 1 to the days, diminish the hours by 12, and write A.M. after it; the remainder will be the required civil time.

Exercises 34.

Express the following civil times astronomically:

(381.) January 19, 5 h. 17 m. 4 s. P.M.
January 27, 3 h. 7 m. 15 s. A.M.

(382.) February 15, 10 h. 14 m. 23 s. A.M.
August 13, 11 h. 16 m. 12 s. P.M.

(383.)	May 11,	7 h. 35 m. 49 s.	A.M.
	September 12,	8 h. 24 m. 16 s.	P.M.
(384.)	June 25,	12 h. 0 m. 0 s.	A.M.
	July 14,	4 h. 12 m. 56 s.	A.M.
(385.)	March 13,	10 h. 12 m. 17 s.	P.M.
	March 27,	11 h. 43 m. 16 s.	A.M.
(386.)	November 1,	7 h. 24 m. 13 s.	A.M.
	November 19,	10 h. 12 m. 26 s.	P.M.

Exercises 35.

Express the following astronomical dates in civil time:

(387.)	January 21,	16 h. 14 m. 3 s.
	February 9,	11 h. 12 m. 14 s.
(388.)	May 14,	7 h. 11 m. 15 s.
	March 12,	23 h. 7 m. 15 s.
(389.)	June 27,	16 h. 11 m. 6 s.
	July 23,	5 h. 17 m. 33 s.
(390.)	March 31,	20 h. 5 m. 12 s.
	March 21,	7 h. 12 m. 24 s.
(391.)	September 12,	5 h. 7 m. 30 s.
	October 20,	17 h. 5 m. 28 s.
(392.)	October 11,	15 h. 13 m. 27 s.
	December 11,	5 h. 12 m. 16 s.

Problem IV.

302. Given the time at any place, and the longitude of that place, to find the corresponding Greenwich time expressed astronomically.

RULE.—Reduce the longitude into time (Art. 299); and if *west, add* it to the astronomical time at the given place, but *subtract* it if *east;* and the sum, or the remainder, will be the required Greenwich time.

Example I.

In longitude 34° 45′ W., on March 15, at 9 h. 7 m. 12 s. A.M.; required the astronomical time at Greenwich.

Astronomical time, March 14,	21 h. 7 m. 12 s.
Longitude in time, W.,	+ 2 h. 19 m.
Astronomical time at Greenwich, March 14,	23 h. 26 m. 12 s.

Example II.

In longitude 87° 13′ E., on September 24, at 3 h. 7 m. 4 s. A.M.; required the astronomical time at Greenwich.

Astronomical time, September 23,	15 h. 7 m. 4 s.
Longitude in time,	− 5 h. 48 m. 52 s.
Astronomical time at Greenwich, Sept. 23,	9 h. 18 m. 12 s.

Exercises 36.

(393.) Find the time at Greenwich corresponding to October 17, 4 h. 23 m. 4 s. A.M., at a place in longitude 43° 12′ W.

(394.) Find the time at Greenwich corresponding to February 4, 9 h. 16 m. 22 s. P.M., at a place in longitude 17° 4′ E.

(395.) Find the time at Greenwich corresponding to May 30, 11 h. 3 m. 4 s. A.M., at a place in longitude 163° 28′ W.

(396.) Find the time at Greenwich corresponding to July 26, 4 h. 0 m. 0 s. A.M., at a place in longitude 120° W.

(397.) Find the time at Greenwich corresponding to December 27, 6 h. 13 m. 51 s. P.M., at a place in longitude 21° 18′ W.

(398.) Find the time at Greenwich corresponding to August 1, 1 h. 0 m. 8 s. P.M., at a place in longitude 53° 4′ E.

(399.) Find the time at Greenwich corresponding to July 18, 5 h. 24 m. 0 s. A.M., at a place in longitude 40° 20′ W.

(400.) Find the time at Greenwich corresponding to June 19, 1 h. 12 m. 40 s. P.M., at a place in longitude 90° 37′ 30″ E.

(401.) Find the time at Greenwich corresponding to July 18, 8 h. 5 m. 20 s. A.M., at a place in longitude 40° 20′ W.

(402.) Find the time at Greenwich corresponding to June 19, 1 h. 12 m. 40 s. P.M., at a place in longitude 75° 36′ 45″ E.

Problem V.

303. To reduce the declination and right ascension of the sun, as given in the *Nautical Almanac*, to any other meridian, and to any given time of the day.

RULE.—Find the time at Greenwich, and the daily change of declination or right ascension; then, as 24 h. : the time past the previous noon at Greenwich : : the daily change : the correction to be applied to the declination or right ascension at the previous noon; which, for the declination, must be added or subtracted according as it is increasing or decreasing. Or, by Diurnal Proportional Logarithms (Table, pages 291, 292), to the logarithm of the daily change add the logarithm of the time past the previous noon at Greenwich; and the sum will be the proportional logarithm of the correction required.

304. If the change for one hour be given, the correction may be found by multiplying the hourly change by the number of hours and parts of an hour past the previous noon at Greenwich.

305. The correction of the sun's latitude, longitude, and semidiameter, and of the equation of time, may all be found by a similar process.

Example.

Find the sun's declination and right ascension at a place in longitude 43° 20′ W., on the 27th October 1866, at 8 h. 35 m. A.M., mean time at the place.

Mean time at the given place, October 26	=	$20^h\ 35^m\ 0^s$		
Longitude in time (Art. 299)	=	2 53 20		
Mean time at Greenwich, October 26	=	23 28 20		
☉'s dec. at mean noon, 26th October 1866	=	12° 28′ 16·6″ S.		
" " 27th October 1866	=	12° 48′ 41·3″ S.		
Change of dec. in 24 h.	=	20′ 24·7″	P.L. =	7038
Proportional logarithm of 23 h. 28 m. 20 s.	=		P.L. =	960
		+ 19′ 57·8″	P.L. =	7998
Hence the ☉'s dec., Oct. 26, 23 h. 28 m. 20 s.	=	12° 48′ 14·4″		

⊙'s R.A. at mean noon, 26th October 1866	=	14^h	2^m	$35{\cdot}49^s$		
" " 27th October 1866	=	14	6	26·40		
Change of R.A. in 24 h.	=		3	50·91	P.L.	= ·79492
Proportional logarithm of 23 h. 28 m. 20 s.	=				P.L.	= 966
			+3	45·83	P.L.	= ·80458
Hence the ⊙'s R.A., 26th Oct. at 23 h. 28 m. 20 s.	=	14	6	21·32		

NOTE.—⊙'s dec. means sun's declination; ⊙'s R.A. means sun's right ascension; and P.L. means proportional logarithms.

Exercises 37.

(403.) Find the sun's declination and right ascension on the 10th October 1866, at 3 h. 43 m. 12 s. P.M., mean time at a place in longitude 36° 17′ W.

(404.) Find the sun's declination and right ascension at a place in longitude 21° 30′ E., on the 28th March 1866, at 3 h. 46 m. P.M. mean time.

(405.) Find the sun's declination and right ascension at New York, in longitude 73° 59′ W., on the 10th March 1866, at 10 h. 7 m. A.M. mean time.

(406.) Find the sun's declination and right ascension at Constantinople, in longitude 28° 55′ 15″ E., on the 20th March 1866, at 5 h. 27 m. 30 s. P.M. mean time.

(407.) Find the sun's declination and right ascension at Calcutta, in longitude 88° 26′ E., on the 5th March 1866, at 2 h. 50 m. 40 s. P.M. mean time.

(408.) Find the sun's declination and right ascension at Boston, United States, in longitude 70° 59′ W., on the 10th October 1866, at 3 h. 14 m. P.M. apparent time.

(409.) Find the sun's declination and right ascension at Port Jackson, Nova Scotia, in longitude 64° 28′ W., on the 19th October 1866, at 7 h. 42 m. 8 s. A.M. apparent time.

(410.) Find the sun's declination and right ascension at Cape St Roque, Brazil, in longitude 35° 40′ W., on the 30th March 1866, at 9 h. 29 m. 39 s. P.M. apparent time.

(411.) Find the sun's declination and right ascension at South Cape, Tasmania, in longitude 146° 49′ E., on March 22, 1866, at 5 h. 13 m. A.M. apparent time.

(412.) Find the sun's declination and right ascension at Port Royal, Jamaica, in longitude 76° 52′ 30″ W., on the 24th October 1866, at 11 h. 30 m. A.M. apparent time.

Problem VI.

306. To reduce the moon's declination and right ascension, as given in the *Nautical Almanac*, to any given meridian and to any time of the day.

RULE.—Find the Greenwich date, and the declination for the preceding hour of that date; then, as 10 minutes is to the time past that hour, so is the change of declination in 10 minutes to the change of declination in the time past that hour, which add to the declination formerly found if the declination be increasing, but subtract if it be decreasing.

307. The moon's right ascension may be found in the same way; and the moon's semidiameter, horizontal parallax, &c., may be reduced by the following proportion—As 12 h. : the time at Greenwich : : the change in the moon's semidiameter in 12 h. : its change in the time at Greenwich; which change being added to, or subtracted from, the moon's semidiameter, &c., at the preceding noon or midnight, according as it is increasing or decreasing, will give the semidiameter, &c., at the time required. Or correct by Table, page 317.

Example I.

Find the declination, right ascension, semidiameter, and horizontal parallax of the moon on the 22d November, 1 h. 10 m. A.M. mean time, at a place in longitude 33° 15′ W.

Solution.

Mean astronomical time at the given place, Nov. 21, 1866	=	$13^h\ 10^m$
Longitude in time	=	2 13
Mean ast. time at Greenwich, Nov. 21	=	15 23

☾'s dec. Nov. 21, 15 h.	= 15° 5′ 48″	☾'s R.A., Nov. 21, 15 h.	= 3ʰ 35ᵐ 54·29ˢ
10 m. : 23 m. :: 70·39″ :	+ 2′ 41·9″	60 m. : 23 m. :: 2 m. 35·86 s. :	+ 59·74
Declination sought	= 15° 8′ 29·9″	R.A. sought	= 3 36 54·03
☾'s semid. at midnight	= 16′ 45·7″	Hor. par. at midnight	= 61′ 24·6″
12 h. : 3 h. 23 m. :: 1·2″ :	0·3″	12 h. : 3 h. 23 m. :: 4·4″	= 1·2″
☾'s semid. sought	= 16′ 46·0″	Hor. par. sought	= 61′ 25·8″

Example II.

Find the moon's declination, right ascension, semidiameter, and horizontal parallax on the 17th November 1866, at 11 h. 52 m. P.M. mean time, in longitude 41° 45′ W.

Solution.

Mean astronomical time at the given place, Nov. 17	= 11ʰ 52ᵐ
Longitude in time	= 2 47
Mean ast. time at Greenwich, Nov. 17	= 14 39

☾'s dec. Nov. 17, 14 h.	= 1° 8′ 52″ S.	☾'s R.A., Nov. 17, 14 h.	= 23ʰ 44ᵐ 36·36ˢ
10 m. : 39 m. :: 108·6″ :	− 7′ 3·5″	60 m. : 39 m. :: 2 m. 11·45 s. :	+ 1 25·44
☾'s dec. sought	= 1° 1′ 48·5″	☾'s R.A. sought	= 23 46 1·80
☾'s semid. at midnight	= 15′ 58·1″	Hor. par. at midnight	= 58′ 30·1″
12 h. : 2 h. 39 m. :: 8·0″ :	+ 1·8″	12 h. : 2 h. 39 m. :: 29·4 s. :	6·5″
☾'s semidiameter sought	= 15′ 59·9″	Hor. par. sought	= 58′ 36·6″

Exercises 38.

(413.) Find the moon's declination and right ascension on the 12th November 1866, at 9 h. 54 m. P.M. mean time, in longitude 14° 30′ E.

(414.) Find the moon's declination and right ascension on the 27th November 1866, at 11 h. 34 m. P.M. mean time, in longitude 27° 45′ E.

(415.) Find the moon's declination and right ascension on the 15th November 1866, at 1 h. 13 m. A.M. mean time, in longitude 36° 30′ W.

(416). Find the moon's declination and right ascension on the

23d November 1866, at 3 h. 23 m. 30 s. A.M. mean time, in longitude 39° 25′ W.

(417.) Find the moon's declination and right ascension on the 28th March 1866, at 9 h. 33 m. P.M. mean time, in longitude 73° 59′ W.

(418.) Find the moon's declination and right ascension on the 31st March 1866, at 5 h. 43 m. A.M. mean time, in longitude 62° 30′ E.

(419.) Find the moon's semidiameter and horizontal parallax on the 27th March 1866, at 7 h. 35 m. P.M. mean time, in longitude 43° 15′ W.

(420.) Find the moon's semidiameter and horizontal parallax on the 15th March 1866, at 11 h. 30 m. P.M. mean time, in longitude 45° 30′ E.

(421.) Find the moon's semidiameter and horizontal parallax on the 27th November 1866, at 1 h. 14 m. A.M. mean time, in longitude 47° 50′ E.

(422.) Find the moon's semidiameter and horizontal parallax on the 20th November 1866, at 10 h. 10 m. 30 s. A.M. mean time, in longitude 139° 45′ E.

Problem VII.

308. To correct the observed altitude of the sun or of a star, for index error of the instrument, dip of the horizon for the height of the eye, for refraction and parallax, and for the semidiameter; to obtain the true altitude of the centre.

RULE.

1. Correct the observed altitude for the index error of the instrument by addition or subtraction, as directed in the question.

2. Subtract the apparent depression of the horizon (page 309) corresponding to the height of the eye.

3. Subtract the mean refraction (page 307) corresponding to

the observed altitude as above corrected; and add the sun's parallax in altitude (page 311).

4. Add the sun's semidiameter to the former result, if the *lower limb* was observed; but subtract it if the *upper limb* was observed.

5. If the observed altitude be that of a star, the corrections for parallax and semidiameter are unnecessary; for the fixed stars have neither parallax nor semidiameter, on account of their extreme distance.

309. NOTE.—The sun's semidiameter is found on the second page of the month of the *Nautical Almanac.* The symbol $\underline{\odot}$, means the sun's lower limb, and $\overline{\odot}$, the sun's upper limb; also, N.A. means the *Nautical Almanac;* A.T. means astronomical time; G.T. means Greenwich astronomical time; and we will also occasionally use ⊙'s l.l. and ⊙'s u.l., for sun's lower limb, and for sun's upper limb.

Example I.

On November 1, the meridian altitude of the $\overline{\odot}$ was observed to be 24° 36′ 15″, the height of the eye being 18 feet, and the index error − 13″; required the true altitude of the sun's centre.

Observed alt. of the ⊙'s u.l.	=	24° 36′ 15″
Index error		− 13″
		24° 36′ 2″
Dip (page 309)		− 4′ 6″
Apparent altitude of the ⊙'s u.l.	=	24° 31′ 56″
Correction for ref. and par. (pages 307 and 311)	=	1′ 57″
		24° 29′ 59″
⊙'s semidiameter		− 16′ 10″
True alt. of the ⊙'s centre	=	24° 13′ 49″

Example II.

On the 30th March 1866, the meridian altitude of the $\underline{\odot}$ was observed to be 40° 15′ 40″, the height of the eye being 20 feet, and the index error + 1′ 18″; required the true altitude of the sun's centre.

Observed alt. of ⊙'s l.l.	=	40° 15′ 40″
Index error		+ 1′ 18″
		40° 16′ 58″
Dip (page 309)		− 4′ 20″
Apparent alt. of ⊙'s l.l.	=	40° 12′ 38″
Correction for refraction and par.		− 1′ 2″
		40° 11′ 36″
⊙'s semidiameter		+ 16′ 2″
True altitude of the ⊙'s centre	=	40° 27′ 38″

Exercises 39.

In each of the following exercises, the true altitude of the sun's centre, or of the star, is required.

	Date.	Observed Altitude.	Index Error.	Height of Eye.
(423.)	Nov. 12, 1866,	⊙ 14° 11′ 45″	+ 1′ 54″	16 feet.
(424.)	Nov. 1, "	⊙ 25° 34′ 20″	− 2′ 16″	18 "
(425.)	Nov. 7, "	⊙ 40° 17′ 34″	+ 1′ 50″	20 "
(426.)	Nov. 20, "	⊙ 27° 18′ 40″	− 2′ 16″	14 "
(427.)	Mar. 6, "	⊙ 58° 19′ 30″	+ 2′ 5″	24 "
(428.)	Mar. 9, "	⊙ 46° 15′ 19″	− 3′ 16″	17 "
(429.)	Mar. 20, "	⊙ 60° 15′ 40″	+ 1′ 40″	22 "
(430.)	Mar. 30, "	⊙ 59° 56′ 20″	− 1′ 15″	18 "
(431.)	June 19, "	✳ 43° 11′ 48″	+ 2′ 5″	15 "
(432.)	Aug. 14, "	✳ 19° 39′ 15″	+ 3′ 12″	19 "
(433.)	July 17, "	✳ 59° 18′ 50″	− 1′ 30″	20 "
(434.)	May 11, "	✳ 65° 12′ 17″	0′ 0″	21 "

Problem VIII.

310. From the observed altitude of the moon's upper or lower limb, her semidiameter, and horizontal parallax, as given in the *Nautical Almanac*, and the latitude of the place of observation, to find the true altitude of her centre.

RULE.

1. To the semidiameter from the *Nautical Almanac* apply

the augmentation corresponding to the altitude (Table II., page 310).

2. From the *horizontal parallax*, as given in the *Nautical Almanac*, subtract the reduction corresponding to the latitude (Table IV., page 309).

3. Correct the observed altitude for index error and dip, then add or subtract the augmented semidiameter, according as the lower or upper limb was observed; and the sum or remainder will be the *apparent* altitude of the moon's centre.

4. Multiply the seconds in the reduced equatorial parallax by the natural cosine of the apparent altitude, and the product will be the seconds in the parallax in altitude; from which subtract the refraction corresponding to the apparent altitude; and the remainder added to the apparent altitude will be the true altitude. Or, the multiplication may be conveniently performed by logarithms.

Example I.

If the observed altitude of the moon's lower limb be 40° 26′ in latitude 58°, her semidiameter, from the *Nautical Almanac*, being 16′ 2″, and horizontal parallax 58′ 47″, the index error of the instrument + 5′ 2″, and the height of the eye 14 feet; what is the true altitude?

☽'s semidiameter	=	16′ 2″	Horizontal parallax	=	58′ 47″	
Aug. (p. 310)		+ 11	Reduction (page 309)		− 6	
Aug. semidiameter	=	16′ 13″	Reduced parallax	=	58′ 41″	
					60	
					3521	
			inverted	=	57757· nat. cos.	
Obs. altitude of ☽	=	40° 26′ 0″			24647	
Index error		+ 5′ 2″			1761	
		40° 31′ 2″			246	
Cor. for dip		− 3′ 37″			25	
		40° 27′ 25″			2	
Aug. semidiameter		+ 16′ 13″			60)2668·1	
Apparent altitude	=	40° 43′ 38″	Parallax in altitude	=	44′ 28″	
		+ 43′ 22″	Refraction, 40° 44′		− 1′ 6″	
True alt. of centre	=	41° 27′ 0″	Correction of alt.	=	43′ 22″	

Example II.

If the observed altitude of the moon's upper limb be 50° 12′ 20″ in latitude 49°, her semidiameter, from the *Nautical Almanac*, being 15′ 49″, and horizontal parallax 57′ 56″, the index error of the instrument − 3′ 15″, and the height of the eye 24 feet; what is the true altitude?

☽'s semidiameter	=	15′ 49″	Horizontal parallax	=	57′ 56″
Aug. (page 310)		+ 13	Reduction (page 309)		− 8
Aug. semidiameter	=	16′ 2″	Reduced parallax	=	57′ 48
					60
Obs. altitude ☽	=	50° 12′ 20″			
Index error		− 3′ 15″	Seconds of par	=	3468
		50° 9′ 5″			
Cor. for dip		− 4′ 45″			
		50° 4′ 20″			
Aug. semidiameter		− 16′ 2″	Log. 3468	=	3·5400791
Apparent alt.	=	49° 48′ 18″	cos. (app. alt.)	=	9·8098678
		+ 36′ 30″	{37′ 18″	=	3·3499469 = 2238·4
True altitude	=	50° 24′ 48″	= {− 48″		

Exercises 40.

In each of the following exercises, the true altitude of the moon's centre is required:

	Lat.		Observed Altitude.	Index Error.	Semidiameter.	Hor. Paral.	Ht. of Eye.
(435.)	38°	☽	42° 5′ 30″	− 1′ 45″	14′ 49″	54′ 17″	19 feet.
(436.)	46°	☽	30° 24′ 50″	+ 3′ 20″	14′ 45″	54′ 3″	22 "
(437.)	58°	☽	40° 48′ 30″	− 4′ 10″	16′ 32″	60′ 36″	16 "
(438.)	61°	☽	19° 43′ 20″	+ 2′ 50″	15′ 49″	57′ 56″	20 "
(439.)	48°	☽	53° 18′ 20″	+ 5′ 2″	15′ 53″	58′ 14″	14 "
(440.)	20°	☽	49° 2′ 48″	− 4′ 26″	16′ 29″	60′ 27″	19 "
(441.)	17°	☽	57° 14′ 23″	− 3′ 11″	14′ 52″	54′ 30″	20 "
(442.)	32°	☽	63° 42′ 4″	+ 4′ 12″	15′ 8″	55′ 29″	12 "
(443.)	10°	☽	20° 4′ 57″	− 2′ 0″	14′ 47″	54′ 14″	16 "
(444.)	39°	☽	6° 3′ 38″	+ 1′ 48″	16′ 16″	59′ 38″	23 "
(445.)	51°	☽	49° 2′ 16″	− 2′ 18″	14′ 54″	54′ 36″	18 "
(446.)	29°	☽	56° 13′ 12″	− 3′ 16″	14′ 47″	54′ 9″	21 "

Problem IX.

311. To find the polar distance of a celestial object.

RULE.

1. When the latitude of the place and the declination of the object are of the same name—that is, are both *north* or both *south*, the polar distance is (90 − dec.).

2. When the latitude of the place and the declination of the object are of opposite names—that is, the one *north* and the other *south*, the polar distance is (90 + dec.); hence, if P.D. stand for polar distance,

Lat. North,	dec. 11° 28′ S.	gives	P.D. = 101° 28′
Lat. North,	dec. 20° 30′ N.	"	P.D. = 69° 30′
Lat. South,	dec. 22° 50′ N.	"	P.D. = 112° 50′
Lat. South,	dec. 23° 28′ S.	"	P.D. = 66° 32′

Problem X.

312. To find the time at which any celestial object culminates, or passes the meridian of a given place, on a given day.

RULE.

1. From the object's right ascension for the noon of the given day, as found in the *Nautical Almanac*, increased, if necessary by 24 hours, subtract the sun's right ascension, and the remainder will be the required time nearly.

2. Find the corresponding G.T. by Problem IV., and for that time compute the sun's right ascension, and that of the given object by Problem V.; then from the object's right ascension thus corrected, subtract that of the sun; and the remainder will be the required time more nearly. If still greater exactness be required, the operations may be repeated until the times found agree.

3. If the object be the moon, find the time of her meridian

passage, from the *Nautical Almanac*, for the given day, and to it apply the correction found in Table, page 310, by *addition*, if the longitude be *west;* but by *subtraction* if the longitude be *east.*

Example I.

At what time will Altair pass the meridian of Constantinople, in longitude 28° 55′ 15″ E., on the 12th November 1866?

✱'s R.A., Jan. 1, 1866 (N.A. 326)	=	$19^h\ 44^m\ 14{\cdot}635^s$	+ 2·927 annual variation.		
2·927 × ·844	=	+ 2·471			
✱'s R.A. at given time	=	19 44 17·106			(A)
⊙'s R.A. at noon (N.A.)	=	15 9 50·72	$+ 4^m\ 5^s$	P.L. =	·76920
Required time nearly	=	4 34 26·39			
Longitude 28° 55′ 15″ E. in time	=	1 55 41			
Greenwich time	=	2 38 45	P.L. (p. 291)	=	·95765
Correction of ⊙'s R.A.	=	27	P.L.	=	1·72685
⊙'s corrected R.A.	=	15 10 17·72			(B)
Required time	=	4 33 59·39		=	(A)−(B)

Example II.

Required the time at which the moon will pass the meridian of 55° W., on November 17, 1866.

☽'s meridian passage, 17th Nov. 1866 (N.A. p. 205)	=	$7^h\ 44{\cdot}4^m$
" " 18th Nov. 1866	=	8 35·2
Difference	=	50·8

To 51 m. and longitude 55 W. (p. 310) corresponds 7·5 m., which, added to 7 h. 44·4 m., gives the time sought = 7 h. 52 m. nearly.

Exercises 41.

At what time will the following celestial objects pass the meridians of the places named in each question on the given day?

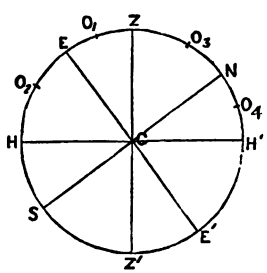
Object.	Date.	Place.	Longitude.
(447.) Fomalhaut,	May 18, 1866,	Cape Hatteras,	75° 34′ 27″ W.
(448.) Aldebaran,	June 1, "	Cape Finisterre,	9° 16′ 0″ W.
(449.) Rigel,	Oct. 10, "	Dondra Head,	80° 43′ 0″ E.
(450.) Sirius,	Mar. 9, "	Cuba,	76° 4′ 30″ W.

Object.	Date.	Place.	Longitude.
(451.) Procyon,	Nov. 20, 1866,	Christiania,	10° 48′ 0″ E.
(452.) Regulus,	Sept. 15, "	Cayenne,	52° 15′ 0″ W.
(453.) Spica,	Jan. 30, "	Botany Bay,	151° 14′ 0″ E.
(454.) Arcturus,	July 16, "	Barbary Point,	16° 31′ 15″ W.
(455.) Moon,	Mar. 1, "	Amber Cape,	49° 25′ 0″ E.
(456.) Moon,	May 28, "	Albany, U.S.,	73° 45′ 0″ W.
(457.) Moon,	July 27, "	Cape Clear,	9° 29′ 0″ W.
(458.) Moon,	Nov. 21, "	Halifax,	63° 35′ 45″ W.

Problem XI.

313. To deduce the latitude from the observed meridian altitude of a known celestial object.

In the annexed figure, let HH′ be the intersection of the plane of the meridian and that of the rational horizon, and EE′ its intersection with the plane of the equator; and if Z be the zenith, then the arc ZE or the angle ECZ = the latitude; but the arcs EZN and ZNH′ are each quadrants; hence, taking ZN from each, there remains ZE = NH′; hence NH′, the altitude of the pole above the horizon, is also the measure of the latitude.



314. Let O_1 be a celestial object having its north declination EO_1; then its altitude is HO_1, ZO_1 is its co-altitude or zenith distance, and the latitude is $ZE = ZO_1 + O_1E$ = co-latitude + dec.; if the object be O_2, having south declination, its altitude is O_2H, and its co-altitude O_2Z, also the latitude $EZ = ZO_2 - O_2E$ = co-latitude − dec. In the same manner, if the object be O_3 or O_4, above or below the pole on the meridian, the latitude of the place is equal to $NH' = H'O_3 - O_3N$, or $= H'O_4 + O_4N$ = altitude ∓ co-dec., according as the object is above or below the pole; hence,

RULE.

1. Find the Greenwich time at the place by Problem IV., and the true declination of the observed body by Problem V. or VI.

2. Find the true altitude of the object's centre by Problem VII.; that is, correct the observed altitude for index error, dip, semi-diameter, refraction, and parallax.

3. Subtract the true altitude from 90°, and mark the remainder N. or S., according as the zenith is north or south of the observed object; under this remainder write the object's true declination, and mark it N. or S., according as it is north or south; then the sum of these two, if they are both north or both south, will be the latitude sought, and of the same name; but if the one be north and the other south, their difference will be the latitude, of the same name as the greater.

4. If the elevated pole and the observed object be on the same side of the zenith, subtract the declination from 90°, and if the object be below the pole, add the remainder to the true altitude; but if the observed object be above the pole, subtract the remainder from the true altitude, and the sum or the remainder will be the latitude sought, of the same name as the elevated pole.

Or, if alt. = the true altitude,
co-alt. = the complement of the true altitude,
dec. = the object's declination,
co-dec. = the complement of the object's declination.

The third above may be concisely expressed thus,

$$\text{Latitude} = \text{co-alt.} \pm \text{dec.}$$

Similarly, the fourth gives,

$$\text{Latitude} = \text{alt.} \pm \text{co-dec.}$$

Example I.

On March 30, 1866, in longitude 65° 17′ W., the observed meridian altitude of the sun's lower limb was 57° 13′ 20″, the

zenith being north of the sun; the index error being − 2′ 24″, and the height of the eye 18 feet; required the latitude of the place of observation.

Ship's date, March		30d 0h 0m 0s apparent time.
Longitude in time		+ 4 21 8
Greenwich time, March	=	30 4 21 8 apparent time.
Declination at apparent noon, 30th March	=	3° 47′ 38″ N, + 58·18″ hourly var.
58·18″ × 4·352 h.		+ 4′ 13″
Reduced declination	=	3° 51′ 51″ N.

Observed altitude of ☉	=	57° 13′ 20″
Index error		− 2′ 24″
		57° 10′ 56″
Correction for dip (p. 309)		− 4′ 6″
		57° 6′ 50″
Refraction − 37″ / Parallax + 4″	=	− 33″
		57° 6′ 17″
Sun's semid.		+ 16′ 2″
Sun's true alt.	=	57° 22′ 19″
		90° 0′ 0″
Zenith distance	=	32° 37′ 41″ N.
Declination	=	3° 51′ 51″ N.
Latitude	=	36° 29′ 32″ N.

Example II.

On November 20, 1866, in latitude 72° 30′ W., the meridian altitude of the moon's lower limb, 47° 12′ N., the index error being + 2′ 15″, and the height of the eye 17 feet; required the latitude of the place.

Moon's meridian passage, Nov. 20, 1866 (N.A.)	=	10h 24·6m + 59m
Correction for longitude, 72° 30′ and 59 m. (p. 320)		+ 11
		10 35·6
Longitude in time, W.		+ 4 50
Time of observation at Greenwich	=	15 25·6

☽'s semidiameter previous midnight	=	16′ 39·2″	and Hor. parall.	=	61′ 1″
At following noon	=	16′ 43·1″		=	61′ 15″
Change in 12 hours		+ 3·9″	Change in 12 h.	=	14″
Change in 3·426 h.		+ 1·1″	Change in 3·426 h.		+ 4″
Moon's semidiameter	=	16′ 40·3″	Hor. par.	=	61′ 5″
Augmentation (p. 310)	=	13″			60
Corrected semidiameter	=	16′ 53″	Seconds of H.P.	=	3665

Observed alt. ☽	=	47° 12′ 0″			
Index error		+ 2′ 15″			
		47° 14′ 15″			
Cor. for dip		− 3′ 59″			
		47° 10′ 16″	Log. 3665	= 3·5640740	
☽'s aug. semid.		+ 16′ 53″	cos.	= 9·8300966	
☽'s apparent alt.	=	47° 27′ 9″	41′ 18″	= 3·3941706	= Log. 2478
Correction		+ 40′ 25″	− 53″	Refraction	
☽'s true alt.	=	48° 7′ 34″			
		90° 0′ 0″			
☽'s zenith dist.	=	41° 52′ 26″ S.			
☽'s dec. (N.A.)	=	11° 52′ 33″ N.			
Latitude	=	30° 0′ 7″ S.			

Example III.

If the meridian altitude of Sirius be 24° 56′ 30″ S., on the 25th May 1879, the index error being − 1′ 50″, and the height of the eye 20 feet; what is the latitude of the place of observation?

✳'s dec. Jan. 1, 1866 (N.A.)	= 16° 32′ 6″		Observed alt.	=	24° 56′ 30″
Correction − 4·643 × 13·4 yrs.	− 1′ 2″		Index error		− 1′ 50″
✳'s true declination	= 16° 31′ 4″ S.				24° 54′ 40″
			Cor. for dip		− 4′ 20″
					24° 50′ 20″
			Cor. for refraction		− 2′ 3″
			✳'s true alt.	=	24° 48′ 17″
					90° 0′ 0″
			✳'s zenith dist.	=	65° 11′ 43″ N.
			✳'s dec.	=	16° 31′ 4″ S.
			Latitude	=	48° 40′ 39″ N.

Exercises 42.

In each of the following exercises the latitude is required from the given meridian altitude of the object:

	Civil Date. 1866.	Longitude. ° ′	Observed Altitude. ° ′ ″	Index Error. ′ ″	Eye. Feet.
(459.)	Jan. 7,	34 17 W.	☉ 53 13 20 S.	+ 2 44	14
(460.)	Jan. 24,	56 13 W.	☉ 48 16 40 S.	− 1 40	15
(461.)	Feb. 17,	39 18 E.	☉ 39 18 30 S.	+ 3 24	18
(462.)	Feb. 27,	34 50 E.	☉ 73 12 40 N.	+ 2 54	16

	Civil Date. 1866.	Longitude. ° ′		Observed Altitude.	°	′	″		Index Error. ′	″	Eye. Feet.
(463.)	Mar. 10,	140	12 E.	☉	53	17	50	N.	−3	40	20
(464.)	Mar. 20,	71	45 W.	☉	24	58	40	S.	+5	16	14
(465.)	April 15,	34	20 E.	☉	43	17	35	N.	+2	25	21
(466.)	April 29,	17	50 W.	☉	84	13	50	S.	−3	20	19
(467.)	May 30,	29	45 W.	☉	27	45	15	N.	+2	30	16
(468.)	May 31,	47	20 E.	☉	41	16	20	N.	−3	5	17
(469.)	June 2,	41	30 W.	☉	78	13	50	S.	+1	50	24
(470.)	June 15,	72	18 W.	☉	72	5	20	S.	+2	10	17
(471.)	June 30,	67	2 W.	☉	12	2	30	N.	+3	7	15
(472.)	July 12,	53	42 E.	☉	79	13	20	S.	−2	18	19
(473.)	July 30,	104	13 E.	☉	42	17	15	N.	+4	20	21
(474.)	July 31,	12	17 W.	☉	85	12	19	S.	−2	40	22
(475.)	Aug. 11,	74	13 E.	☉	45	13	40	N.	+1	50	23
(476.)	Aug. 20,	35	17 W.	☉	77	20	30	N.	−2	10	18
(477.)	Nov. 21,	45	30 W.	☽	75	12	20	S.	+3	20	16
(478.)	Nov. 22,	93	10 E.	☽	54	18	30	N.	+1	40	18
(479.)	Dec. 23,	24	17 W.	☽	48	13	40	N.	−3	12	19
(480.)	Dec. 20,	34	20 W.	☽	72	3	30	N.	+2	10	16
(481.)	Dec. 21,	45	0 E.	☽	24	7	40	N.	−7	40	12
(482.)	Jan. 1,	1870,		Spica	36	17	40	S.	−3	10	16
(483.)	June 2,	1886,		"	71	12	20	N.	+2	50	19
(484.)	June 5,	1890,		Vega	69	48	30	S.	−2	10	20
(485.)	Dec. 31,	1870,		"	50	1	40	N.	+1	45	23
(486.)	Dec. 29,	1877,		Rigel	37	5	45	S.	−2	50	24
(487.)	Jan. 1,	1892,		"	59	18	16	S.	+2	20	12
(488.)	June 3,	1869,		Arcturus	70	5	10	N.	−1	10	14
(489.)	April 1,	1898,		"	43	17	25	N.	+2	35	18

CHAPTER XI.

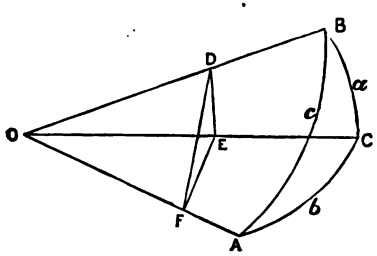
SPHERICAL TRIGONOMETRY.

315. A *sphere* is a solid generated by the revolution of a semicircle about its diameter, and is such that all points on its surface are equally distant from a point within it, called the centre, which is the middle point of the fixed diameter round which the generating semicircle revolves.

316. A *spherical triangle* is a figure formed on the surface of a sphere by the mutual intersection of three great circles. The intercepted parts of each of the three great circles between the other two, form the three sides of the triangle.

317. The angles of a spherical triangle are the plane angles contained by tangents to each of the arcs at the point of their intersection; or they are the inclination of the planes of the great circles which contain the angles.

Let ABC be a spherical triangle formed by the portions AC, CB, and BA of three great circles contained between each of their intersections with the other two; and let the lengths of these in degrees be called a, b, and c; also let O be the centre of the sphere, so that OB, OC, and OA



are all equal; then the arc a is the measure of the angle BOC, b is the measure of the angle AOC, and c is the measure of BOA.

318. Let the angle BCA be a right angle, and consequently the plane OBC perpendicular to the plane OAC; and in OB take any point D; and in the plane OBC draw DE perpendicular to OC, which will therefore be perpendicular to every straight line passing through E in the plane AOC; draw through E, EF perpendicular to OA, and join DF: then the angles DEO, DEF, EFO are right angles. Again, since OFE is a right angle,

$$OF^2 + FE^2 = OE^2; \text{ to each add } DE^2,$$

then $$OF^2 + FE^2 + DE^2 = OE^2 + DE^2;$$

but $$OE^2 + DE^2 = OD^2, \text{ and } FE^2 + DE^2 = FD^2;$$

hence, substituting, we have $OF^2 + FD^2 = OD^2$; and therefore (Euc. I. 48) OFD is a right angle; also EF and DF being each perpendicular to OA, the common intersection of the planes BOA and COA, the angle DFE is the measure of the angle A of the triangle; and therefore, angle A = $\angle$ DFE.

319. Now, from the four right-angled triangles DEF, FEO, OED, and DOF, by applying Articles 122, 123, 124, and 125 of Plane Trigonometry, we obtain the following important relations between the sides and angles of a right-angled spherical triangle,

First, $\frac{OF}{OD} = \frac{OF}{OE} \cdot \frac{OE}{OD}$; that is, $\cos. c = \cos. b \cos. a.$ (1)

Second, $\frac{DE}{OD} = \frac{DE}{DF} \cdot \frac{DF}{OD}$; that is, $\sin. a = \sin. A \sin. c.$ (2)

Similarly, $\sin. b = \sin. B \sin. c.$

Third, $\frac{EF}{OF} = \frac{EF}{DF} \cdot \frac{DF}{OF}$; that is, $\tan. b = \cos. A \tan. c.$ (3)

Similarly, $\tan. a = \cos. B \tan. c.$

Fourth, $\frac{FE}{ED} = \frac{FE}{OE} \cdot \frac{OE}{ED}$; that is, $\cot. A = \sin. b \cot. a.$ (4)

Similarly, $\cot. B = \sin. a \cot. b.$

Fifth, Taking the product of the two expressions in (4), and observing that cot. a sin. a = cos. a, and that cot. b sin. b = cos. b, and that cos. a cos. b is, by (1), equal to cos. c; we have

$$\text{cos. } c = \text{cot. A cot. B.} \qquad (5)$$

Sixth, Dividing the first expression in (2) by the second in (3), and the second in (2) by the first in (3), and observing that sin. $a \div$ tan. a = cos. a, that sin. $b \div$ tan. b = cos. b, and that from (1) cos. $c \div$ cos. a = cos. b, and cos. $c \div$ cos. b = cos. a; we obtain, after a little reduction,

$$\left.\begin{array}{l} \text{cos. A} = \text{sin. B cos. } a, \\ \text{and} \qquad \text{cos. B} = \text{sin. A cos. } b. \end{array}\right\} \qquad (6)$$

320. The above *ten* results can be reduced to the following very symmetrical forms, from which Napier derived his rules for the circular parts—namely:

$$\left.\begin{array}{l} \text{From (2) and (4) we have} \\ \qquad \text{sin. } a = \text{sin. A sin. } c = \text{cot. B tan. } b. \\ \qquad \text{sin. } b = \text{sin. B sin. } c = \text{cot. A tan. } a. \\ \text{From (1) and (5) we have} \\ \qquad \text{cos. } c = \text{cos. } a \text{ cos. } b = \text{cot. A cot. B.} \\ \text{and from (6) and (3) we have} \\ \qquad \text{cos. A} = \text{sin. B cos. } a = \text{tan. } b \text{ cot. } c. \\ \qquad \text{cos. B} = \text{sin. A cos. } b = \text{tan. } a \text{ cot. } c. \end{array}\right\} \qquad (7)$$

NAPIER'S RULES FOR THE CIRCULAR PARTS.

321. If in a right-angled spherical triangle the right angle be left out, and the two sides containing it, with the complements of the two acute angles and of the side opposite the right angle, be called the *five* circular parts; and if these parts be taken three and three, in all the ten ways that this can be done, and one of these parts be called the *middle* part, the other two will either be *adjacent* to it, or they will be each separated from it by another part, in which case they are called *opposite* parts;

then the *two* following rules contain all the *ten* expressions in (7):

RULE I.—Sin. (middle part) = the product of the *cosines* of the *opposite* parts.

RULE II.—Sin. (middle part) = the product of the *tangents* of the *adjacent* parts.

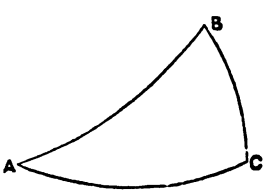
322. In applying these rules, it is necessary to observe that the cos. tan. and cot. of an angle greater than 90° is negative; hence, from (1) and (5), it appears that c is greater than 90°, when a and b or A and B are one greater and the other less than 90°; and that c is less than 90°, when a and b or A and B are both less or both greater than 90°. Also from (4), it appears that A and B are greater or less than 90°, according as a and b are greater or less than 90°; or the sides are of the same affection as the opposite angles.

323. Some may perhaps find it most convenient to associate the results obtained above with the corresponding definitions in Plane Trigonometry; the following can easily be derived from the above ten formulæ:

In Plane Right-angled Triangles.	In Spherical Right-angled Triangles.	
$\text{Sin. A} = \frac{a}{c}$. $\quad \text{sin. B} = \frac{b}{c}$.	$\text{Sin. A} = \frac{\text{sin. } a}{\text{sin. } c}$. $\quad \text{sin. B} = \frac{\text{sin. } b}{\text{sin. } c}$.	(8)
$\text{Cos. A} = \frac{b}{c}$. $\quad \text{cos. B} = \frac{a}{c}$.	$\text{Cos. A} = \frac{\text{tan. } b}{\text{tan. } c}$. $\quad \text{cos. B} = \frac{\text{tan. } a}{\text{tan. } c}$.	(9)
$\text{Tan. A} = \frac{a}{b}$. $\quad \text{tan. B} = \frac{b}{a}$.	$\text{Tan. A} = \frac{\text{tan. } a}{\text{sin. } b}$. $\quad \text{tan. B} = \frac{\text{tan. } b}{\text{sin. } a}$.	(10)
Sin. A = cos. B. $\quad$ sin. B = cos. A.	$\text{Sin. A} = \frac{\text{cos. B}}{\text{cos. } b}$. $\quad \text{sin. B} = \frac{\text{cos. A}}{\text{cos. } a}$.	(11)
$c^2 = b^2 + a^2$	$\text{cos. } c = \text{cos. } b \text{ cos. } a$	(12)
$1 = \text{cot. A cot. B}$.	$\text{cos. } c = \text{cot. A cot. B}$	(12)

Example I.

In the right-angled spherical triangle ABC, right-angled at C, let angle A = 50° 20′, and BC or $a = 39°\ 40'$; to find the angle B, and the sides AC = b, and AB = c.



Solution.

In solving this or any question in right-angled Spherical Trigonometry, we must take each of the sought parts along with the two given parts; this will form three groups of three terms each —namely, A, B, a, Aca, and Aba. In each of these we take either a *known* or an *unknown* part for the *middle* part, and then apply Napier's Rules; thus, in the group A, B, a, the complement of A is the middle part, and a and the complement of B are opposite parts; for B is separated from A by the side c, and a is separated from A by the side b: and therefore,

$$\text{cos. A} = \text{sin. B cos. } a\text{; wherefore, sin B} = \frac{\text{cos. A}}{\text{cos. } a}.$$

In the second group, Aca, a is the middle part, and the complements of A and c are the opposite parts; and therefore,

$$\text{sin. } a = \text{sin. A sin. } c \text{ .'. sin. } c = \frac{\text{sin. } a}{\text{sin. A}}.$$

In the third group, Aba, b is the middle part, and a and the complement of A are the adjacent parts; and therefore,

$$\text{sin. } b = \text{tan. } a \text{ cot. A}.$$

To find B, sin. B $= \frac{\text{cos. A}}{\text{cos. } a}$

Log. cos. A + 10	=	19·8050385
Log. cos. a	=	9·8863616
Log. sin. 56° 1′ 12·3″	=	9·9186769

$\therefore \angle$ B = 56° 1′ 12·3″

To find b, sin. b = tan. a cot. A.

Log. tan. a	=	9·9186769
Log. cot. A	=	9·9186769
Log. sin. 43° 26′ 34″	=	9·8373538

$\therefore b$ = 43° 26′ 34″

To find c, sin. c $= \frac{\text{sin. } a}{\text{sin. A}}$

Log. sin. a + 10	=	19·8050385
Log. sin. A	=	9·8863616
Log. sin. 56° 1′ 12·3″	=	9·9186769

$\therefore c$ = 56° 1′ 12·3″

Check sin. b = sin. c sin. B.

Log. sin. c	=	9·9186769
Log. sin. B	=	9·9186769
sin. b	=	9·8373538

The same as before.

NOTE.—The check here introduced is to find one of the three unknown parts in terms of the other two; and if the

result agree with that found from the two given quantities, the working is concluded to be correct.

Example II.

In a right-angled spherical triangle ABC, right-angled at C, there are given $AB = c = 81° 30'$, and $BC = a = 56° 11'$; to find the other parts.

Solution.

First (figure to last Example), Take B along with c and a; then B being the middle part, c and a are adjacent parts; and by Napier's second Rule,

$$\cos. B = \tan. a \cot. c.$$

Second, Let A be taken along with c and a; then a is the middle part, A and c are opposite parts; and by Napier's first Rule,

$$\sin. a = \sin. A \sin. c \therefore \sin. A = \frac{\sin. a}{\sin. c}.$$

Third, Let b be taken along with c and a; then c is the middle part, and a and b are opposite parts; and by Napier's first Rule,

$$\cos. c = \cos. a \cos. b \therefore \cos. b = \frac{\cos. c}{\cos. a}.$$

To find B; $\cos. B = \tan. a \cot. c.$

Log. tan. a	$= 10·1740140$
Log. cot. c	$= 9·1744988$
Log. cos. ($B = 77° 6' 30''$)	$= 9·3485128$

To find b; $\cos. b = \frac{\cos. c}{\cos. a}$.

Log. cos. c + 10	$= 19·1697021$
Log. cos. a	$= 9·7454943$
Log. cos. b	$= 9·4242078$

$\therefore b = 74° 35' 53''$

To find A; $\sin. A = \frac{\sin. a}{\sin. c}$.

Log. sin. a + 10	$= 19·9195083$
Log. sin. c	$= 9·9952033$
Log. sin. ($A = 57° 8' 43''$)	$= 9·9243050$

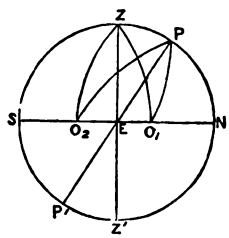
Check $\cos. B = \cos. b \sin. A.$

Log. cos. b	$= 9·4242078$
Log. sin. A	$= 9·9243050$
Log. cos. B	$= 9·3485128$

The same value as above.

324. If, in the adjoining figure, SZPN represent a meridian,

SEN the horizon, Z the zenith, P the north pole, and O_1 or O_2 any celestial object rising or setting, according as the point E represents the east or west point of the horizon; then PO_1 or PO_2 will represent the object's polar distance, according as its declination is north or south; and NO_1 or NO_2 will be its azimuth, EO_1 or EO_2 will be its amplitude, according as the object's declination is north or south. Since Z is the zenith, ZN, ZO_1, ZE, and ZO_2 are all quadrants; and hence the arcs NO_1, NE, and NO_2 are respectively the measures of the angles NZO_1, NZE, and NZO_2, which are therefore also the measures of the azimuths of the points O_1, E, and O_2.



Again, in the triangle PON (where O may be either O_1 or O_2), PO is the polar distance, PN the latitude of the place whose zenith is Z; and if the object be the sun, the angle OPN is the measure of the apparent time from midnight, and the angle PNO is a right angle; then, if the sun's declination and the latitude of the place of observation be known, this triangle is the same as that solved in the last example, and the arc NO is the azimuth, and the angle OPN is the measure of the time from midnight; hence we have the following:

325. To find the apparent time of sunrising or of sunsetting.

RULE.—To the log. *tangent* of the latitude add the log. *tangent* of the declination, and the sum, rejecting 10 from the index, will be the log. *cosine* of the apparent time from midnight, which is less or greater than 90°, according as the declination is of the same name as the elevated pole, or of an opposite name from it.

326. To find the sun's azimuth at rising or setting in terms of his declination, and the latitude of the place of observation.

RULE.—To the log. *sine* of the declination add the log. *secant* of the latitude, and the sum, rejecting 10 from the index, will be the log. *cosine* of the azimuth, which is less or greater than 90°,

according as the declination is of the same name as the elevated pole, or of an opposite name from it.

327. To find the latitude, having given the sun's declination and amplitude.

RULE.—To the log. *sine* of the declination add the log. *cosecant* of the amplitude, and the sum, rejecting 10 from the index, will be the log. *cosine* of the latitude.

Exercises 43.

(490.) At a place in latitude 51° 16′ N., the sun's declination being 14° 30′ N.; it is required to find the apparent time of sunrise, and his azimuth at that time.

(491.) At what time will the sun rise at Edinburgh on the 21st June, and what will be his azimuth—the latitude of Edinburgh being 55° 57′ N. and the sun's declination 23° 27′ 14″ N.?

(492.) At what time will the sun rise at Liverpool, in latitude 53° 22′ N., when the sun's declination is 15° 12′ N., and what will then be his amplitude?

(493.) At what time will the sun rise at Portsmouth, in latitude 50° 47′ N., when the sun's declination is 23° 27′ S., and what will be his amplitude?

(494.) At a certain place when the sun's declination is 20° 15′ N., his amplitude is 29° 12′ N.; find the latitude of the place and his time of rising.

(495.) At what time will the sun rise at Edinburgh, in latitude 55° 57′ N., when the sun's declination is 23° 27′ 12″ S., and what will then be its amplitude?

(496.) In a right-angled triangle ABC, right-angled at C, there are given the two sides a and b, 55° 18′ and 36° 54′ respectively; find c, A, and B.

(497.) In a right-angled triangle ABC, there are given angle A = 73° 16′, and angle B = 95° 36′; to find the three sides a, b, and c.

(498.) In a right-angled triangle ABC, there are given the angle $B = 87° 16'$, and the side $b = 70° 5'$; to find the angle A and the sides a and c.

(499.) In a right-angled triangle ABC, there are given $a = 54° 26$, and $c = 89° 11'$; to find the angles at A and B and the third side b.

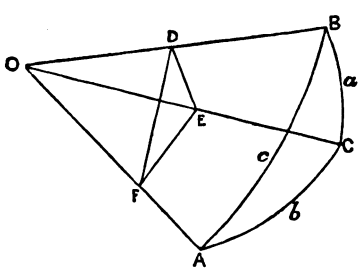
(500.) In a right-angled triangle ABC, there are given the side $a = 75° 18'$, and the angle $B = 49° 35'$; to find the angle A and the sides b and c.

(501.) In a right-angled triangle ABC, there are given the side $c = 120° 50'$, and the angle at $A = 60° 15'$; to find the angle B and the sides a and b.

NOTE.—In each of the six previous questions apply the following CHECK: Find one of the required quantities in terms of the other two required quantities, and if the work be correct, the value so found will be the same as before.

OBLIQUE-ANGLED SPHERICAL TRIGONOMETRY.

328. Let ABC be any spherical triangle, and O the centre of the sphere; take any point E in OC, and from it draw EF perpendicular to OA; and from F in the plane BOA draw FD at right angles to OA; then the angle EFD being the measure of the inclination of the planes COA, BOA, is equal to the spherical angle A.



Now, in the triangles DFE and DOE, which have the side DE common, and the angle DEF = the spherical angle at A, and the angle DOE = a, since it is measured by the side BC = a, we obtain

$DE^2 = DF^2 + EF^2 - 2DE \cdot EF \cos. A$ from the triangle DFE;
and $DE^2 = DO^2 + EO^2 - 2DO \cdot EO \cos. a$ " " DOE.

Subtracting the first of these values from the second, gives

$$0 = 2OF^2 + 2DE \cdot EF \cos. A - 2DO \cdot EO \cos. a,$$

which, being divided by $2DO \cdot EO$, and $\cos. a$ transposed to the first side, gives

$$\cos. a = \frac{OF}{OE} \cdot \frac{OF}{DO} + \frac{EF}{OE} \cdot \frac{DE}{DO} \cos. A,$$

which, by Plane Trigonometry, gives

$$\left.\begin{array}{l} \cos. a = \cos. b \cos. c + \sin. b \sin. c \cos. A. \\ \text{Similarly, } \cos. b = \cos. c \cos. a + \sin. c \sin. a \cos. B, \\ \text{and } \cos. c = \cos. a \cos. b + \sin. a \sin. b \cos. C. \end{array}\right\} \quad (13)$$

329. Again, by transposition, and dividing both sides by $\sin. b \sin. c$, we find

$$\left.\begin{array}{l} \cos. A = \dfrac{\cos. a - \cos. b \cos. c}{\sin. b \sin. c}. \\ \text{Similarly, from the second,} \\ \cos. B = \dfrac{\cos. b - \cos. c \cos. a}{\sin. c \sin. a}; \\ \text{and from the third,} \\ \cos. C = \dfrac{\cos. c - \cos. a \cos. b}{\sin. a \sin. b}. \end{array}\right\} \quad (14)$$

330. Subtracting the square of each side of the first in (14) from 1, and remembering that $1 - \cos.^2 A = \sin.^2 A$, we obtain

$$\sin.^2 A = 1 - \left(\frac{\cos. a - \cos. b \cos. c}{\sin. b \sin. c}\right)^2 = \frac{\sin.^2 b \sin.^2 c - (\cos. a - \cos. b \cos. c)^2}{\sin.^2 b \sin.^2 c}$$

$$= \frac{\{\sin. b \sin. c - \cos. b \cos. c + \cos. a\} \{\sin. b \sin. c + \cos. b \cos. c - \cos. a\}}{\sin.^2 b \sin.^2 c}$$

$$= \frac{\{\cos. a - \cos. (b + c)\} \{\cos. (b - c) - \cos. a\}}{\sin.^2 b \sin.^2 c}$$

$$= \frac{2 \sin. \frac{1}{2}(a + b + c) \sin. \frac{1}{2}(b + c - a) \times 2 \sin. \frac{1}{2}(a + b - c) \sin. \frac{1}{2}(a + c - b)}{\sin.^2 b \sin.^2 c}$$

Extracting the square root of both sides, and dividing by sin. A, we have

$$\frac{\sin. A}{\sin. a} = \frac{2}{\sin. a \sin. b \sin. c} \sqrt{\sin. \tfrac{1}{2}(a+b+c) \sin. \tfrac{1}{2}(b+c-a) \sin. \tfrac{1}{2}(a+c-b) \sin. \tfrac{1}{2}(a+b-c)}$$

Or putting $\frac{1}{2}(a + b + c) = s$, and consequently $\frac{1}{2}(b + c - a) = s - a$, $\frac{1}{2}(a + c - b) = s - b$, and $\frac{1}{2}(a + b - c) = s - c$, it becomes the following:

$$\frac{\sin.\ A}{\sin.\ a} = \frac{2}{\sin.\ a \sin.\ b \sin.\ c}\sqrt{\sin.\ s \sin.\ (s-a) \sin.\ (s-b) \sin.\ (s-c)} \cdot = m \text{ say}$$

The second side, being a symmetrical function of the three sides, will also be equal to $\frac{\sin.\ B}{\sin.\ b}$, or to $\frac{\sin.\ C}{\sin.\ c}$; and hence we have the following result,

$$\frac{\sin.\ A}{\sin.\ a} = \frac{\sin.\ B}{\sin.\ b} = \frac{\sin.\ C}{\sin.\ c} = m. \tag{15}$$

331. This proves that *the sines of the angles of a spherical triangle are proportional to the sines of the opposite sides.*

332. Again, add 1 to both sides of the first equation in (14), and remember that $1 + \cos.\ A = 2\ \cos.^2 \frac{1}{2}A$ (238, *p*); we have

$$1 + \cos.\ A = 1 + \frac{\cos.\ a - \cos.\ b \cos.\ c}{\sin.\ b \sin.\ c}$$

$$= \frac{\sin.\ b \sin.\ c - \cos.\ b \cos.\ c + \cos.\ a}{\sin.\ b \sin.\ c}$$

$$= \frac{\cos.\ a - \cos.\ (b+c)}{\sin.\ b \sin.\ c}$$

$$\therefore 2 \cos.^2 \tfrac{1}{2}A = \frac{2 \sin.\ \frac{1}{2}(a+b+c) \sin.\ \frac{1}{2}(b+c-a)}{\sin.\ b \sin.\ c}$$

Hence dividing by 2, using the notation given above, and extracting the square root of both sides,

$$\left.\begin{aligned} \cos.\ \tfrac{1}{2}A &= \sqrt{\frac{\sin.\ s \sin.\ (s-a)}{\sin.\ b \sin.\ c}}. \\ \text{Similarly,}\quad \cos.\ \tfrac{1}{2}B &= \sqrt{\frac{\sin.\ s \sin.\ (s-b)}{\sin.\ a \sin.\ c}}, \\ \text{and}\quad \cos.\ \tfrac{1}{2}C &= \sqrt{\frac{\sin.\ s \sin.\ (s-c)}{\sin.\ a \sin.\ b}}. \end{aligned}\right\} \tag{16}$$

333. Next, subtracting both sides of the first equation in (14) from 1, and remembering that $1 - \cos.\ A = 2\ \sin.^2 \frac{1}{2}A$ (238, *q*), we have

$$1 - \cos.\ A = 1 - \frac{\cos.\ a - \cos.\ b \cos.\ c}{\sin.\ b \sin.\ c}$$

$$= \frac{\sin.\ b \sin.\ c + \cos.\ b \cos.\ c - \cos.\ a}{\sin.\ b \sin.\ c}$$

$$= \frac{\cos.\ (b-c) - \cos.\ a}{\sin.\ b \sin.\ c}$$

$$\therefore 2 \sin.^2 \tfrac{1}{2}A = \frac{2 \sin.\ \frac{1}{2}(a+b-c) \sin.\ \frac{1}{2}(a+c-b)}{\sin.\ b \sin.\ c}$$

Hence dividing by 2, using the notation given above, and extracting the square root of both sides,

$$\left.\begin{aligned} &\text{sin. } \tfrac{1}{2}A = \sqrt{\frac{\text{sin. } (s-b) \text{ sin. } (s-c)}{\text{sin. } b \text{ sin. } c}}. \\ \text{Similarly, } &\text{sin. } \tfrac{1}{2}B = \sqrt{\frac{\text{sin. } (s-a) \text{ sin. } (s-c)}{\text{sin. } a \text{ sin. } c}}, \\ \text{and } &\text{sin. } \tfrac{1}{2}C = \sqrt{\frac{\text{sin. } (s-a) \text{ sin. } (s-b)}{\text{sin. } a \text{ sin. } b}}. \end{aligned}\right\} \quad (17)$$

334. Dividing the corresponding sides of the first equation in (17) by those in the first equation in (16), gives

$$\frac{\text{sin. } \frac{1}{2}A}{\text{cos. } \frac{1}{2}A} = \text{tan. } \tfrac{1}{2}A = \sqrt{\frac{\text{sin. } (s-b) \text{ sin. } (s-c)}{\text{sin. } b \text{ sin. } c} \times \frac{\text{sin. } b \text{ sin. } c}{\text{sin. } s \text{ sin. } (s-a)}}$$

$$\left.\begin{aligned} \text{or } &\text{tan. } \tfrac{1}{2}A = \sqrt{\frac{\text{sin. } (s-b) \text{ sin. } (s-c)}{\text{sin } s \text{ sin. } (s-a)}}. \\ \text{Similarly, } &\text{tan. } \tfrac{1}{2}B = \sqrt{\frac{\text{sin. } (s-a) \text{ sin. } (s-c)}{\text{sin. } s \text{ sin. } (s-b)}}, \\ \text{and } &\text{tan. } \tfrac{1}{2}C = \sqrt{\frac{\text{sin. } (s-a) \text{ sin. } (s-b)}{\text{sin. } s \text{ sin. } (s-c)}}. \end{aligned}\right\} \quad (18)$$

335. From any of the expressions in (16), (17), or (18), an angle of a spherical triangle may be calculated when the three sides of the triangle are known; but when two or more angles are required, it will be more convenient to use a different form of the expression for the tangent of the half angle, which can be derived in the following manner: Multiply both numerator and denominator of the second side of the expressions in (17)—the first by sin. $(s-a)$, the second by sin. $(s-b)$, and the third by sin. $(s-c)$; then extracting the root of the squares of sin. $(s-a)$, sin. $(s-b)$, and sin. $(s-c)$, thus introduced, we obtain the three following very convenient expressions for the tangents of half the angles:

$$\left.\begin{aligned}
\tan.\tfrac{1}{2}A &= \frac{1}{\sin.(s-a)}\sqrt{\frac{1}{\sin. s}\sin.(s-a)\sin.(s-b)\sin.(s-c)},\\
\tan.\tfrac{1}{2}B &= \frac{1}{\sin.(s-b)}\sqrt{\frac{1}{\sin. s}\sin.(s-a)\sin.(s-b)\sin.(s-c)},\\
\text{and}\\
\tan.\tfrac{1}{2}C &= \frac{1}{\sin.(s-c)}\sqrt{\frac{1}{\sin. s}\sin.(s-a)\sin.(s-b)\sin.(s-c)}.
\end{aligned}\right\} \quad (19)$$

336. *Napier's Analogies.*—Divide the *first* of (18) by the *second*, and we find

$$\frac{\tan.\frac{1}{2}A}{\tan.\frac{1}{2}B} = \frac{\sin.(s-b)}{\sin.(s-a)}.$$

Regarding this as a proportion we have by mixing

$$\frac{\tan.\frac{1}{2}A + \tan.\frac{1}{2}B}{\tan.\frac{1}{2}A - \tan.\frac{1}{2}B} = \frac{\sin.(s-b) + \sin.(s-a)}{\sin.(s-b) - \sin.(s-a)}$$

Or

$$\frac{\sin.\frac{1}{2}A\cos.\frac{1}{2}B + \cos.\frac{1}{2}A\sin.\frac{1}{2}B}{\sin.\frac{1}{2}A\cos.\frac{1}{2}B - \cos.\frac{1}{2}A\sin.\frac{1}{2}B} = \frac{2\sin.\frac{1}{2}\{2s-(a+b)\}\cos.\frac{1}{2}(a-b)}{2\cos.\frac{1}{2}\{2s-(a+b)\}\sin.\frac{1}{2}(a-b)} = \frac{\sin.\frac{1}{2}c\cos.\frac{1}{2}(a-b)}{\cos.\frac{1}{2}c\sin.\frac{1}{2}(a-b)}$$

$$\therefore \frac{\sin.\frac{1}{2}(A+B)}{\sin.\frac{1}{2}(A-B)} = \frac{\tan.\frac{1}{2}c}{\tan.\frac{1}{2}(a-b)}$$

Or $\sin.\frac{1}{2}(A+B) : \sin.\frac{1}{2}(A-B) :: \tan.\frac{1}{2}c : \tan.\frac{1}{2}(a-b)$, (*a*)
which is the first of Napier's Analogies.

337. Again, the product of the *first* and *second* of (18) gives

$$\frac{\tan.\frac{1}{2}A\ \tan.\frac{1}{2}B}{1} = \frac{\sin.(s-c)}{\sin. s}.$$

which, by mixing, gives

$$\frac{1-\tan.\frac{1}{2}A\tan.\frac{1}{2}B}{1+\tan.\frac{1}{2}A\tan.\frac{1}{2}B} = \frac{\sin. s - \sin.(s-c)}{\sin. s + \sin.(s-c)}$$

$$\text{Or } \frac{\cos.\frac{1}{2}A\cos.\frac{1}{2}B - \sin.\frac{1}{2}A\sin.\frac{1}{2}B}{\cos.\frac{1}{2}A\cos.\frac{1}{2}B + \sin.\frac{1}{2}A\sin.\frac{1}{2}B} = \frac{2\cos.\frac{1}{2}(2s-c)\sin.\frac{1}{2}c}{2\sin.\frac{1}{2}(2s-c)\cos.\frac{1}{2}c} = \frac{\cos.\frac{1}{2}(a+b)\sin.\frac{1}{2}c}{\sin.\frac{1}{2}(a+b)\cos.\frac{1}{2}c}$$

$$\therefore \frac{\cos.\frac{1}{2}(A+B)}{\cos.\frac{1}{2}(A-B)} = \frac{\tan.\frac{1}{2}c}{\tan.\frac{1}{2}(a+b)}$$

Or $\cos.\frac{1}{2}(A+B) : \cos.\frac{1}{2}(A-B) :: \tan.\frac{1}{2}c : \tan.\frac{1}{2}(a+b)$, (*b*)
which is the second of Napier's Analogies.

338. To prove the other two analogies—namely,

$$\cos.\tfrac{1}{2}(a+b) : \cos.\tfrac{1}{2}(a-b) :: \cot.\tfrac{1}{2}C : \tan.\tfrac{1}{2}(A+B),$$
and $$\sin.\tfrac{1}{2}(a+b) : \sin.\tfrac{1}{2}(a-b) :: \cot.\tfrac{1}{2}C : \tan.\tfrac{1}{2}(A-B).$$

$$\tan.\tfrac{1}{2}(A+B) = \frac{\tan.\tfrac{1}{2}A + \tan.\tfrac{1}{2}B}{1 - \tan.\tfrac{1}{2}A \tan.\tfrac{1}{2}B}$$

$$= \frac{\sqrt{\frac{\sin.(s-b)\sin.(s-c)}{\sin.s \sin.(s-a)}} + \sqrt{\frac{\sin.(s-a)\sin.(s-c)}{\sin.s \sin.(s-b)}}}{1 - \frac{\sin.(s-c)}{\sin.s}}$$

Multiplying numerator and denominator of this expression by sin. s, and reducing the roots in the numerator to a common denominator, we obtain,

$$\tan.\tfrac{1}{2}(A+B) = \frac{\sin.(s-b) + \sin.(s-a)}{\sin.s - \sin.(s-c)} \sqrt{\frac{\sin.s \sin.(s-c)}{\sin.(s-a)\sin.(s-b)}}$$

$$= \frac{2 \sin.\tfrac{1}{2}c \cos.\tfrac{1}{2}(a-b)}{2 \cos.\tfrac{1}{2}(a+b) \sin.\tfrac{1}{2}c} \cot.\tfrac{1}{2}C$$

$$\therefore \tan.\tfrac{1}{2}(A+B) = \frac{\cos.\tfrac{1}{2}(a-b)}{\cos.\tfrac{1}{2}(a+b)} \cot.\tfrac{1}{2}C \qquad (c)$$

Similarly, it may be proved that

$$\tan.\tfrac{1}{2}(A-B) = \frac{\sin.(s-b) - \sin.(s-a)}{\sin.s + \sin.(s-c)} \sqrt{\frac{\sin.s \sin.(s-c)}{\sin.(s-a)\sin.(s-b)}}$$

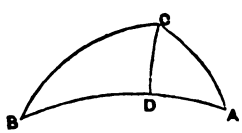
$$= \frac{2 \cos.\tfrac{1}{2}c \sin.\tfrac{1}{2}(a-b)}{2 \sin.\tfrac{1}{2}(a+b) \cos.\tfrac{1}{2}c} \cot.\tfrac{1}{2}C$$

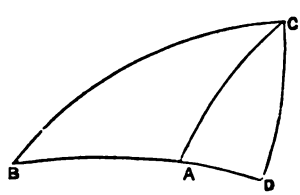
$$= \frac{\sin.\tfrac{1}{2}(a-b)}{\sin.\tfrac{1}{2}(a+b)} \cot.\tfrac{1}{2}C. \qquad (d)$$

339. Collecting now the four formulæ (a), (b), (c), and (d) into one result, gives

$$\left.\begin{array}{l} \sin.\tfrac{1}{2}(A+B) : \sin.\tfrac{1}{2}(A-B) :: \tan.\tfrac{1}{2}c : \tan.\tfrac{1}{2}(a-b), \ (a) \\ \cos.\tfrac{1}{2}(A+B) : \cos.\tfrac{1}{2}(A-B) :: \tan.\tfrac{1}{2}c : \tan.\tfrac{1}{2}(a+b), \ (b) \\ \sin.\tfrac{1}{2}(a+b) : \sin.\tfrac{1}{2}(a-b) :: \cot.\tfrac{1}{2}C : \tan.\tfrac{1}{2}(A-B), \ (d) \\ \cos.\tfrac{1}{2}(a+b) : \cos.\tfrac{1}{2}(a-b) :: \cot.\tfrac{1}{2}C : \tan.\tfrac{1}{2}(A+B). \ (c) \end{array}\right\} (20)$$

340. Let ABC be any spherical triangle, and CD an arc





drawn through C at right angles to AB or AB produced; it is

required to prove that the cosines of the sides are proportional to the cosines of the adjacent segments of the base; and that the tangents of the angles at the base are proportional to the sines of the alternate segments of the base.

In the right-angled triangle BDC, cos BC = cos. BD . cos. CD; and in the triangle ACD, cos. AC = cos. AD . cos. CD.

$$\therefore \frac{\text{cos. BC}}{\text{cos. AC}} = \frac{\text{cos. BD . cos. CD}}{\text{cos. AD . cos. CD}} = \frac{\text{cos. BD}}{\text{cos. AD}};$$

or cos. BC : cos. AC :: cos. BD : cos. AD. (21)

Again, from the triangle BDC, sin. BD = tan. CD cot. B; and from the triangle ADC, sin. AD = tan. CD cot. A.

$$\therefore \frac{\text{sin. BD}}{\text{sin. AD}} = \frac{\text{tan. CD cot. B}}{\text{tan. CD cot. A}} = \frac{\text{cot. B}}{\text{cot. A}} = \frac{\text{tan. A}}{\text{tan. B}};$$

or tan. A : tan. B :: sin. BD : sin. AD. (22)

341. The formulæ of Spherical Trigonometry now established will be sufficient to solve all the problems in Nautical Astronomy where only one triangle is required; and where the consideration of two or more triangles is involved, the combination of the various cases will give the necessary results.

Exercises 44.

(502.) In an oblique-angled spherical triangle there are given the three sides $a = 56° 40'$, $b = 83° 13'$, and $c = 114° 30'$; to find the angles.

(503.) In an oblique-angled spherical triangle there are given the three sides $a = 100°$, $b = 50°$, and $c = 60°$; to find the angles.

(504.) In an oblique-angled spherical triangle there are given the three sides $a = 70°$, $b = 75°$, and $c = 85°$; to find the three angles.

(505.) In an oblique-angled spherical triangle there are given the side $b = 120° 30' 30''$, $c = 70° 20' 20''$, and the angle $A = 50° 10' 10''$; to find the angles at B and C and the third side.

(506.) In an oblique-angled spherical triangle there are given

$b = 99° 40' 48''$, $c = 100° 49' 30''$, and the angle $A = 65° 33' 10''$; to find the angles at B and C and the third side.

(507.) In an oblique-angled spherical triangle there are given $a = 38° 30'$, $b = 70°$, and the angle $C = 31° 34' 26''$; to find the angles A and B and the third side.

(508.) In an oblique-angled spherical triangle there are given the angle $A = 39° 23'$, and $B = 33° 45' 3''$, and the side $c = 68° 46' 2''$; to find a and b and the angle C.

(509.) In an oblique-angled spherical triangle there are given $a = 63° 50'$, $b = 80° 19'$, and the angle $A = 51° 30'$; to find the angles B and C and the third side c.

(510.) Required the distance between Procyon and Capella, the right ascension and declination of Procyon, 7 h. 32 m. 20 s. and 5° 34′ 6″ N., and of Capella, 5 h. 6 m. 52 s. and 45° 51′ 32″ N.

342. NOTE.—This and the three following questions may be solved by the formulæ (13) Art. (328), but it is better to change that expression to a new form, thus,

$$\begin{aligned}\cos. a &= \cos. b \cos. c + \sin. b \sin. c \cos. A, \text{ add and subtract } \sin. b \sin. c\\ &= \cos. b \cos. c + \sin. b \sin. c - \sin. b \sin. c\,(1 - \cos. A)\\ &= \cos.(b - c) - 2 \sin. b \sin. c \sin.^2 \tfrac{1}{2}A.\end{aligned}$$

Again, subtract and add $\sin. b \sin. c$, and we easily find

$$\begin{aligned}\cos. a &= \cos. b \cos. c - \sin. b \sin. c + \sin. b \sin. c\,(1 + \cos. A)\\ &= \cos.(b + c) + 2 \sin. b \sin. c \cos.^2 \tfrac{1}{2}A; \text{ and therefore}\end{aligned}$$

$$\cos. a = \cos.(b + c) + 2 \sin. b \sin. c \cos.^2 \tfrac{1}{2}A = \cos.(b - c) - 2 \sin. b \sin. c \sin.^2 \tfrac{1}{2}A;$$

with similar expressions for the other sides.

(511.) Required the distance between Rigel and Arcturus, the right ascension and declination of Rigel being 5 h. 8 m. 8 s. and 8° 21′ 37″ S., and of Arcturus, 14 h. 9 m. 35 s. and 19° 52′ 34″ N.

(512.) Required the distance between Spica Virginis and Vega, the right ascension and declination of Spica being 13 h. 18 m. 12 s. and 10° 27′ 19″ S., and of Vega, 18 h. 32 m. 26 s. and 38° 39′ 42″ N.

(513.) When the sun's polar distance is 74° 18′, and the time from apparent noon 2 h. 16 m. = 34°; what is his true altitude at a place in latitude 56° 58′?

Problem XII.

343. Given the sun's observed altitude and the time from apparent noon, on a given day; to find the latitude of the place of observation.

In the figure to Art. (340), let BA represent a portion of a meridian of the place of observation; B being the pole and A the zenith, and C the place of the sun; and consequently BC = the sun's polar distance, and AC = the complement of the sun's corrected altitude, and the angle CBD = the time from apparent noon.

In the triangle CBD, right-angled at D,

$$\cos. CBD = \tan. BD \cot. CB;$$

$$\therefore \tan. BD = \cos. CBD \tan. CB = \cos. CBD \cot. \text{dec.} \quad \text{(arc 1)}$$

and by (21), cos. BC : cos. AC :: cos. BD : cos. AD,

or cos. BC : sin. alt. :: cos. BD : cos. AD;

$$\therefore \cos. AD = \cos. BD \sin. \text{alt.} \sec. BC = \cos (\text{arc } 1) \sin. (\text{alt.}) \operatorname{cosec}. \text{dec.} \quad \text{(arc 2)}$$

RULE.

1. To the log. cosine of the time from apparent noon add the log. tangent of the polar distance, or the log. cotangent of the sun's declination; the sum, rejecting 10 from the index, will be the log. tangent of arc 1 = BD; greater or less than 90°, according as the polar distance is greater or less than 90°.

2. To the log. cosine of arc 1 add the log. sine of the true observed altitude, and the cosecant of the sun's declination; the sum, rejecting 20 from the index, will be the log. cosine of arc 2 = AD, always less than 90°.

3. If the sun and pole be on opposite sides of the prime vertical, add 90° to arc 2, and from the sum subtract arc 1, the remainder will be the latitude of the place; but if the sun and the pole be on the same side of the prime vertical, subtract the

sum of arc 1 and arc 2 from 90°, and the remainder will be the latitude of the place of observation.

Or Latitude = 90° ± arc 2 − arc 1.

Example I.

At 12 m. 43 s. from apparent noon, when the sun's corrected declination was 9° 18′ 20″ S., the observed altitude of his lower limb was 53° 17′ 15″ S., the index error being + 1′ 25″, the height of the eye being 24 feet, and the sun's semidiameter being 16′ 12″; required the latitude of the place of observation.

Observed alt. ⊙'s l. l.	=	53° 17′ 15″	⊙'s declination	=	9° 18′ 20″ S.
Index error and dip		− 3′ 20″			90° 0′ 0″
		53° 13′ 55″	Polar distance	=	99° 18′ 20″
Refraction and paral.		− 38″			
		53° 13′ 17″	Time from apparent noon	=	12ᵐ 43ˢ
⊙'s semidiameter	=	16′ 12″	" in degrees	=	3° 10′ 45″
⊙'s true altitude	=	53° 29′ 29″			

Log. cos. 3° 10′ 45″	=	9·9993311				
Log. cot. 9° 18′ 20″	=	10·7855381		cosec.	=	10·7912913
Log. tan. 99° 19′ 11″	=	10·7848692	= arc 1	cos.	=	9·2093635
				Log. sin. 53° 29′ 29″	=	9·9051304
90° + arc 2	=	126° 23′ 30″ N.		Log. cos. 36° 23′ 30″	=	9·9057852 = arc 2
− arc 1	=	99° 19′ 11″				
∴ Latitude	=	27° 4′ 19″ N.				

Example II.

At 20 m. 16 s. from apparent noon, when the sun's corrected declination was 14° 11′ 34″ N., the observed altitude of his lower limb was 40° 13′ 30″ N., the index error being + 3′ 15″, the height of the eye being 18 feet, and the sun's semidiameter 15′ 55″; required the latitude of the place of observation.

Observed alt. ⊙'s l. l.	=	40° 13′ 30″ N.	⊙'s declination	=	14° 11′ 34″
Index error and dip		− 51″			90° 0′ 0″
		40° 12′ 39″	Polar distance	=	104° 11′ 34″
Refraction and paral.		− 1′ 2″			
		40° 11′ 37″	Time from apparent noon	=	20ᵐ 16ˢ
⊙'s semidiameter		+ 15′ 55″	" in degrees	=	5° 4′
⊙'s true altitude	=	40° 27′ 32″			

Log. cos. 5° 4′	= 9·9982997					
Log. cot. 14° 11′ 34″	= 10·5970430			cosec.	= 10·6105058	
Log. tan. 104° 14′ 46″	= 10·5953427	= arc 1		cos.	= 9·3910896	
			Log. sin. 40° 27′ 32″	=	9·8121793	
90° + arc 2	= 139° 21′ 40″		Log. cos. 49° 21′ 40″	=	9·8137747	= arc 2
− arc 1	= 104° 14′ 46″					
∴ Latitude	= 35° 6′ 54″ S.					

Exercises 45.

(514.) At 13 m. 16 s. from apparent noon, the true corrected altitude of the sun's centre was 72° 18′, and his polar distance 94° 15′; find the latitude of the place of observation.

(515.) At 8 m. 44 s. from apparent noon, the true corrected altitude of the sun's centre was 54° 18′, and his polar distance 81° 17′ 30″; find the latitude of the place of observation.

(516.) At 24 m. 28 s. from apparent noon, the true corrected altitude of the sun's centre being 60° 19′, and his polar distance 70° 43′; find the latitude of the place of observation.

(517.) At 18 m. 20 s. from apparent noon, the true corrected altitude of the sun's centre was 18° 43′, and his polar distance 112° 15′; find the latitude of the place of observation.

(518.) At 22 m. 40 s. from apparent noon, the true corrected altitude of the sun's centre being 29° 15′ 25″, and his polar distance 105° 39′; what is the latitude of the place of observation?

(519.) At 16 m. 48 s. from apparent noon, the true corrected altitude of the sun's centre being 68° 15′, and his polar distance 81° 19′; what is the latitude of the place of observation?

(520.) At 30 m. 12 s. from apparent noon, the true corrected altitude of the sun's centre was 48° 53′ 30″, and his polar distance 97° 18′; find the latitude of the place of observation.

(521.) At 8 m. 56 s. from apparent noon, the true altitude of the sun's centre was 29° 52′, and his polar distance 113° 28′; find the latitude of the place of observation.

(522.) At 21 m. 36 s. from apparent noon, the true altitude of

the sun's centre was 59° 56′ 45″, and his polar distance 95° 11′ 40″; find the latitude of the place of observation.

(523.) At 25 m. 28 s. from apparent noon, the true altitude of the sun's centre was 75° 13′ 30″, and his polar distance 88° 10′ 40″; find the latitude of the place of observation.

(524.) At 7 m. 48 s. from apparent noon, the true corrected altitude of the sun's centre was 59° 17′ 30″, and his polar distance 112° 59′ 10″; find the latitude of the place of observation.

(525.) At 36 m. 12 s. from apparent noon, the true corrected altitude of the sun's centre was 17° 55′ 20″, and his polar distance 110° 56′ 12″; find the latitude of the place of observation.

(526.) At 7 m. 36 s. from apparent noon, the true corrected altitude of the sun's centre was 58° 12′ 32″, and his polar distance 79° 18′ 27″; find the latitude of the place of observation.

Problem XIII.

344. To find the apparent time at a place whose latitude is known, from an observed altitude of the sun, when at a distance from the meridian on a given day, the Greenwich time being known nearly.

RULE.

1. From the Greenwich time find the sun's polar distance at the instant of observation, by means of the *Nautical Almanac.*

2. Correct the sun's observed altitude for index error, dip, refraction, parallax, and semidiameter; and call the result the true altitude.

3. Under the true altitude write the latitude of the place of observation, and the sun's polar distance; add the three together, and take half the sum; from the half sum subtract the true altitude.

4. Take the secant of the latitude, the cosecant of the polar distance (omitting 10 from the index of each), the cosine of the half sum, and the sine of the remainder; add the four logarithms together, and take half the sum; the quotient will be the log. sine of half the time from apparent noon in degrees, minutes, and seconds. Multiply this arc by 8, calling the product of the minutes seconds of time, and the product of the degrees minutes of time; the result will be the time from apparent noon at the time of observation.

5. Correct this result for the equation of time, and the mean time at the place will be found.

345. 6. The difference between the mean time thus found and the mean Greenwich time will be the longitude of the place of observation; *east*, if the time found be faster than the Greenwich time, and *west*, if the time found be slower than the Greenwich time.

Example I.

If, on the 12th August 1866, at 8 h. 7 m. 6 s. A.M. by watch, the watch shewing Greenwich time nearly, in latitude 43° 17′ N., longitude by account 14° 19′ W., the altitude of the sun's l. l. be 21° 45′, and increasing, index error + 54″, height of the eye 16 feet; required the true and apparent time at the place, and its true longitude, on the supposition that the watch shewed true mean Greenwich time.

Observed alt. of ⊙	=	21° 45′ 0″
Index error and dip		− 2′ 58″
		21° 42′ 2″
Refraction and paral.		− 2′ 15″
		21° 39′ 47″
⊙'s semidiameter		+ 15′ 50″
True altitude	=	21° 55′ 37″
Given latitude	=	43° 17′ 0″
Polar distance	=	74° 58′ 6″
Sum		2)140° 10′ 43″

⊙'s dec., Aug. 11, at mean noon		15° 16′ 56·3″ N.
Daily change		− 17′ 56·3″ P.L. = ·12642
Time past noon		20ʰ 7·1ᵐ P.L. = 7662
		15′ 2·3″ P.L. = ·20304
Corrected declination		15° 1′ 54″ N.
		90° 0′ 0″
Polar distance		74° 58′ 6″
sec.	=	0·1378852
cosec.	=	0·0151206

½ Sum	= 70° 5′ 21″	cos.	=	9·5321902
½ Sum − altitude	= 48° 9′ 44″	sine	=	9·8721775
				2)19·5573735
	36° 55′ 22·5″	sine	=	9·7786867
	8			

Ap. time from noon = 4ʰ 55ᵐ 23ˢ
24 0 0

August 11 = 19 4 37
Equat. of time + 4 51

August 11 = 19 9 28 = mean time.

Mean time at Greenwich = 20ʰ 7ᵐ 6ˢ
Mean time at the place = 19 9 28

Longitude in time = 57 38
Longitude in = 14° 24′ 30″ W.

Example II.

Suppose that on the 10th of October 1866, at 8 h. 21 m. A.M., per watch, in the latitude of 51° 30′ N. and longitude 97° E. from Greenwich by account, the altitude of the sun's lower limb was 13° 32′, height of the eye 14 feet; required the apparent time of observation.

Time at the place, Oct. 9	= 20ʰ 21ᵐ	⊙'s dec., Oct. 9	=	6° 16′ 51·8″ S.
Longitude in time, 97°	− 6 28	Daily change		+ 22′ 48″ P.L. = ·02228
Greenwich time of observation, Oct. 9	= 13 53	Time past noon	=	13ʰ 53ᵐ P.L. = ·23772
				+ 13′ 11·2″ P.L. = ·26000
		True declination	=	6° 30′ 3″ S.
Alt. of ⊙'s l. l.	= 13° 32′ 0″	Hence polar dist.	=	96° 30′ 3″
Correction for dip	− 3′ 37″			
	13° 28′ 23″			
Ref. and paral.	− 3′ 46″			
	13° 24′ 37″			
⊙'s semidiameter	+ 16′ 4″			
True altitude	= 13° 40′ 41″			
Latitude	= 51° 30′ 0″	sec.	=	0·2058504
Polar distance	= 96° 30′ 3″	cosec.	=	0·0028014
Sum	2)161° 40′ 44″			
½ Sum	= 80° 50′ 22″	cos.	=	9·2019473
½ Sum − altitude	= 67° 9′ 41″	sin.	=	9·9645433
				2)19·3751424
	29° 8′ 48″	sin.	=	9·6875712
	8			

24ʰ − (3ʰ 53ᵐ 10ˢ 24ˢ) = 20ʰ 6ᵐ 50ˢ = 8ʰ 6ᵐ 50ˢ A.M.

Time by watch 8 21

∴ The watch was fast for apparent time 14 10

Problem XIV.

346. To find the apparent time, and also the azimuth of the sun, from an observed altitude of the same when at a considerable distance from the meridian, the latitude of the place of observation being known, and also the Greenwich time nearly.

RULE.

1. From the Greenwich time find the sun's polar distance at the instant of observation, by means of the *Nautical Almanac.*

2. Correct the sun's observed altitude for index error, dip, refraction, parallax, and semidiameter; and call the result the true altitude. Subtract the true altitude from 90°, which will give the sun's zenith distance.

3. Subtract the latitude from 90°, which will give the co-latitude, or distance of the pole from the zenith. The polar distance, the co-latitude, and the sun's zenith distance form the three sides of a spherical triangle, of which the angle at the pole is the measure of the time from noon, and the angle at the zenith is the measure of the sun's azimuth; these can now be found by means of the formulæ in Art. 335 (19), which give the following rule:

4. Write the sun's zenith distance, his polar distance, and the co-latitude all under each other, then add the three together, and take half their sum; from this half sum subtract each of the three sides in succession.

5. Take the log. cosecant of the half sum, and the log. sine of each of the three remainders, add the four together, and take half the sum, which call a constant.

6. From this constant subtract the log. sine of the remainder found by subtracting from the half sum of the sides the sun's zenith distance; and the remainder will be the logarithmic tangent of half the time from noon, or the logarithmic cotangent of half the time from midnight.

7. From the same constant subtract the log. sine of the

remainder found by subtracting the polar distance from half the sum of the three sides, and the remainder will be the logarithmic tangent of half the azimuth.

Example.

On the 12th June 1866, in latitude 55° 55′ N., at 5 h. 27 m. 20 s. P.M. mean Greenwich time, the bearing of the sun was W. 38° N. by compass, the observed altitude of his lower limb was 15° 57′ 45″, the index error being − 1′ 16″, height of the eye 18 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination at that instant being 23° 10′ 36″ N., and his semidiameter 15′ 47″.

Observed alt. ⊙	=	15° 57′ 45″			90° 0′ 0″
Index error and dip		− 5′ 22″	Dec.	=	23° 10′ 36″
		15° 52′ 23″	P. distance	=	66° 49′ 24″
Refraction and paral.		− 3′ 10″			
		15° 49′ 13″			90° 0′ 0″
⊙'s semidiameter		+ 15′ 47″	Latitude	=	55° 55′ 0″
True alt. of centre	=	16° 5′ 0″	Co-latitude	=	34° 5′ 0″
		90° 0′ 0″			
True zenith distance	=	73° 55′ 0″	= a		
Polar distance	=	66° 49′ 24″	= b		
Co-latitude	=	34° 5′ 0″	= c		
Sum		2)174° 49′ 24″	= $2s$		
s	=	87° 24′ 42″	cosec.	=	10·0004433
$s - a$	=	13° 29′ 42″	sin.	=	9·3680274
$s - b$	=	20° 35′ 18″	sin.	=	9·5461119
$s - c$	=	53° 19′ 42″	sin.	=	9·9042129
					2)38·8187955
					19·4093977 = Const.
Tan. 47° 43′ 30″	=	tan. ½(time from noon)		=	10·0413703 = Const. − sin. $(s - a)$
Tan. 36° 7′ 37″	=	tan. ½ azimuth		=	9·8632858 = Const. − sin. $(s - b)$

∴ The apparent time from noon = 47° 43′ 30″ × 8 = 6h 21m 48s
Equation of time − 29
Mean time at the place = 6 21 19
Mean time at Greenwich = 5 27 20
Longitude in time = 53 59
∴ Longitude in = 13° 29′ 45″ E.

True azimuth = 36° 7′ 37″ × 2 = N. 72° 15′ 14″ W.
Observed azimuth = N. 52° 0′ 0″ W.
Variation of the compass = 20° 15′ 14″ W.

Exercises 46.

(527.) In latitude 36° 39′ S., the sun's declination being 9° 27′ 15″ N., the altitude of the sun's lower limb in the morning was observed to be 10° 33′ 20″, the index error being + 1′ 15″, the height of the eye 20 feet, and the sun's semidiameter 15′ 58″; required the apparent time.

(528.) In latitude 36° 21′ S., the sun's declination being 8° 44′ 50″ N., the observed altitude of the sun's l. l. in the morning was 10° 47′ 30″, index error + 2′ 15″, height of the eye 17 feet, and the sun's semidiameter 16′ 5″; required the apparent time.

(529.) In latitude 29° 25′ N., the sun's declination being 23° 20′ N., the observed altitude of the sun's ⊙ l. in the afternoon 15° 29′ 30″, the index error being + 1′ 20″, height of the eye 16 feet, and the sun's semidiameter 15′ 47″; required the apparent time of observation.

(530.) In latitude 3° 31′ N., the sun's declination being 20° 3′ S. the observed altitude of the sun's l. l. in the afternoon was 38° 41′ 30″, index error − 1′ 12″, height of the eye 20 feet, and the sun's semidiameter 15′ 50″; required the apparent time of observation.

(531.) In latitude 13° 17′ N., the sun's declination being 22° 10′ S., the observed altitude in the morning of the sun's l. l. was 36° 26′ 40″, index error + 2′ 50″, height of the eye 18 feet, and the sun's semidiameter 15′ 48″; required the apparent time of observation.

(532.) In latitude 21° 36′ S., the sun's declination being 3° 37′ S., the observed altitude of the sun's l. l. in the morning was 35° 48′ 20″, index error − 3′ 15″, height of the eye 17 feet, and the sun's semidiameter 16′ 7″; required the apparent time.

(533.) In latitude 61° 12′ N., the sun's declination being 17° 43′ N., the observed altitude of the sun's l. l. in the afternoon was 12° 47′ 15″, index error − 1′ 40″, height of

the eye 19 feet, and sun's semidiameter 15′ 52″; required the apparent time of observation.

(534.) In latitude 48° 30′ N., the sun's declination being 15° 18′ 20″ N., the observed altitude of the sun's lower limb in the morning was 15° 11′ 17″, index error + 54″, the height of the eye being 15 feet, and the sun's semidiameter 15′ 54″; required the apparent time at the place of observation.

(535.) On the 1st August 1866, in latitude 34° 15′ N., at 3 h. 17 m. 20 s. P.M. mean Greenwich time, the bearing of the sun by compass was N. 62° 40′ W., the observed altitude of his lower limb 17° 50′, index error − 2′ 20″, height of the eye 20 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination at the instant being 17° 59′ 54″ N., his semidiameter 15′ 48″, and the equation of time + 6 m. 2 s.

(536.) On the 7th of September 1866, in latitude 38° 4′ S., at 2 h. 15 m. 16 s. P.M. mean Greenwich time, the bearing of the sun by compass being N. 88° 30′ E., the observed altitude of his l. l. was 12° 43′ 40″, index error + 2′ 46″, height of the eye 15 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination at the instant being 6° 1′ 58″ N., semidiameter 15′ 55″, and the equation of time 2 m. 6 s. to be subtracted from the apparent time.

(537.) On the 12th August 1866, in latitude 50° 10′ N., at 9 h. 12 m. 45 s. P.M. mean Greenwich time, the bearing of the sun by compass was S. 70° 12′ W., the observed altitude of the sun's l. l. was 10° 17′ 33″, index error + 2′ 12″, and the height of the eye 21 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination at the instant being 14° 52′ 1″ N., his semidiameter 15′ 50″, and the equation of time 4 m. 45 s. to be added to the apparent time.

(538.) On the 19th September 1866, in latitude 24° 30′ N., at 8 h. 7 m. 34 s. P.M. mean Greenwich time, the bearing of the sun by compass was S. 78° 54′ E., the observed altitude of the sun's l. l. 12° 13′ 20″, index error − 1′ 7″, and the height of the eye 17 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination at the instant being 1° 20′ 37″ N., his semidiameter 15′ 58″, and the equation of time 6 m. 22 s. to be subtracted from the apparent time.

(539.) On the 4th October 1866, in latitude 43° 12′ N., at 4 h. 37 m. 12 s. P.M. mean Greenwich time, the bearing of the sun by compass being due west, and the observed altitude of the sun's l. l. 10° 4′ 28″, index error + 3′ 15″, and the height of the eye 21 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination being then 4° 26′ 10″ S., his semidiameter 16′ 2″, and the equation of time 11 m. 36 s. to be subtracted from the apparent time.

(540.) On the 11th November 1866, in latitude 12° 5′ S., at 10 h. 12 m. 14 s. P.M. mean Greenwich time, the bearing of the sun by compass being S. 54° 30′ W., and the observed altitude of his u. l. 12° 17′, index error − 2′ 11″, height of the eye 18 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination being then 17° 34′ 3″ S., his semidiameter 16′ 12″, and the equation of time 15 m. 46 s. to be subtracted from the apparent time.

(541.) On the 22d June 1866, in latitude 55° 59′ N., at 7 h. 12 m. 40 s. A.M. mean Greenwich time, the bearing of the sun by compass was due east, the observed altitude of the sun's l. l. 20° 12′ 30″, index error − 2′ 17″, and the height of the eye 18 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination being then 23° 27′ 12″ N.,

his semidiameter 15′ 46″, and the equation of time 1 m. 33 s. to be added to apparent time.

(542.) On the 3d December 1866, in latitude 24° 58′ S., at noon, mean Greenwich time, the bearing of the sun by compass was S. 76° 30′ E., the observed altitude of his l. l. 15° 12′ 36″, index error − 3′ 12″, and the height of the eye 20 feet; required the apparent and mean time at the place of observation, the longitude of the place, and the variation of the compass—the sun's declination being then 22° 7′ 36″ S., his semidiameter 16′ 16″, and the equation of time 10 m. 1 s. to be subtracted from the apparent time.

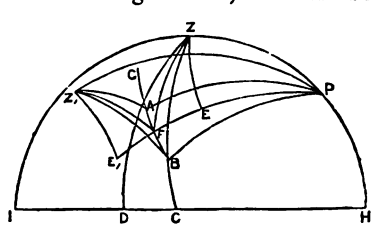
Problem XV.

347. To find the latitude from two observed altitudes of the sun, and the time elapsed between the observations; and also to find the longitude of the place of observation when the time is taken by a chronometer which gives the true Greenwich time when each observation was taken.

INVESTIGATION OF METHOD.

Let Z or Z_1 be the zenith, IDCH the horizon, P the pole, A and B the places of the sun at the two times of observation, AD and BC the true altitudes; AZ, BZ, or AZ_1, BZ_1 the true zenith distances; AP and BP the polar distances, which, in the practice of this problem, may be considered as equal.

Join AB, and bisect AB in F, and join PF; PF will be perpendicular to AB; then the two triangles AFP, BFP will be right-angled at F, and equal in all respects. To PF, or PF produced, let the perpendicular ZE or Z_1E_1 be drawn; and join ZF or Z_1F_1. Then APB will be the elapsed interval of time between the observations, APF half the elapsed time, ZP the co-latitude, ZPA the sun's



meridian distance when the greater altitude was taken, and ZBP his meridian distance at the time of taking the less altitude, and ZPF his meridian distance at the middle time between the observations. Also, since PFA is a right angle, ZFP or ZFE is the complement of ZFA or ZFB.

In the right-angled spherical triangle PFA, we have

$$\text{sin. AP . sin. APF} = \text{sin. AF} = A_1, \text{ always acute;}$$

and cos. AP . sec. AF = cos. PF = A_2, of the same affection as AP;

$$\text{cos. ZA, or sin. AD} = \text{cos. AF . cos. FZ} + \text{sin. AF . sin. FZ . sin. ZFE;}$$

and cos. ZB, or sin. BC = cos. AF . cos. FZ − sin. AF . sin. FZ . sin. ZFE.

Subtracting the last two equations, we have

$$\text{sin. AD} - \text{sin. BC} = 2 \text{ sin. AF . sin. FZ . sin. ZFE,}$$

or $$\text{sin. AD} - \text{sin. BC} = 2 \text{ sin. AF . sin. ZE.}$$

Whence, $$\text{sin. ZE} = \frac{\text{sin. AD} - \text{sin. BC}}{2 \text{ sin. AF}}$$

$$= \frac{2 \text{ cos. } \frac{1}{2}(\text{AD} + \text{BC}) \text{ . sin. } \frac{1}{2}(\text{AD} - \text{BC})}{2 \text{ sin. AF}};$$

$$\therefore \text{ sin. ZE} = \text{cosec. AF . cos. } \tfrac{1}{2}(\text{AD} + \text{BC}) \text{ . sin. } \tfrac{1}{2}(\text{AD} - \text{BC}) = A_3.$$

Adding the same two equations, we have

$$\text{sin. AD} + \text{sin. BC} = 2 \text{ cos. AF . cos. FZ,}$$

or $$\text{sin. AD} + \text{sin. BC} = 2 \text{ cos. AF . cos. FE . cos. ZE.}$$

Whence, $$\text{cos. FE} = \frac{\text{sin. AD} + \text{sin. BC}}{2 \text{ cos. AF . cos. ZE}}$$

$$= \frac{2 \text{ sin. } \frac{1}{2}(\text{AD} + \text{BC}) \text{ . cos. } \frac{1}{2}(\text{AD} - \text{BC})}{2 \text{ cos. AF . cos. ZE}};$$

$$\therefore \text{ cos. FE} = \text{sin. } \tfrac{1}{2}(\text{AD} + \text{BC}) \text{ . cos. } \tfrac{1}{2}(\text{AD} - \text{BC}) \text{ . sec. AF . sec. ZE} = A_4,$$

$$\text{PF} \mp \text{FE} = \text{EP} = \overline{A}_5,$$

or $$A_2 \mp A_4 = A_5,$$

sin. latitude = cos. PZ = cos. ZE . cos. PE = cos. A_3 . cos. A_5.

348. Next, to find the longitude of the place of observation. In the triangle ZEP,

$$\text{sin. ZE} = \text{sin. ZP} \;.\; \text{sin. ZPE} = \text{cos. lat.} \;.\; \text{sin. ZPE};$$

$$\therefore \text{sin. ZPE} = \text{sin. } A_3 \;.\; \text{sec. latitude} = A_6,$$

the sine of the apparent time from noon of the middle time between the altitudes.

349. Correct the observed altitudes so as to find the true altitudes (Prob. VII.); and if the ship has changed her position between the observations, mark the angle contained between the ship's course and the direction of the sun when the least altitude was taken; with this angle as a course, and the distance run as a distance, enter the Traverse Table, and in the Dif. Lat. column will be found the correction in minutes, which is to be added when the sun's bearing at the least altitude and the direction sailed make an acute angle, but subtracted when the angle so formed is obtuse; this correction will reduce the least observed altitude to what it would have been if observed at the same place as the greater. Find also the sun's polar distance at the middle time between the observations, which is PA or PB in the above figure; find the time between the observations, and take its half, which reduce to degrees and minutes, &c., and call the result half the elapsed time; then proceed to compute the latitude of the place where the greater altitude was taken, and also the time from apparent noon by the following

RULE.

1. Add the log. *sine* of half the elapsed time to the log. *sine* of the polar distance; and the sum, rejecting 10 from the index, will be the log. *sine* of A_1, arc first, which is always acute.

2. Add the log. *secant* of A_1 to the log. *cosine* of the polar distance; and the sum, rejecting 10 from the index, will be the log. *cosine* of A_2, arc second, which will be acute or obtuse, like the polar distance.

3. Add together the log. *cosecant* of A_1, the log. *cosine* of half the sum of the true altitudes, and the log. *sine* of half their difference; and the sum, rejecting 20 from the index, will be the log. *sine* of A_3, arc third, which is always acute.

4. Add together the log. *secant* of A_1, the log. *sine* of half the sum of the true altitudes, the log. *cosine* of half their difference, and the log. secant of A_3; and the sum, rejecting 30 from the index, will be the log. *cosine* of A_4, which is always acute.

5. The difference of arc second, A_2, and arc fourth, A_4, is arc fifth, A_5, when the zenith and the elevated pole are on the same side of the great circle passing through the places of the sun at the times of observation; otherwise, their sum is arc fifth, A_5.

6. Add the log. *cosine* of arc third, A_3, to the log. *cosine* of arc fifth, A_5; and the sum, rejecting 10 from the index, will be the *sine* of the latitude.

7. To the log. *secant* of the latitude add the log. *sine* of arc third, A_3, formerly found; and the sum, rejecting 10 from the index, will be the log. *sine* of arc sixth, A_6, the apparent time from noon at the *middle* time.

8. To the apparent time thus found, apply the equation of time (*Nautical Almanac*, page 1 of each month); the difference between the mean time thus found and the *middle time*, as shewn by chronometer, will be the longitude in time, which will be *east*, if the time at the ship is *faster*, but *west*, if the time at the ship is *slower*, than the middle Greenwich time.

Example I.

At sea, in north latitude, January 9, 1866, at 23 h. 18 m. 40 s. apparent Greenwich time, the observed altitude of the sun's l. l. was 10° 27′ 30″, and the bearing of its centre by azimuth compass SE. $\frac{3}{4}$ S.; and 1 h. 40 m. 10 s. afterwards, the observed altitude of his lower limb was 17° 6′ 40″, the index error being −35″, and the height of the eye 15 feet above the level of the

sea; the ship's course during the elapsed time was SSE., at the rate of 10 miles an hour; required the latitude and longitude of the ship at the time of observing the greater altitude.

Solution.

January 9,	$23^h\ 18^m\ 40^s$	=	Greenwich time of first observation.
Add	50 5	=	half-elapsed time = 12° 31′ 15″
January 10,	0 8 45	=	middle time.

⊙'s declination, January 10, noon	=	21° 56′ 56″ S.
Correction for 8 m. 45 s. = 22·55″ × ·146		− 3″
⊙'s declination at middle time	=	21° 56′ 53″ S.
		90° 0′ 0″
⊙'s polar distance at middle time	=	111° 56′ 53″

⊙'s bearing at the first observation = SE. ¼ S.	=	3¼ points.
Ship's course, SSE.	=	2 points.
Contained angle	=	1¼ points.

Time elapsed between the observations = 1 h. 40 m. 10 s.; and the rate of sailing was 10 miles an hour, and therefore

$1^h : 1^h\ 40^m\ 10^s :: 10 : 16{\cdot}69$ = 17 miles nearly = the distance run between the two observations.

Now, to course 1¼ points, and distance 17 miles, the difference of latitude is 16·5 miles, and the departure 4·1, hence the *reduction of altitude* due to the ship's change of place is 16′ 30″, which is additive to the least altitude, because the contained angle is less than 8 points.

First altitude ☉	= 10° 27′ 30″	Second altitude ☉	= 17° 6′ 40″
Index error and dip	− 4′ 20″	Index error and dip	− 4′ 20″
	10° 23′ 10″		17° 2′ 20″
Refraction and paral.	− 4′ 54″	Refraction and paral.	− 2′ 56″
	10° 18′ 16″		16° 59′ 24″
⊙'s semidiameter	+ 16′ 18″	⊙'s semidiameter	+ 16′ 18″
Correction for run	+ 16′ 30″	True altitude	= 17° 15′ 42″
True altitude	= 10° 51′ 4″		

True second altitude	= 17° 15′ 42″		
True first altitude	= 10° 51′ 4″		
Sum of altitudes	= 28° 6′ 46″	∴ ½ sum =	14° 3′ 23″
Difference of altitudes	= 6° 24′ 38″	∴ ½ dif. =	3° 12′ 19″

To compute the Latitude and Longitude.

H.E.T.	= 12° 31′ 15″	sin.	9·3360484				
P. dist.	= 111° 56′ 53″	sin.	9·9673247	cos.	9·5725996		
A_1	= 11° 36′ 1″	sin.	9·3033731	sec.	0·0089626	cosec.	0·6966269
A_2	= 112° 25′ 49″			cos.	9·5815622		
				sec. A_1	0·0089626		
½ Sum	= 14° 3′ 23″			sin.	9·3853859	cos.	9·9867973
½ Dif.	= 3° 12′ 19″			cos.	9·9993201	sin.	8·7475165
A_3	= 15° 38′ 55″			sec.	0·0164035	sin.	9·4309407
A_4	= 75° 6′ 11″			cos.	9·4100721	cos. A_3	9·9835965
A_5	= 37° 19′ 38″	= $A_2 - A_4$				cos.	9·9004684
Latitude	= 49° 58′ 13″			sec.	0·1916642	sin.	9·8840649
				sin. A_3	9·4309407		
A_6	= 24° 47′ 43″			sin.	9·6226049		
	4						
	1h 39m 11s						
	24 0 0						
Jan. 9	22 20 49	apparent middle time at ship.					
Jan. 10	0 8 45	apparent middle time at Greenwich.					
Long.	1 47 56	in time.					
∴ Long.	= 26° 59′ 0″	W.					

Example II.

At sea, in latitude 43° 30′ S., by account April 14, 1866, at 19 h. 18 m. 24 s. apparent Greenwich time, the observed altitude of the sun's l. l. was 35° 54′; and at 22 h. 8 m. 26 s. apparent Greenwich time, the observed altitude of the same limb was 28° 42′ 15″, and the bearing of the sun's centre, by azimuth compass, NW. ¾ N.; the height of the eye being 24 feet, and the ship's course during the elapsed time SW., at the rate of 9 miles an hour; required the ship's latitude at the time of taking the greater altitude, and her longitude at the middle time.

Solution.

⊙'s bearing at second observation, NW. ¼ N.	=	W. 4¼ points N.
Ship's course, SW.	=	W. 4 points S.
Contained angle	=	8¼ points

Time elapsed between the observations, 2 h. 50 m. 2 s.; and therefore

1h : 2h 50m 2s : : 9 miles : 25·5 miles, the distance run.

Now, to course 7¼ points, and distance 25·5 miles, the difference of latitude is 3·7 miles, the reduction of altitude, which is additive to the last altitude, because the angle between the course sailed and the sun's direction is less than 8 points, since the observation was made in the afternoon.

Time of observing the less altitude, April 14	=	$22^h\ 8^m\ 26^s$
Time of observing the greater	=	19 18 24
Elapsed time	=	2 50 2

$\therefore$ H.E.T. $= 1^h\ 25^m\ 1^s = 21°\ 15'\ 15''$

Middle time between the observations $= 20^h\ 43^m\ 25^s$ = ½ sum times.

⊙'s dec. at apparent noon (N.A.), April 14	=	9° 25′ 39·5″ N.
Correction for 20 h. 43 m. 25 s. at 53·7″ per hour		+ 18′ 32·7″
⊙'s corrected declination at middle time	=	9° 44′ 12″ N.

Polar distance at middle time = 90° + dec. = 99° 44′ 12″

First obs. altitude ☉	=	35° 54′ 0″	Second obs. altitude ☉	=	28° 42′ 15″
Dip		− 4′ 45″	Dip		− 4′ 45″
		35° 49′ 15″			28° 37′ 30″
Refraction and paral.		− 1′ 11″	Refraction and paral.		− 1′ 37″
		35° 48′ 4″			28° 35′ 53″
⊙'s semidiameter		+ 15′ 58″	⊙'s semidiameter		+ 15′ 58″
					28° 51′ 51″
First true altitude	=	36° 4′ 2″	Cor. for change of place		+ 3′ 42″
Second true altitude	=	28° 55′ 33″	Second true altitude	=	28° 55′ 33″
Sum of altitudes	=	64° 59′ 35″	$\therefore$ ½ sum =		32° 29′ 47½″
Dif. of altitudes	=	7° 8′ 29″	$\therefore$ ½ dif. =		3° 34′ 14½″

To compute the Latitude and Longitude.

H.E.T.	= 21° 15′ 15″	sin.	9·5593150				
P. dist.	= 99° 44′ 12″	sin.	9·9936987	cos.	9·2281954		
A_1	= 20° 56′ 1″	sin.	9·5530137	sec.	0·0296554	cosec.	0·4469863
A_2	= 100° 25′ 56″			cos.	9·2578508		
				sec. A_1	0·0296554		
½ Sum alts.	= 32° 29′ 47·5″			sin.	9·7301752	cos.	9·9260460
½ Dif. alts.	= 3° 34′ 14·5″			cos.	9·9991561	sin.	8·7943151
A_3	= 8° 27′ 13″			sec.	0·0047443	sin.	9·1673474
A_4	= 54° 31′ 15″			cos.	9·7637310		
						cos. A_3	9·9952557
A_5	= 45° 54′ 41″ = $A_2 - A_4$					cos.	9·8424657
Latitude	= 43° 29′ 19″			sec.	0·1393559	sin.	9·8377214
				sin. A_3	9·1673474		
A_6	= 11° 41′ 26″			sin.	9·3067023		

4

$0^h\ 46^m\ 45^s\ 44^t$ apparent middle time from noon.

24 0 0

24 46 45·7 apparent middle time at the ship.

− 20 43 27 apparent middle time at Greenwich.

Longitude 4 3 18·7 in time.

60

4)243 18 42

Longitude = 60° 49′ 46½″ E. at middle time.

Exercises 47.

(543.) Being at sea in latitude 46° 30′ N. by account, when the sun's declination was 11° 17′ N., at 10 h. 2 m. A.M., the sun's corrected central altitude was 46° 55′; and at 11 h. 27 m. A.M., his corrected central altitude was 54° 9′; required the true latitude, and the time from apparent noon when the greater altitude was taken.

(544.) Being at sea in latitude 47° 19′ N. by account, when the sun's declination was 12° 16′ N., at 10 h. 24 m. A.M. per watch, the sun's correct central altitude was 49° 9′; and at 1 h. 14 m. P.M., his corrected central altitude was 51° 59′; required the true latitude, and the apparent time from noon when the greater altitude was taken.

(545.) Being at sea in latitude by account 50° 40′ N., when the sun's declination was 20° 0′ S., at 10 h. 17 m. A.M. per watch, the sun's corrected central altitude was found to be 17° 13′; and at 11 h. 17 m. A.M. per watch, his corrected central altitude was found to be 19° 41′; required the latitude in, and the apparent time of taking the greater altitude.

(546.) Being at sea in latitude 60° N. by account, when the sun was on the equator, and consequently had no declination, at 1 h. 0 m. P.M. per watch, his corrected central altitude was found to be 28° 53′; and at 3 h. 0 m. P.M. per watch, his corrected central altitude was 20° 42′; required the true latitude, and the error of the watch from apparent time at the place.

(547.) Being at sea in latitude by account 7° 40′ N., the sun's declination being 22° 47′ S., at 7 h. 25 m. 40 s. A.M., the true altitude of his centre was 22° 30′, and at 10 h. 31 m. 48 s. A.M., his true central altitude was 63° 40′; required the true latitude, and the apparent time from noon when the greater altitude was taken.

(548.) In a ship running N. by E. $\frac{3}{4}$ E. per compass, at the rate of 9 knots an hour, at 10 h. 0 m. A.M. per watch,

the sun's corrected central altitude was found to be 13° 18′ bearing S. ¾ E. by compass; and at 1 h. 40 m. P.M. per watch, his central altitude was found to be 14° 15′, the latitude by account being 49° 17′ N., and the sun's declination 23° 28′ S.; required the true latitude, and the error of the watch for apparent time at the place.

(549.) Sailing NE. ½ E. by compass, at the rate of 9 knots an hour, at 0 h. 31 m. 40 s. P.M. per watch, the altitude of the sun's lower limb was 28° 20′ above the horizon of the sea, the eye being elevated 20 feet above the surface of the water, and the sun's bearing by compass S. by W.; and at 2 h. 58 m. 20 s. P.M. by watch, the altitude of the sun's lower limb was 16° 41′ above the horizon, the eye being elevated as before, and the index error in both observations nothing, and the sun's semidiameter 16′ 9″, the latitude by account at the last observation was 52° 2′ N., and the sun's declination 13° 17′ S.; required the true latitude at taking the last observation, and the error of the watch for apparent time.

(550.) Being at sea in latitude by account 39° 28′ N., when the sun's declination was 20° 41′ N., at 11 h. 30 m. 15 s. A.M. per watch, the altitude of the sun's lower limb was observed to be 68° 18′ 45″; and at 0 h. 26 m. 28 s. P.M., was 70° 58′, the height of the eye being 21 feet above the surface of the sea, the index error + 1′ 17″, and the sun's semidiameter 15′ 46″; required the true latitude of the place of observation, and the error of the watch for apparent time.

(551.) Being at sea in latitude 50° 40′ N. by account, at 10 h. 17 m. 30 s. A.M. per watch, the altitude of the sun's lower limb was observed to be 17° 4′ 15″; and at 11 h. 17 m. 30 s., the same limb was 19° 31′ 15″, the sun's declination being 20° 2′ S., the index error − 2′ 10″, the height of the eye 21 feet, and the sun's semidiameter 16′ 14″—both observations being taken at the same place; required the true latitude of the place of observation, and the error of the watch for apparent time.

Problem XVI.

350. To find the *latitude* and *longitude* of a place by construction on the terrestrial globe, from two observed altitudes of the sun, the Greenwich time of observation being known. Also to find the same by Captain Sumner's method.

RULE.

(*a.*) Find from the Greenwich time the sun's declination and right ascension, and the places on the earth to which the sun is vertical at the first and second observations.

(*b.*) Correct the observed altitudes for index error, dip, refraction, parallax, and semidiameter; and take the complements of each, which will give the sun's zenith distance at the time of each observation.

(*c.*) If the ship has sailed during the interval between the observations, reduce the first altitude to what it would have been if observed at the same place as the second, before taking its zenith distance.

(*d.*) Measure the first corrected zenith distance on the equator with a pair of compasses; and with one foot in the place to which the sun was vertical when the observation was taken, describe with the other a circle on the surface of the globe; this circle will pass through all places where the sun's altitude was the same at the time of taking the first observation.

(*e.*) In the same manner, measure the second zenith distance on the equator, and with one foot of the compasses in the place to which the sun was vertical at the time of taking the second observation, describe with the other a circle on the surface of the globe, which will pass through all places where the sun's altitude was the same when the second altitude was taken. These two circles, both passing through the place of observation, will therefore intersect in it, and its position on the globe will therefore be found.

(*f.*) Bring this point to the brazen meridian, and on it will be

found the latitude in, and on the equator will be found the longitude required.

351. REMARK I.—If the two circles should not intersect but only touch, then the sun has been on the same vertical at both observations, and as the point of contact of two circles cannot be found with the same accuracy as their point of intersection, this case gives the latitude and longitude with the least certainty. Should the circles neither intersect nor touch, this will prove that some error has been introduced, either in taking the altitudes or in finding the zenith distances, or the place to which the sun is vertical when the observations were taken.

352. REMARK II.—To prevent marking and spoiling the globe, it may be advisable to use two pair of compasses, and having adjusted the one pair to the first zenith distance, and the other to the second, then, placing one foot of each pair in the point to which the sun was vertical when the observation was taken to which they have been adjusted, turn the other feet round till they coincide; the point on the globe where this coincidence takes place is the place of observation, which bring to the brazen meridian as before, and read off the latitude and longitude required.

353. REMARK III.—If the latitude alone is sought, it will only be necessary to take, for the positions to which the sun is vertical at the times of observation, two positions having the same latitude as the sun's declination, and whose difference of longitude is equal to the interval of time between the observations. With these two positions, so related, perform the operations as directed above, and the latitude will be found with as much accuracy as if the places as centres had been those to which the sun was vertical at the times of observation, but nothing can be inferred from this in reference to the longitude of the place of observation.

354. REMARK IV.—In finding the latitude alone, as directed in 353, it is not necessary to know the Greenwich time, except for finding the sun's declination at the time of observation, and for this an approximate time will be sufficient; so that, knowing the approximate Greenwich time, and having a watch which will measure with sufficient accuracy the interval of time between the observations, the latitude of a place may be determined without knowing the accurate Greenwich time; this may be applied to find the latitude when the mariner cannot depend on his chronometer, so as to enable him also to find his longitude.

355. REMARK V.—A little reflection will enable any one to see that the circles described on the globe as above will cut a variety of different parallels of latitude, and will touch one to the north and another to the south of the place to which the sun is vertical, and in the points where the meridian on which he is cuts these parallels. The circles will also touch two meridians, one to the east and the other to the west of the place to which the sun is vertical, and nearly on the parallel of latitude which corresponds to his declination. The circles will either cut in two points or touch in one, and either of the points of intersection may be the position of the ship, but as the one will always be to the north and the other to the south of the sun's place, there will be no difficulty in ascertaining which is to be taken, except when the sun passes at noon very near the zenith of the place; and this last is an unfavourable position for finding the latitude by double altitudes.

356. REMARK VI.—If we now suppose that the observations were made first in one latitude and then in another, and from these observed altitudes and the declination of the sun, calculate the time from noon when the observation was taken, that latitude which is farthest from the parallel to which the sun is vertical will give the least time from noon; and if this be done with two latitudes and the same altitude, two different longitudes will be obtained, one on each of the parallels; these, being laid down on the chart, will give two points, one on each latitude, which being joined will give a line of equal altitudes. If now, with the other altitude and the same latitudes as before, the calculations be repeated, two other longitudes on the same parallels will be obtained, which, being joined on the chart,

will give another line of equal altitudes; but as each of these lines will pass through the place of observation, their intersection will be the place of the ship, which may therefore be laid down on the chart, and thus both the latitude and longitude of the ship may be found by construction on the chart if the Greenwich time be accurately known, but the latitude very nearly though the Greenwich time be only known approximately. This remark contains the elementary principles of Sumner's new method of finding a ship's place at sea.

VERIFICATION OF THE DOUBLE ALTITUDE PROBLEM BY SUMNER'S METHOD.

357. RULE.—1. Find the Greenwich date of each observation.

2. From the *Nautical Almanac* find the sun's declination for each date.

3. From the observed find the true altitudes.

4. Assume two latitudes differing by about a degree or less, and each differing little from the latitude by account; these may be conveniently taken—the first, the latitude by account wanting the minutes, and the second the next higher degree.

5. Find the apparent time from noon with each assumed latitude, first with one altitude and then with the other, as follows:

(*a*.) With the 1st alt., the *less* assumed lat., and the sun's P.D. at the time of taking the first observation.

(*b*.) With the 2d alt., the *less* assumed lat., and the sun's P.D. at the time of taking the second observation.

(*c*.) With the 1st alt., the *greater* assumed lat., and the sun's P.D. at the time of taking the first observation.

(*d*.) With the 2d alt., the *greater* assumed lat., and the sun's P.D. at the time of taking the second observation.

6. Obtain the *elapsed time* corresponding to *each assumed latitude* as follows: If one observation be A.M. and the other P.M., take the sum of the times from noon, or hour angle found by (*a*) and (*b*); if both observations be A.M., or both P.M., take the difference of the hour angles of (*a*) and (*b*), and call the sum or the remainder the *elapsed time* for the *less latitude*.

Proceed in the same manner, using the *hour angles* of (*c*) and (*d*) to find the *elapsed time* for the greater assumed latitude.

7. The *true elapsed time* is the difference of the times shewn by the watch at the first and second observation.

8. Take the *difference* between the *elapsed time* for the *less*, and that for the *greater* assumed latitude, and call it dif. A.

9. Also, take the *difference* between the elapsed time for the less assumed latitude and the *true* elapsed time, and call it dif. B.

10. Then state the following proportion, which may be computed by the table of Proportional Logarithms (page 275)—Dif. A : dif. B :: difference of the assumed latitudes : correction to be applied to the *less* lat.

358. *N.B.*—If the elapsed time for the greater lat. and the true elapsed time be either both *greater* or both *less* than the elapsed time for the less assumed latitude, the correction found above is to be *added* to the less assumed lat. to get the true latitude; but if one is greater and the other less, the correction is to be subtracted from the less assumed latitude.

Example I.

Verify by Sumner's method the first example in double altitudes (page 272).

It was there given that the *true elapsed time* = 1 h. 40 m. 10 s.

That the first alt., corrected and reduced to what it would have been if observed at the second station, was	10° 51′ 4″
That the second corrected alt. was	17° 15′ 42″
That the sun's P.D. at the middle time was	111° 56′ 53″
Change of dec. in 50 m. = 22·55 × $\frac{5}{6}$ h. =	19″
Which, being added, gives P.D. at 1st. obser.	111° 57′ 12″
And being subtracted, gives P.D. at 2d obser.	111° 56′ 34″

Assume now first lat. = 49°, and second lat. = 50°; then we have

	(a)			(b)		
	10° 51′ 4″			17° 15′ 42″		
sec.	49° 0′ 0″	=	·1830571	49° 0′ 0″	=	·1830571
cosec.	111° 57′ 12″	=	·0326914	111° 56′ 34″	=	·0326592
	2)171° 48′ 16″			2)178° 12′ 16″		
cos.	85° 54′ 8″	=	8·8540555	89° 6′ 8″	=	8·1950284
sin.	75° 3′ 4″	=	9·9850474	71° 50′ 26″	=	9·9778118
			2)19·0548514			2)18·3885565
sin.	19° 41′ 4″	=	9·5274257	8° 59′ 56″	=	9·1942782
subt.	8° 59′ 56″					

10° 41′ 8″ × 8 = 1ʰ 25ᵐ 29ˢ = elapsed time for the less lat.

	(c)			(d)		
	10° 51′ 4″			17° 15′ 42″		
sec.	50° 0′ 0″	=	·1919325	50° 0′ 0″	=	·1919325
cosec.	111° 57′ 12″	=	·0326914	111° 56′ 34″	=	·0326592
	2)172° 48′ 16″			2)179° 12′ 16″		
cos.	86° 24′ 8″	=	8·7976263	89° 36′ 8″	=	7·8415144
sin.	75° 33′ 4″	=	9·9860416	72° 20′ 26″	=	9·9790366
			2)19·0082918			2)18·0451427
sin.	18° 37′ 7″	=	9·5041459	6° 2′ 47″	=	9·0225713
subt.	6° 2′ 47″					

12° 34′ 20″ × 8 = 1ʰ 40ᵐ 35ˢ = elapsed time for greater lat.

Elapsed time for less lat.	1ʰ 25ᵐ 29ˢ			1ʰ 25ᵐ 29ˢ
Elapsed time for greater lat.	1 40 35	True elapsed time	1 40 10	
∴ Dif. A =	15 6	Dif. B =	14 41	

Hence (10) 15ᵐ 6ˢ : 14ᵐ 41ˢ :: 6′ : the correction to be added (358) to the less assumed latitude to get the true latitude.

Ar. co. prop. log.	15ᵐ 6ˢ	= 8·92370
Prop. log.	14ᵐ 41ˢ	= 1·08845
Prop. log.	6′ 0″	= ·47712
Prop. log.	58′ 21″	= 0·48927

Less assumed lat. = 49°

∴ Latitude in = 49° 58′ 21″,

being 8″ greater than that found by double altitudes on page 274.

Example II.

Verify by Sumner's method the second example in double altitudes (page 274).

Since the sun's declination is changing at the rate of 53·7″ per hour, the sun's polar distance at the time of taking the first observation would be 53·7″ × 1·42 = 1′ 16″ less than at the middle time, and at taking the second observation it would be as much greater.

Polar distance at middle time (page 275)	99° 44′ 12″
Change in 1 h. 25 m.	1′ 16″
Polar distance for first observation	99° 42′ 56″
Polar distance for second observation	99° 45′ 28″

To find the Hour Angles for assumed Lat. 43°.

(*a*)			(*b*)		
1st obs. alt.	36° 4′ 2″		2d obs. alt.	28° 55′ 33″	
Latitude	43° 0′ 0″	sec. ·1358725	Latitude	43° 0′ 0″	sec. ·1358725
Polar dist.	99° 42′ 56″	cosec. ·0062739	Polar dist.	99° 45′ 28″	cosec. ·0063288
	2)178° 46′ 58″			2)171° 41′ 1″	
	89° 23′ 29″	cos. 8·0262091		85° 50′ 30″	cos. 8·8604153
	53° 19′ 27″	sin. 9·9041893		56° 54′ 57″	sin. 9·9231764
		2)18·0725448			2)18·9257930
	6° 14′ 28″ =	sin. 9·0362724		16° 52′ 40″ =	sin. 9·4628965
				6° 14′ 28″	
		Sum of hour angles	=	23° 7′ 8″	
				8	
		1st elapsed time	=	3h 4m 57s	

To find the Hour Angles for assumed Lat. 44°.

(*c*)			(*d*)		
1st obs. alt.	36° 4′ 2″		2d obs. alt.	28° 55′ 33″	
Latitude	44° 0′ 0″	sec. ·1430659	Latitude	44° 0′ 0″	sec. ·1430659
Polar dist.	99° 42′ 56″	cosec. ·0062739	Polar dist.	99° 45′ 28″	cosec. ·0063288
	2)179° 46′ 58″			2)172° 41′ 1″	
	89° 53′ 29″	cos. 7·2777514		86° 20′ 30″	cos. 8·8048655
	53° 49′ 27″	sin. 9·9069862		57° 24′ 57″	sin. 9·9256221
		2)17·3340774			2)18·8798823
Sin.	2° 39′ 46″ =	8·6670387	sin.	15° 59′ 6″ =	9·4399411
				2° 39′ 46″	
		Sum of hour angles	=	18° 38′ 52″	
				8	
		2d elapsed time	=	2h 29m 11s	

1st elapsed time	3 h. 4 m. 57 s.		1st elapsed time	3 h. 4 m. 57 s.	
2d " "	2 h. 29 m. 11 s.		True " "	2 h. 50 m. 2 s.	
Dif. A =	35 m. 46 s.		Dif. B =	14 m. 55 s.	

35 m. 46 s. : 14 m. 55 s. : : 60′ : 25′ 2″

Add	43°	0′	0″	
Latitude in	43°	25′	2″	by Sumner's method.
Latitude in	43°	29′	16″	by double altitudes.
Difference	0°	4′	14″	

359. REMARK.—This example shews that all positions of the sun are not equally favourable for finding the latitude by this method; the reason is, that when the sun is near the meridian his altitude changes very slowly, and a very small error on an observed altitude makes then a considerable difference in the time calculated from it. That the sun was near the meridian when the observation was made is shewn by the sum of the altitude, the latitude, and the sun's polar distance being nearly 180°, which they would be if the observation had been made when the sun was on the meridian. In finding the latitude by double altitudes or by Sumner's method, the observations should therefore be taken, as far as practicable, from the meridian; and in making the above calculations, the cosine of the half sum should always be carefully corrected for the seconds in the angle when it exceeds 80°, as it is then changing very rapidly.

360. For additional exercises in this method, verify the exercises given in double altitudes, extending from Exercise 543 to 551 inclusive.

Problem XVII.

To compute the equation of equal altitudes.

361. The equation of equal altitudes is a correction for the change of the sun's declination, to be applied to the middle time between the instants by a chronometer, at which, on

a given day, the sun has equal altitudes, to find the time by the chronometer when he is upon the meridian. It is, in fact, half the difference between the forenoon and afternoon hour angles when the sun on the same day has equal altitudes.

362. To compute this correction, take the logs. A and B, answering to the elapsed time, from the Table, page 293; and to each add the logs. of the seconds in the change of the sun's declination between the noon of the day preceding and the noon of the day following the given one. Also, to log. A add the log. tangent of the latitude; and to log. B the log. cotangent of the sun's polar distance; and the sums, rejecting tens from the indices, are the logarithms of the two parts of the equation of equal altitudes. The part under log. A is + when the polar distance is increasing, and − when it is decreasing; and the part under log. B is − when the polar distance is acute and increasing, or obtuse and decreasing, and + when the polar distance is obtuse and increasing, or acute and decreasing.

363. REMARK.—The polar distance used in the calculation, may be taken as that of the noon of the given day Greenwich time, without reduction, to the meridian.

Example.

On July 25, 1866, in latitude 54° 20′ N., at 8 h. 59 m. 4 s. A.M., and at 3 h. 0 m. 40 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

	$20^h\ 59^m\ 4^s$	⊙'s dec., July 24	19° 52′ 57″ N.
	27 0 40	" " 26	19° 27′ 6″ N.
Interval =	6 1 36	Change in two days	25′ 51″ = 1551″
		⊙'s polar dist., July 25 =	70° 19′ 49″

To 6 h. 1 m. 36 s. in Table, page 293.

Log. A	= 7·7707	Log. B	= 7·6188
Log. 1551	= 3·1906	Log. 1551	= 3·1906
Lat. 54° 20′ tan.	= 10·1441	Polar dist. 70° 20′ cot.	= 9·5531
	1·1054	− 2·30	= 0·3625

+ 12·75
− 2·30
+ 10·45 equation of equal altitudes.

Exercises 48.

(552.) On August 16, 1866, in latitude 50° 10′ N., at 9 h. 12 m. 35 s. A.M., and at 2 h. 50 m. 12 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(553.) On October 12, 1866, in latitude 36° 20′ N., at 8 h. 50 m. 20 s. A.M., and at 3 h. 12 m. 10 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(554.) On June 10, 1866, in latitude 55° 56′ N. at 9 h. 5 m. 30 s. A.M., and at 2 h. 58 m. 20 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(555.) On November 11, 1866, in latitude 51° 12′ N., at 9 h. 20 m. 10 s. A.M., and at 2 h. 36 m. 50 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(556.) On November 11, 1866, in latitude 51° 12′ S., at 9 h. 20 m. 10 s. A.M., and at 2 h. 36 m. 50 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(557.) On May 8, 1866, in latitude 35° 17′ S., at 8 h. 50 m. 24 s. A.M., and at 3 h. 16 m. 12 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(558.) On September 30, 1866, in latitude 10° 55′ S., at 10 h. 1 m. 50 s. A.M., and at 2 h. 5 m. 40 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(559.) On March 3, 1866, in latitude 50° 16′ N., at 10 h. 50 m. 10 s. A.M., and at 1 h. 30 m. 4 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

(560.) On April 25, 1866, in latitude 30° 12′ S., at 10 h. 5 m. 40 s. A.M., and at 2 h. 1 m. 10 s. P.M., the sun had equal altitudes; required the equation of equal altitudes.

Problem XVIII.

To find the error of a chronometer by equal altitudes of the sun.

364. RULE.—Take several altitudes of the sun's upper or

lower limb, two or three hours before noon, and note the time of each observation by the chronometer; again, in the afternoon, note the successive times by the chronometer when he has the same altitudes. Take the mean of the times at each set of observations, adding 24 hours to the mean of those taken in the afternoon; half the sum of these will be the middle time between the observations, and the difference between them will be the elapsed interval of time. With this interval, the latitude, polar distance, and change of declination, compute the equation of equal altitudes by last problem, and apply it with the proper sign to the middle time; the result will be the time shewn by the chronometer at *apparent* noon; and the difference between this time and 24 hours will be the error of the chronometer for *apparent* time at the place. To this apply the equation of time, with its proper sign from the *Nautical Almanac* (page 1), and the result will be the error of the chronometer for *mean* time at the place of observation; from which and the longitude of the place, the error for Greenwich *mean* time may be found.

Example I.

On July 25, 1866, in lat. 54° 20′ N. and longitude 21° 10′ E., equal altitudes of the ☉ were observed as under; required the error of the chronometer for *apparent* and *mean* time at the place of observation, and also for *mean* time at Greenwich.

	Times A.M.	☉'s alts.	Times P.M.	Equat. of time + $0^h\ 6^m\ 12{\cdot}6$ +
				Correct. for long. 0 0 0
	$8^h\ 58^m\ 30^s$	31° 30′	$3^h\ 0^m\ 6^s$	
	0 58 47	31° 35′	3 0 23	Reduced E. T. 0 6 13
	0 59 4	31° 40′	3 0 40	Time ap. noon 24 0 0
Mean	8 58 47		3 0 23	Mean T. at noon 24 6 13
	12 0 0		24 0 0	Long. in time E. −1 24 40
Mean times	20 58 47		27 0 23	Mean T. at Gr. = 22 41 33
	27. 0 23			
Sum	47 59. 10	half sum $23^h\ 59^m\ 35^s$		
Dif.	6 1 36	interval between the mean observations		

The equation of equal altitudes is, by Example, Art. (322), about + 10 s., which, being added to 23 h. 59 m. 45 s., gives the apparent time of noon by the chronometer,

23 h. 59 m. 45 s. Hence the chronometer was slow 15 s. for apparent time at the place of observation.

Again, the mean time at noon was	=	$24^h\ 6^m\ 13^s$
And the time at noon by chronometer was	=	23 59 45
Therefore the chronometer was for mean time at place	=	0 6 28 slow.
Again, the mean time at Gr. at apparent noon was	=	$22^h\ 41^m\ 33^s$
And the time by chronometer was	=	23 59 45
∴ Chronometer was fast for mean Greenwich time	=	1 18 12

Example II.

On March 5, 1866, in lat. 49° 8′ S. and long. 20° 13′ W., the following times by chronometer were obtained when the sun had equal altitudes:

At the A.M. observation, March 4, 21 h. 22 m. 30 s.

At the P.M. observation, March 5, 2 h. 13 m. 15 s.

Required the error of the chronometer on apparent time at the ship, on mean time at the ship, and on mean time at Greenwich.

Ship's date, app. T., March 5	=	$0^h\ 0^m\ 0^s$	March 5, Eq. T. (N.A. p. 1)	=	$0^h\ 11^m\ 42{\cdot}68^s$
Long. in time	+	1 20 52	Cor. for 1 h. 21 m.		− 7·8
Gr. date, app. T., March 5	=	1 20 52	Corrected Eq. T.		+ 11 40·9
March 5, ⊙'s dec. (N.A. p. 1)	=	6° 0′ 59″ S.	Mar. 4, dec. (N.A. p. 1)	=	6° 24′ 6″ S.
Cor. for 1 h. 21 m.		− 1′ 18″	Mar. 6, " "	=	5° 37′ 46″ S.
True declination	=	5° 59′ 41″	Change in two days	=	46′ 20″
		90° 0′ 0″			60
Polar distance	=	84° 0′ 19″			2780″

First time by chronometer, March 4 = $21^h\ 22^m\ 30^s$

Second " " " 5 = 2 13 15

Interval between the observations = 5 50 45 ∴ ½ interval $2^h\ 55^m\ 22{\cdot}5^s$

To 5 h. 50 m. 45 s. corresponds in Table, page 293.

Log. A	=	7·7680	Log. B	=	7·6260
Log. 2780	=	3·4440	Log. 2780	=	3·4440
Log. 49° 8′ tan.	=	10·0629	Log. 84° 0′ 19″ cot.	=	9·0216
+ 18·83	=	1·2749	− 1·23	=	0·0916
− 1·23					
+ 17·6 equation of equal altitudes.					

First time by chronometer	=	4^d	21^h	22^m	30^s	
½ interval	=	0	2	55	22·5	
Middle time by chronometer, March	=	5	0	17	52·5	
Equation of equal altitudes					+ 17·6	
Middle time by chronometer at ap. noon	=	5	0	18	10	

∴ The chronometer is fast for apparent time at ship 18 m. 10 s.

Apparent noon at ship, March	=	5^d	0^h	0^m	0^s	
Equation of time			+	11	41	
Mean time at ship at apparent noon	=	5	0	11	41	(*a*)
Longitude in time, W.			+ 1	20	52	
Mean time at Greenwich, March	=	5	1	32	33	(*b*)
Mean time by chron. corrected, March	=	5	0	18	10	(*c*)
Chronometer slow on mean time at Gr.	=	0	1	14	23	(*b*) − (*c*)
Chronometer fast for mean time at ship	=	0	0	6	29	(*c*) − (*a*)

Exercises 49.

(561.) On April 3, 1866, in lat. 34° 50′ S. and long. 12° 44′ W., the following times by chronometer were obtained when the sun had equal altitudes:

At the A.M. observation, April 2, 20 h. 13 m. 50 s. A.T.

At the P.M. observation, April 3, 2 h. 5 m. 10 s. A.T.

Required the error of the chronometer on apparent time at the place, on mean time at the place, and on mean time at Greenwich.

(562.) On May 17, 1866, in lat. 50° 11′ N. and long. 20° 45′ E., the following times by chronometer were obtained when the sun had equal altitudes:

At the A.M. observation, May 16, 22 h. 5 m. 15 s. A.T.

At the P.M. observation, May 17, 3 h. 10 m. 45 s. A.T.

Required the error of the chronometer for apparent time at the place, for mean time at the place, and for mean time at Greenwich.

(563.) On May 27, 1866, in lat. 25° 17′ N. and long. 37° 20′ W., the following times by chronometer were obtained when the sun had equal altitudes:

At the A.M. observation,	At the P.M. observation,
May 26, 23 h. 40 m. 16 s. A.T.	May 27, 3 h. 53 m. 14 s. A.T.

Required the error of the chronometer on apparent time at the place, on mean time at the place, and on mean time at Greenwich.

(564.) On June 24, 1866, in lat. 57° 10′ N. and long. 42° 18′ W., the following times were obtained by chronometer when the sun had equal altitudes:

At the A.M. observation,	At the P.M. observation,
June 24, 0 h. 23 m. 12 s. A.T.	June 24, 5 h. 53 m. 32 s. A.T.

Required the error of the chronometer on apparent time at the place, on mean time at the place, and on mean time at Greenwich.

(565.) On July 30, 1866, in lat. 36° 17′ S., and long. 14° 38′ E., the following times were obtained when the sun had equal altitudes by the chronometer:

At the A.M. observation,	At the P.M. observation,
July 29, 20 h. 12 m. 18 s. A.T.	July 30, 1 h. 7 m. 16 s. A.T.

Required the error of the chronometer for apparent and mean time at the place, and for mean time at Greenwich.

(566.) On August 12, 1866, in lat. 51° 30′ N. and long. 12° 24′ W., the following times by chronometer were obtained when the sun had equal altitudes:

At the A.M. observation,	At the P.M. observation,
Aug. 11, 22 h. 17 m. 10 s. A.T.	Aug. 12, 3 h. 10 m. 16 s. A.T.

Required the error of the chronometer for apparent and for mean time at the place, and for mean time at Greenwich.

(567.) On September 6, 1866, in lat. 36° 12′ N. and long. 5° 18′ E., the following times by chronometer were obtained when the sun had equal altitudes:

At the A.M. observation,	At the P.M. observation,
Sept. 1, 21 h. 15 m. 28 s. A.T.	Sept. 2, 2 h. 29 m. 34 s. A.T.

Required the error of the chronometer for apparent and for mean time at the place, and for mean time at Greenwich.

(568.) On November 27, 1866, in lat. 43° 20′ S. and long. 80° 15′ 30″ E., the following times were obtained by chronometer when the sun had equal altitudes:

At the A.M. observation,	At the P.M. observation,
Nov. 26, 16 h. 50 m. 30 s. A.T.	Nov. 26, 23 h. 4 m. 36 s. A.T.

Required the error of the chronometer for apparent and for mean time at the place, and for mean time at Greenwich.

Problem XIX.

365. To compute the true and apparent altitude of any celestial object.

1. If the object be the sun, the time from apparent noon is the meridian distance; but if the object be the moon or a star, to the apparent time add the sun's right ascension for the given instant, and the difference between the sum and the right ascension of the object at the same instant is the meridian distance of the object.

2. Reduce the meridian distance into degrees, &c.; take the object's declination (N.A.), and the latitude of the place of observation, and proceed by the following

RULE.

366. Add together the log. cosine of the meridian distance, the log. cosine of the declination, and the log. cosine of the latitude of the place; from the sum omit 30 in the index, and take the natural number corresponding to the result, which call A_1.

Then to the log. sine of the latitude add the log. sine of the object's declination, and rejecting 20 from the index, take out the natural number corresponding to the result, which call A_2. A_1 is + or − according as the meridian distance is acute or obtuse, and A_2 is + or − according as the latitude and

declination are of the same or opposite names—that is, according as they are either both north or both south, or the one north and the other south.

Lastly, take the algebraic sum of A_1 and A_2, and the result will be the *natural sine* of the true altitude; to this, if necessary, apply the ordinary corrections of altitude with opposite signs to obtain the apparent altitude.

367. REMARK.—If the *mean time* at the place of observation be given, it must be converted into *apparent time* by applying the equation of time (N.A. p. 2), before it is used in finding the meridian distance.

368. The proof of the above rule is contained in Art. 328 (13), where the complements of the sides there given are the quantities here involved, and the angle of the triangle there given is the meridian distance here used; hence, as there given, we have

$$\text{cos. (obj. zenith dist.)} = \text{cos. colat. cos. P.D.} + \text{sin. colat. sin. P.D. cos. (mer. dist.)};$$

but cosine of the object's zenith distance = sine of its altitude, and cos. colat. and sin. colat. = sin. lat. and cos. lat.; also cos. P.D. and sin. P.D. = sin. decl. and cos. decl.; hence we have

sin. alt. = sin. lat. sin. decl. + cos. lat. cos. decl. cos. (mer. dist.), which wrought by logarithms, gives the above rule.

Example I.

Given the meridian distance of an object 3 h. 12 m. 30 s., its declination 12° 34′ S., and the latitude of the place 39° 12′ N., to find its true altitude.

Solution.

Meridian distance 3 h. 12 m. 30 s. = 48° 7′ 30″

Log. cos. 48° 7′ 30″	=	9·8244563			
Log. cos. 39° 12′	=	9·8892706	Log. sin.	=	9·8007372
Log. cos. 12° 34′	=	9·9894692	Log. sin.	=	9·3376099
$\therefore A_1 = +\cdot5048892$		1·7031961	$\therefore A_2 = -\cdot1375141$		1·1383471
$A_2 = -\cdot1375141$					

·3673751 = nat. sine of 21° 33′ 13″

369. REMARK.—Here A_2 is negative, because the declination is south and the latitude north, and therefore of opposite names; A_1 can only be negative when the time from noon is greater than *six hours*, which can only take place when the declination and latitude are both north or both south; hence when A_1 is *negative*, A_2 is always positive. If, in the example above, the declination and latitude had been of the same name, A_2 would have been positive; hence, adding A_1 and A_2, we have nat. sine of the altitude ·6424033, and therefore the true altitude = 39° 58′ 16″. If the final result were negative, it would shew that the object was below the horizon, and the angle corresponding to the negative natural sine would be the distance of the object *below* the horizon at the proposed time.

Example II.

Required the moon's true and apparent altitude on the 20th October, at 7 h. 12 m. 20 s. P.M. apparent time, at a place in lat. 56° 57′ N. and long. 13° 12′ W.

October 20,	$7^h\ 12^m\ 20^s$	apparent time at ship
	+ 52 48	long. in time
October 20,	8 5 8	apparent time at Greenwich.

⊙'s R.A. (N.A. p. i.)	☽'s R.A. (N.A. p. ix.)	☽'s Dec.	☽'s Hor. Paral.
$13^h\ 39^m\ 42{\cdot}7^s$	$23^h\ 0^m\ 16{\cdot}35^s$	4° 36′ 1″ S.	58′ 13·8″
+ 1 16·3 cor.	11·34 cor.	− 55″ cor.	+ 2·3″ cor.
13 40 59 = ⊙'s R.A.	23 0 27·69	4° 35′ 6″ S.	58′ 16″
8 5 8 app. time			60
21 46 7 R.A. of meridian		3·5435714 = log.	3496
23 0 28 ☽'s R.A.		9·9512858 = cos.	26° 38′
1 14 21 ☽'s meridian distance		3·4948572 = log.	3125″ paral. in alt.
60		∴ par. in alt. =	52′ 5″
4)74 21 0			

Mer. dist.	18 35 15	cos.	=	9·9767341		
Lat.	56 57 0 N.	cos.	=	9·7366918	sin.	9·9233450
Decl.	4 35 6 S.	cos.	=	9·9986080	sin.	8·9027531
Log.	·5152690		=	1·7120339	log. ·0670036 =	2·8260981
	− ·0670036					
	·4482654 = nat. sine of 26° 37′ 58″ true altitude					
				− 52′ 5″ cor. for parallax		
				25° 45′ 53″		
				+ 1′ 58″ cor. for refraction		
				25° 47′ 51″ = ☽'s apparent altitude		

NOTE.—In calculating the parallax in altitude in the above, the cosine of the true altitude was used instead of the cosine of the apparent altitude; if now the cosine of the apparent altitude thus obtained be substituted in the calculation, the parallax in altitude will be found to be 52′ 35″, which being 30″ more than that used, it is necessary to subtract 30″ from the result obtained; hence, 25° 47′ 51″ − 30″ = 25° 47′ 21″, the true apparent altitude.

Exercises 50.

(569.) Find the true altitude of the sun's centre, and the apparent altitude of his lower limb, in lat. 12° 35′ N. and long. 43° 7′ E., it being apparent noon at Greenwich, and the sun's declination at the instant 12° 35′ S., and semidiameter 16′ 8″.

(570.) Find the true altitude of the sun's centre, and the apparent altitude of his lower limb, at a place in latitude 50° 20′ N. and longitude 17° 9′ W.—the apparent time at Greenwich being then 3 h. 58 m. 40 s. P.M., and the sun's declination 10° 34′ N., and semidiameter 15′ 52″.

(571.) Find the true altitude of the sun's centre, and the apparent altitude of his upper limb, at a place in lat. 47° 30′ S. and long. 16° 36′ E., at 8 h. 27 m. 36 s. A.M., apparent Greenwich time—the sun's declination being 21° 17′ N., and semidiameter 15′ 46″.

(572.) Find the true altitude of the sun's centre, and the apparent altitude of his upper limb, at a place in lat. 2° 12′ S. and long. 120° 15′ E., at 3 h. 12 m. A.M. apparent Greenwich time—the sun's declination being 23° 28′ N., and his semidiameter 15′ 46″.

(573.) Find the true altitude of the sun's centre, and the apparent altitude of his lower limb, at a place in lat. 50° 12′ N. and long. 40° 25′ W., at 11 h. 13 m. 40 s. A.M. apparent Greenwich time—the sun's declination being 8° 17′ N., and semidiameter 15′ 54″.

(574.) Find the altitude of Aldebaran, right ascension 4 h. 28 m. 17 s., and declination 18° 52′ 57″ N., on the 19th November 1866, at 6 h. 34 m. 36 s. A.M. apparent time, the latitude of the place of observation being 54° 36′ N., and its longitude 8° 43′ W., and the sun's R.A. 15 h. 37 m. 48 s.

(575.) Find the true and apparent altitude of Rigel on December 30, 1866, at 5 h. 29 m. 30 s. P.M. apparent time, at a place in latitude 51° 37′ N. and longitude 17° 12′ W.—the R.A. of Rigel being then 5 h. 8 m. 8·77 s., and declination 8° 21′ 33·5″ S.

(576.) Find the true and apparent altitude of Arcturus on the 1st December 1866, at 5 h. 50 m. 20 s. A.M. apparent time, at a place in latitude 12° 15′ S. and longitude 39° 43′ 30″ E.—the right ascension and declination of Arcturus being then 14 h. 9 m. 35·47 s. and 19° 52′ 36″ N.

(577.) Find the position of the moon above or below the horizon, at a place in lat. 37° 20′ N. and long. 30° 45′ W., on September 24, 1866, at 6 h. 7 m. 35 s. apparent time at the place in the afternoon—the moon's declination being then (*Nautical Almanac*, p. x.) 1° 44′ 58″ N., and meridian distance 91° 13′ 30″ E.

(578.) The conditions being all the same as in the last question, except that the latitude is south instead of north, find how far the moon's centre is below the rational horizon.

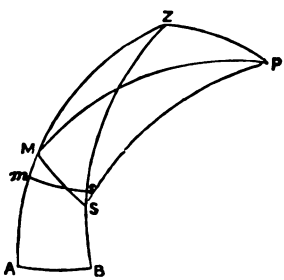
Problem XX.

To clear the lunar distance from the effects of parallax and refraction.

370. The position of the heavenly bodies as seen from the surface of the earth is different from what it would be as seen from the earth's centre, this apparent change of position being the effect of parallax and refraction—the former causing all bodies, except the fixed stars, to appear lower than they would if seen from the centre of the earth; while the latter causes all bodies to appear higher than they really are. The parallax of the moon is always greater than the refraction, and hence the moon's altitude as observed is always less than her true altitude; but in all other bodies the refraction of the atmosphere is always greater than their parallax, and

hence their observed altitude is always greater than their true altitude.

371. To investigate formulæ for clearing the lunar distance. Let P represent the pole, Z the zenith, AB a portion of the horizon, *m* and *s* the apparent places of the moon, sun, or star, and M and S their true places; then A*m* = the apparent altitude of the moon, and Z*m* its apparent zenith distance; B*s* = the apparent altitude of the sun or star, and Z*s* its apparent zenith distance; also AM = the true altitude of the moon, and ZM = her true zenith distance; BS = the true altitude of the sun or star, and ZS = its true zenith distance. We have likewise *ms* = the apparent distance, and MS = the true distance.



The ordinary corrections for dip, index error, and semidiameter being applied to the observed altitudes, give the apparent altitudes; and their complements give the apparent zenith distances; which, being corrected for parallax and refraction, give the true zenith distances; also, the observed distance being corrected for the semidiameters of the objects, gives *ms*, the apparent distance. Now, our object is, from knowing in the two spherical triangles, *m*Z*s* and MZS, the apparent and true zenith distances, and the apparent distance, to determine MS, the true distance. For this purpose let A*m* = *a*, B*s* = *a'*, AM = A, BS = A', *ms* = *d*, and MS = D; D being therefore the quantity whose value is to be determined in terms of the other five. According to this notation, it is then to be observed that

d = the apparent distance,
D = the true distance,
a and *a'* = the apparent altitudes,
A and A' = the true altitudes,

where *a*, A refer to the altitude of the moon, and *a'*, A' refer to

the altitude of the sun or star. The zenith distances of the bodies are therefore to be indicated by the complements of a, a', A, and A'.

By Spherical Trigonometry, Art. 328 (14), the triangle MZS gives

$$\text{cos. Z} = \frac{\text{cos. D} - \text{sin. A sin. A'}}{\text{cos. A cos. A'}} \tag{1}$$

and the triangle mZs gives in like manner

$$\text{cos. Z} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'} \tag{2}$$

Equating these two values of cos. Z, we have the following equation from which to determine D, namely,

$$\frac{\text{cos. D} - \text{sin. A sin. A'}}{\text{cos. A cos. A'}} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'}$$

from which we obtain the following value of cos. D,

$$\text{cos. D} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'} \text{ cos. A cos. A'} + \text{sin. A sin. A'}$$

But since $\quad \text{cos. } (a + a') = \text{cos. } a \text{ cos. } a' - \text{sin. } a \text{ sin. } a'$

we have $\quad \text{cos. } (a + a') - \text{cos. } a \text{ cos. } a' = - \text{sin. } a \text{ sin. } a'$

and therefore

$$\text{cos. D} = \frac{\text{cos. } d + \text{cos. } (a + a') - \text{cos. } a \text{ cos. } a'}{\text{cos. } a \text{ cos. } a'} \text{ cos. A cos. A'} + \text{sin. A sin. A'}$$

$$= \left\{\frac{\text{cos. } d + \text{cos. } (a + a')}{\text{cos. } a \text{ cos. } a'} - 1\right\} \text{ cos. A cos. A'} + \text{sin. A sin. A'}$$

$$= \frac{\text{cos. } d + \text{cos. } (a + a')}{\text{cos. } a \text{ cos. } a'} \text{ cos. A cos. A'} - \text{cos. A cos. A'} + \text{sin. A sin. A'}$$

$$\text{Or cos. D} = \left\{\text{cos. } d + \text{cos. } (a + a')\right\} \frac{\text{cos. A cos. A'}}{\text{cos. } a \text{ cos. } a'} - \text{cos. } (\text{A} + \text{A}') \tag{A}$$

But Art. (240, *t*) cos. d + cos. $(a + a')$ = 2 cos. $\frac{1}{2}\{(a + a') + d\}$ cos. $\frac{1}{2}\{(a + a') - d\}$

Wherefore, by substitution,

$$\text{cos. D} = \frac{2 \text{ cos. } \frac{1}{2}\{(a + a') + d\} \text{ cos. } \frac{1}{2}\{(a + a') - d\}}{\text{cos. } a \text{ cos. } a'} \text{ cos. A cos. A'} - \text{cos. } (\text{A} + \text{A}') \tag{B}$$

If now each side of the above equations, (1) and (2), be subtracted from 1, we obtain two values of $1 - \text{cos. Z}$, which being equated give

$$1 - \frac{\text{cos. D} - \text{sin. A sin. A'}}{\text{cos. A cos. A'}} = 1 - \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'}$$

$$\text{Or} \quad \frac{\text{cos. A cos. A'} + \text{sin. A sin. A'} - \text{cos. D}}{\text{cos. A cos. A'}} = \frac{\text{cos. } a \text{ cos. } a' + \text{sin. } a \text{ sin. } a' - \text{cos. } d}{\text{cos. } a \text{ cos. } a'}$$

whence $\quad \frac{\text{cos. } (\text{A} \sim \text{A}') - \text{cos. D}}{\text{cos. A cos. A'}} = \frac{\text{cos. } (a \sim a') - \text{cos. } d}{\text{cos. } a \text{ cos. } a'}$

from which $\quad \text{cos. D} = \frac{\text{cos. } d - \text{cos. } (a \sim a')}{\text{cos. } a \text{ cos. } a'} \text{ cos. A cos. A'} + \text{cos. } (\text{A} \sim \text{A}') \tag{C}$

Again, equating the second sides of (1) and (2), since their first sides are equal, we obtain

$$\frac{\text{cos. D} - \text{sin. A sin. A}'}{\text{cos. A cos. A}'} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'}$$

But
$$\frac{\text{cos. A cos. A}'}{\text{cos. A cos. A}'} = \frac{\text{cos. } a \text{ cos. } a'}{\text{cos. } a \text{ cos. } a'}$$

and adding these equals to the former, since

$$\text{cos. A cos. A}' - \text{sin. A sin. A}' = \text{cos. (A + A}') \quad \text{(Art. 237, 2)}$$

we have
$$\frac{\text{cos. D} + \text{cos. (A + A}')}{\text{cos. A cos. A}'} = \frac{\text{cos. } d + \text{cos. } (a + a')}{\text{cos. } a \text{ cos. } a'}$$

$$\therefore \text{cos. D} = \frac{\text{cos. A cos. A}'}{\text{cos. } a \text{ cos. } a'} \left\{ \text{cos. } d + \text{cos. } (a + a') \right\} - \text{cos. (A + A}').$$

But
$$\text{cos. } d + \text{cos. } (a + a') = 2 \text{ cos. } \tfrac{1}{2}\{(a + a') + d\} \text{ cos. } \tfrac{1}{2}\{(a + a') - d\}$$
$$\text{cos. (A + A}') = 2 \text{ cos.}^2 \tfrac{1}{2}\text{(A + A}') - 1$$

and
$$\text{cos. D} = 1 - 2 \text{ sin.}^2 \tfrac{1}{2}\text{D}.$$

Substituting these values in the above equation, and changing the signs, it becomes

$$\text{Sin.}^2 \tfrac{1}{2}\text{D} = \text{cos.}^2 \tfrac{1}{2}\text{(A + A}') - \frac{\text{cos. } \frac{1}{2}\{(a + a') + d\} \text{ cos. } \frac{1}{2}\{(a + a') - d\} \text{ cos. A cos. A}'}{\text{cos. } a \text{ cos. } a'}$$

$$= \text{cos.}^2 \tfrac{1}{2}\text{(A + A}') \left\{ \frac{1 - \text{cos. } \frac{1}{2}\{(a + a') + d\} \text{ cos. } \frac{1}{2}\{(a + a') - d\} \text{ cos. A cos. A}'}{\text{cos. } a \text{ cos. } a' \text{ cos.}^2 \frac{1}{2}\text{(A + A}')} \right\}$$

$$= \text{cos.}^2 \tfrac{1}{2}\text{(A + A}') \text{ cos.}^2 \theta,$$

if the latter part of the last factor be made $\text{sin.}^2 \theta$, or the whole factor $\text{cos.}^2 \theta$; and

$$\therefore \text{sin. } \tfrac{1}{2}\text{D} = \text{cos. } \tfrac{1}{2}\text{(A + A}') \text{ cos. } \theta \qquad \text{(D)}$$

which result is known as the formula of Borda.

372. The lunar distance can now be accurately cleared from the effects of parallax and refraction by either of the four formulæ above marked (A), (B), (C), and (D); the formula (D) can be calculated entirely by logarithmic sines and cosines, but the other three require the use of both natural and logarithmic sines and cosines. There are many other methods of clearing the lunar distance; but most of them require the use of tables specially calculated for the purpose, and the results are not generally so trustworthy as those obtained by the four methods demonstrated above.

373. From the formula marked (D) above we derive the following practical

RULE.

1. Write the apparent central distance, the apparent altitude of the moon, and the apparent altitude of the sun or star all under each other, and find their sum; take half this sum, and the difference between this half-sum and the apparent distance.

2. Under this difference write the true altitude of the moon, and then the true altitude of the sun or star, and find their sum; under which, but leaving two blank lines, write the half sum of the true altitudes.

3. We have thus obtained *ten* arcs, with which proceed as follows: Take the log. *secants* of the second and third arcs, omitting 10 from the index of each, and the log. *cosines* of the fifth, sixth, seventh, and eighth arcs, and placing these six logarithms all under each other, add them together, and take half the sum.

4. Under this half-sum write the log. cosine of arc tenth, which write again immediately after itself; subtract this log. cosine from the half-sum, and the remainder will be the sine of an arc, whose log. cosine write immediately after it, and under the second log. cosine of half the true altitudes.

5. Add these two logarithmic cosines together, omitting 10 from the index, and the sum will be the logarithmic sine of half the distance sought, which distance being doubled, the true distance will be obtained.

Example I.

Given the apparent altitude of the moon's centre 22° 15′, the apparent altitude of the sun's centre 21° 35′, and the apparent distance of their centres 119° 20′ 34″, and the moon's horizontal parallax 58′; to find their true central distance.

1st Solution, by the Rule.

1. To correct the moon's altitude.

Moon's horizontal parallax	=	0° 58′ 0″		sine	=	8·2271335
Moon's apparent altitude (a)	=	22° 15′ 0″		cos.	=	9·9663954
Moon's parallax in alt.	=	0° 53′ 41″		sine	=	8·1935289
Moon's refraction	=	0° 2′ 19″				
Moon's correction		+ 51′ 22″				
Moon's true altitude (A)	=	23° 6′ 22″				
Sun's apparent altitude (a')	=	21° 35′ 0″				
Sun's refraction		− 2′ 16″				
Sun's true altitude (A′)	=	21° 32′ 44″				

d	=	119° 20′ 34″			
a	=	22° 15′ 0″	sec.	0·0336046	
a'	=	21° 35′ 0″	sec.	0·0315714	
		2)163° 10′ 34″			
$\frac{1}{2}(a + a' + d)$	=	81° 35′ 17″	cos.	9·1652124	
$\frac{1}{2}\{(a + a') \sim d\}$	=	37° 45′ 17″	cos.	9·8979783	
A	=	23° 6′ 22″	cos.	9·9636838	
A′	=	21° 32′ 44″	cos.	9·9685417	
				2)39·0605922	
(A + A′)	=	44° 39′ 6″		19·5302961	
$\frac{1}{2}$(A + A′)	=	22° 19′ 33″	cos.	− 9·9661589	+ cos. 9·9661589
∴ θ	=	21° 30′ 11″	sin.	9·5641372	cos. 9·9686688
∴ $\frac{1}{2}$D	=	59° 23′ 23·6″			sin. 9·9348277
		2			
∴ True distance	=	118° 46′ 47·2″			

2d Solution, by Formula (B).

In this solution, the calculations are the same as by the rule up to the sum of the first six logarithms, to which add the logarithm of 2 = ·3010300, omit 40 from the index, and take out the natural number corresponding to the result; from this number subtract the natural cosine of (A + A′), and the remainder will be the cosine of the true distance; if the remainder be negative, take it out as a natural sine, and add 90° to the result, and the sum will be the true distance sought.

Sum of first six logs.	=	39·0605922	
Log. 2	=	·3010300	
		$\bar{1}$·3616222	= log. ·2299441
Nat. cos. (A + A′)		=	·7113926
Nat. cos. 118° 46′ 48″		=	− ·4814485
∴ True distance	=	118° 46′ 48″	

DETERMINATION OF D BY FORMULA (A).

				Logarithms.		
d	= 119° 20′ 34″	Nat. cos.	= − ·4900334			
a	= 22° 15′ 0″			sec.	=	0·0336046
a'	= 21° 35′ 0″			sec.	=	0·0315714
$(a+a')$	= 43° 50′ 0″	Nat. cos.	= + ·7213574			
		Natural number	= ·2313240		=	$\bar{1}$·3642207
A	= 23° 6′ 22″			cos.	=	9·9636838
A′	= 21° 32′ 44″			cos.	=	9·9685417
		Natural number	= ·2299440			$\bar{1}$·3616222
A + A′	= 44° 39′ 6″	Nat. cos.	= − ·7113926			
Nat. cos.	= 118° 46′ 48″		− ·4814486			

∴ D = 118° 46′ 48″

DETERMINATION OF D BY FORMULA (C).

				Logarithms.		
d	= 119° 20′ 34″	Nat. cos.	= − ·4900334			
a	= 22° 15′ 0″			sec.	=	0·0336046
a'	= 21° 35′ 0″			sec.	=	0·0315714
$(a-a')$	= 0° 40′ 0″	Nat. cos.	= ·9999323			
		Natural number	= 1·4899657		=	0·1731763
A	= 23° 6′ 22″			cos.	=	9·9636838
A′	= 21° 32′ 44″			cos.	=	9·9685417
		Natural number	= −1·4810770			·1705778
$(A-A')$	= 1° 33′ 38″	Nat. cos.	= + ·9996292			
Nat. cos.	= 118° 46′ 48″		− ·4814478			

∴ D = 118° 46′ 48″; the same as before.

Example II.

If the moon's apparent altitude be 6° 25′, that of a fixed star 37° 40′, their apparent distance 50° 27′ 36″, and the moon's horizontal parallax 55′ 14″; required the true distance.

1st Solution, by the Rule or Formula (D).

Moon's horizontal parallax	= 0° 55′ 14″	= 3314″	log.	3·5203525	
Moon's apparent altitude (a)	= 6° 25′ 0″	=	cos.	9·9972708	
Moon's parallax in alt.	+ 54′ 53″	3293″	=	3·5176233	
Moon's refraction	− 7′ 57″				
Moon's correction	+ 46′ 56″				
Moon's true alt. (A)	= 7° 11′ 56″				
Star's apparent alt. (a')	= 37° 40′ 0″				
Star's correction	− 1′ 14″				
Star's true alt. (A′)	= 37° 38′ 46″				

d	=	50° 27′ 0″				
a	=	6° 25′ 0″	sec.	=	0·0027292	
a'	=	37° 40′ 0″	sec.	=	0·1015056	
		2)94° 32′ 0″				
$\frac{1}{2}\{(a+a')+d\}$	=	47° 16′ 0″	cos.	=	9·8316056	
$\frac{1}{2}\{(a+a')-d\}$	=	3° 11′ 0″	cos.	=	9·9993293	
A	=	7° 11′ 56″	cos.	=	9·9965629	
A′	=	37° 38′ 46″	cos.	=	9·8986146	
					2)39·8303472	
(A + A′)	=	44° 50′ 42″			19·9151736	
$\frac{1}{2}$(A + A′)	=	22° 25′ 21″	cos.	−	9·9658582	+ cos. 9·9658582
		∴ (θ = 62° 51′ 14·6″)	sin.		9·9493154	cos. 9·6593110
		Log. sin. ($\frac{1}{2}$D = 24° 56′ 45·5″)				= 9·6250692
					2	
					49° 53′ 31″	
		Add 36″ omitted in d			0° 0′ 36″	
		∴ D	=		49° 54′ 7″	

2d Solution, by Formula (A).

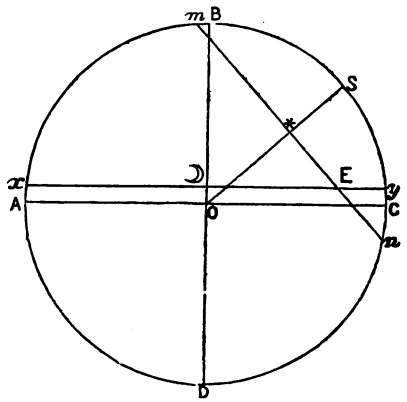
				Logarithms.		
d	= 50° 27′ 36″	Nat. cos. =	·6366167			
a	= 6° 25′ 0″			sec.	=	0·0027292
a'	= 37° 40′ 0″			sec.	=	0·1015096
$a+a'$	= 44° 5′ 0″	Nat. cos. =	·7183287			
		Natural number =	1·3549454	log.	=	0·1319217
A	= 7° 11′ 56″			cos.	=	9·9965629
A′	= 37° 38′ 46″			cos.	=	9·8986146
		Natural number =	1·3531128			0·1311340
A + A′	= 44° 50′ 42″	Nat. cos. =	− ·7090172			
		Nat. cos. (D = 49° 54′ 7″) =	·6440956			

∴ D = 49° 54′ 7″ the true distance sought.

TO CLEAR THE LUNAR DISTANCE BY CONSTRUCTION.

374. Take the chord of 60°, or sine of 90°, from the sector or any other scale of chords or sines, and with it as radius describe the circle ABCD; draw the diameter AC; and at right angles to it draw OB, which call the moon's line; make the arc BS equal to the apparent distance; join OS, and call it the sun or star's line, as the case may be. From the centre B, with a radius equal to the chord of the moon's zenith distance,

cut the circle in x and y; join xy, and at its intersection with OB mark ☽; then from the centre S, with a radius equal to the chord of the sun or star's zenith distance, cut the circle in m and n; join mn, and where it cuts the line xy mark E. Measure the distance ☽ E, and apply it to the line of chords, and mark the number of degrees thus obtained, and call the *degrees* of the result *minutes*.



Multiply this result by the moon's horizontal parallax, and divide the product by 62, if the line ☽ E be to the right of OB, but divide by 53 if the line ☽ E be to the left of OB. Lastly, if the line ☽ E be to the right of OB, *subtract* the correction from the apparent distance; but if ☽ E be to the left of OB, *add* the correction to the apparent distance, and the remainder or sum will be the true distance nearly.

The figure in the text is constructed for the conditions of the second example above; and the line ☽ E measures $37\frac{3}{4}$ nearly, which, being multiplied by the horizontal parallax, and the product divided by 62, gives $33'\ 34''$ to be subtracted from the apparent distance, leaving the true distance $49°\ 54'\ 2''$, or $5''$ less than the true distance as found by calculation.

This method by construction may be used as a convenient check on any of the methods of calculation given above; but great care must be taken to construct the figure accurately, or the results obtained will be erroneous.

Exercises 51.

(579.) Given the moon's app. alt. 63° 12′, the sun's app. alt. 60° 5′, and their app. central distance 51° 3′ 21″, also the moon's hor. par. 58′ 30″; to find the true central distance.

(580.) Given the moon's app. alt. 28° 7′, the sun's app. alt. 31° 14′, and their app. central distance 43° 12′ 16″, also the moon's hor. par. 54′ 29″; to find the true central distance.

(581.) Given the moon's app. alt. 39° 18′, the sun's app. alt. 86° 3′, and their app. central distance 46° 45′, also the moon's hor. par. 53′ 51″; to find the true central distance.

(582.) Given the moon's app. alt. 9° 26′, the sun's app. alt. 60° 33′, and their app. central distance 70° 36′ 16″, also the moon's hor. par. 59° 57′; to find their true central distance.

(583.) Given the sun's app. alt. 72° 26′, the moon's app. alt. 18° 30′, and their app. central distance 81° 2′ 28″, also the moon's hor. par. 60′ 58″; to find their true central distance.

(584.) Given the sun's app. alt. 8° 26′, the moon's app. alt. 19° 24′, and their app. central distance 120° 18′ 26″, also the moon's hor. par. 57′ 14″; to find the true central distance.

(585.) Given the star's app. alt. 29° 47′, the moon's app. alt. 57° 22′, and their app. central distance 27° 35′, also the moon's hor. par. 60′ 3″; to find their true distance.

(586.) Given the star's app. alt. 34° 28′, the moon's app. alt. 10° 42′, and their app. distance 49° 18′ 38″, also the moon's hor. par. 61′ 11″; to find their true distance.

(587.) Given the star's app. alt. 43° 27′, the moon's app.

alt. 40° 9′, and their app. distance 18° 21′ 35″, also the moon's hor. par. 60′ 20″; to find their true distance.

(588.) Given the star's app. alt. 53° 13′, the moon's app. alt. 57° 32′, and their app. distance 60° 13′ 49″, also the moon's hor. par. 60′ 52″; to find their true distance.

Problem XXI.

To find the longitude by lunar observation.

375. In order to solve this problem it is necessary to know the latitude of the place of observation, and the Greenwich mean time nearly, to enable us, from the *Nautical Almanac*, to take out the declinations and right ascensions of the objects, and also their semidiameters, the equation of time, and the moon's horizontal parallax, &c.

376. The observation can best be made by three observers, —the first to take the altitude of the sun or star, the second to take the altitude of the moon, and the third to take the distance, all three at the same instant. The observations being thus taken, derive from them by the previous rules:

1. The apparent altitudes and distance of the centres, by correcting for the index error, the dip, and semidiameters of the objects.

2. From the apparent altitudes derive the true altitudes by correcting for refraction and parallax.

3. From the apparent distance, the apparent and true altitudes, derive by last problem the true distance.

4. From the true distance thus obtained find the *mean time* at Greenwich by means of the *Nautical Almanac* (pages xiii. to xviii. of each month), where the distance is given for every third hour.

5. From the altitude of the sun or star, the latitude of the place, and the object's declination, find its meridian distance by the problem for finding the time from noon (Prob. XIII. and XIV.); if the object be the sun, this meridian distance will be

the apparent time from noon, which being corrected for the equation of time, will give the mean time at the place of observation; but if it be a star, apply it by addition or subtraction to the star's right ascension, by which will be found the right ascension of the meridian, the difference between which and the sun's right ascension will be the apparent time from noon at the place; or from the right ascension of the meridian, increased, if necessary, by 24 hours, subtract the sun's right ascension, and the remainder will be the apparent time past the previous noon.

6. The difference between the time at the place and the corresponding time at Greenwich, is the measure of the longitude in time, which can be reduced to degrees.

7. If the time at the place be faster than the time at Greenwich, the longitude is *east;* but if the time at the place be slower than the time at Greenwich, the longitude of the place is *west.*

NOTE.—In finding the mean Greenwich time as directed in (4) above, if the true distance found be not in the *Nautical Almanac,* take one that is less and the adjacent one greater, and find their difference a; take also the difference between the true distance and that corresponding to the earliest hour b, and state the following proportion—$a : b :: 3$ h. : the time to be added to the earliest hour to obtain the true Greenwich mean time of observation.

Example I.

If on the 17th October, in latitude 48° 12′ N., about 5 h. 15 m. P.M. mean Greenwich time, the observed altitude of the sun's lower limb be 11° 2′, of the moon's lower limb 16° 12′, the height of the eye being 20 feet, and the index error + 1′ 14″, at the same time that another observer found the distance of their nearest limbs to be 99° 10′ 50″; required their true central distance, and the longitude of the place of observation.

Moon's observed alt.	=	16° 12′ 0″	Sun's observed alt.	=	11° 2′ 0″
Index error		+ 1′ 14″	Index error		+ 1′ 14″
		16° 13′ 14″			11° 3′ 14″
Semidiam. augmented		+ 15′ 24″	Sun's semidiameter		+ 16′ 6″
		16° 28′ 38″			11° 19′ 20″
Dip		− 4′ 20″			− 4′ 20″
App. alt. of moon's centre	=	16° 24′ 18″ (*a*)	App. alt. of sun's centre	=	11° 15′ 0″ (*a′*)
Moon's correction		+ 50′ 43″	Refraction and par.		− 4′ 39″
Moon's true alt. A		17° 15′ 1″	⊙'s true alt. A′	=	11° 10′ 21″

To find the true Distance, and mean Greenwich time.

		Observed distance	=	99° 10′ 50″	
		☽'s aug. semidiameter	=	0° 15′ 24″	
		⊙'s semidiameter	=	0° 16′ 6″	
		App. distance (*d*)	=	99° 42′ 20″	
d	=	99° 42′ 20″			
a	=	16° 24′ 18″	sec.	0·0180504	
a′	=	11° 15′ 0″	sec.	0·0084261	
		2)127° 21′ 38″			
½(*a* + *a′* + *d*)	=	63° 40′ 49″		9·6467759	
½{(*a* + *a′*) ~ *d*}	=	36° 1′ 31″		9·9078184	
A	=	17° 15′ 1″		9·8800118	
A′	=	11° 10′ 21″		9·9916904	
				2)39·5527730	
(A + A′)	=	28° 25′ 22″		19·7763865	
½(A + A′)	=	14° 12′ 41″	− cos.	9·9865015	+ cos. 9·9865015
∴ *θ*	=	38° 3′ 22″	sin.	9·7898850	cos. 9·8961995
∴ ½D	=	49° 45′ 25″			sin. 9·8827010
		2			
Hence D	=	99° 30′ 50″ = the true distance.			

		III. h.		VI. h.	
But (N.A. p. xv.) D	=	98° 20′ 20″	and	99° 46′ 38″	
True distance	=	99° 30′ 50″		98° 20′ 20″	
Change in time sought	=	1° 10′ 30″		1° 26′ 18″	change in 3 h.

Prop. log.	1° 10′ 30″	=	·40708
Prop. log.	1° 26′ 18″	=	·31926
Prop. log. of	2^h 27^m 3^s	=	·08782
First given time	3 0 0		
Mean time of obser.	5 27 3 = mean Greenwich time.		

To find the Apparent and Mean Time at the Ship.

Sun's true alt. (A′)	= 11° 10′ 21″			
Latitude	= 48° 12′ 0″	sec.	=	0·1761787
P.D. (N.A.)	= 99° 21′ 28″	cosec.	=	0·0058183
	2)158° 43′ 49″			
	79° 21′ 54″	cos.	=	9·2661182
	68° 11′ 33″	sin.	=	9·9677525
				2)19·4158677
	30° 41′ 32″	sin.	=	9·7079338
	8			
App. time at ship	= 4ʰ 5ᵐ 32ˢ·3			
Equat. of time (N.A. p. i.)	− 14 36			
Mean time at ship	= 3 50 56			
Mean Gr. time of obser.	= 5 27 21			
Longitude in time	= 1 36 25 = 24° 6′ 15″ W.			

Example II.

On October 15, 1866, in lat. 24° 35′ S., when a chronometer, which was known to shew Greenwich mean time nearly, indicated 18 h. 16 m. 20 s., the moon's apparent altitude was 51° 39′, the apparent altitude of Fomalhaut, east of the meridian, was 73° 28′, the apparent distance of the moon's centre and Fomalhaut was 53° 2′ 39″; required the true distance, the error of the chronometer for mean Greenwich time, and the longitude of the place of observation; having given the moon's horizontal parallax 54′ 51″, the R.A. of Fomalhaut 22 h. 50 m. 17 s., its declination 30° 20′ 9″ S., and the sun's R.A. 13 h. 23 m. 51 s.

NOTE.—All the data given at the end of the question are taken from the *Nautical Almanac* for 1866, and should be taken out by the student for himself; but are here given to make the question intelligible to those who may not have that almanac at hand.

Solution.

To find the true altitudes of the moon and star.

Moon's horizontal parallax	= 0° 54′ 51″	= 3291″	log.	3·5173279
Moon's apparent alt. (*a*)	= 51° 39′ 0″		cos.	9·7927163
Moon's parallax in alt.	= 0° 34′ 2″	= 2042″	log.	3·3100342
Moon's refraction	− 44″			
Moon's correction	+ 33′ 18″			
Moon's true alt. (A)	= 52° 12′ 18″			
Star's apparent alt. (*a′*)	= 73° 28′ 0″			
Refraction	− 15″			
Star's true alt. (A′)	= 73° 27′ 45″			

To find the True Distance.

d	=	53° 2′ 39″			
a	=	51° 39′ 0″	sec.	0·2072837	
a'	=	73° 28′ 0″	sec.	0·5458061	
		2)178° 9′ 39″			
$\frac{1}{2}(a + a' + d)$	=	89° 4′ 49½″	cos.	8·2054498	
$\frac{1}{2}\{(a + a') \sim d\}$	=	36° 2′ 10½″	cos.	9·9077577	
A	=	52° 12′ 18″	cos.	9·7873458	
A′	=	73° 27′ 45″	cos.	9·4543002	
				2)38·1079433	
A + A′	=	125° 40′ 3″		19·0539716	
½(A + A′)	=	62° 50′ 1½″	− cos.	9·6595111	+ cos. 9·6595111
∴ θ	=	14° 21′ 34″	sin.	9·3944605	cos. 9·9862157
∴ ½D	=	26° 15′ 5″			sin. 9·6457268
		2			
D	=	52° 30′ 10″	= the true distance sought.		
But		52° 37′ 50″	= their distance at 18 h.	=	52° 37′ 50″
Difference (δ)		0° 7′ 40″	and their dist. at 21 h.	=	51° 19′ 44″
			Difference for 3 h. (α)	=	1° 18′ 6″

Hence $\alpha : \delta$ :: 3 h. : the time past 18 h.

Or prop. log. (δ − 0° 7′ 40″)	=	1·37067
− prop. log. (α − 1° 18′ 6″)	=	·36262
Prop. log. 0ʰ 17ᵐ 40ˢ	=	1·00805
Add 18 0 0		
∴ 18 17 40	=	the mean Gr. time of observation.
But 18 16 20	=	the time shewn by chronometer.
Therefore 0 1 20	=	the error of the chronometer for mean Greenwich time—that is, the chronometer is slow for mean Greenwich time 1 m. 20 s.

Lastly, to find the meridian distance of Fomalhaut, and from it the R.A. of the meridian, and thence the longitude of the place of observation. Apply Problem XIII.

Altitude A′	=	73° 27′ 45″			
Latitude	=	24° 35′ 0″	sec.	=	0·0412655
Polar distance	=	59° 39′ 51″	cosec.	=	0·0639490
		2)157° 42′ 36″			
		78° 51′ 18″	cos.	=	9·2862153
		5° 23′ 33″	sin.	=	8·9730262
					2)18·3644560
		8° 45′ 2″	sin.	=	9·1822280
		8			

Therefore 1ʰ 10ᵐ 0ˢ = the meridian distance of the star.

But	22	50	17	= Fomalhaut's R.A.
∴	21	40	17	= R.A. of the meridian,

since the star is to the east of the meridian of the place.

But	13^h	23^m	51^s	= the sun's R.A.
∴	8	16	26	= the apparent time at the place.
	− 14		18	= equation of time.
	8	2	8	= mean time at the place.
But	18	17	45	= mean time at Greenwich.
Hence	10	15	37	= longitude in time.
	60			
	4)615	37	0	
	153°	54′	15″	= the longitude of the place,

which is west, since the time at Greenwich is faster than the time at the place.

Exercises 52.

(589.) On the 13th October 1866, in lat. 40° 33′ N., the apparent distance of the centres of the sun and moon was 58° 2′ 30″, the sun's apparent altitude was 18° 12′, the moon's apparent altitude was 30° 50′, when a chronometer shewed 11 h. 52 m. mean Greenwich time; required the true distance of the sun and moon, the error of the chronometer, and the longitude of the place of observation—there being given the sun's declination 7° 58′ 44″ S., the moon's declination 17° 51′ 50″ S., and her horizontal parallax 54′ 18″.

(590.) If, on October 30, 1866, in lat. 43° 4′ S., the apparent distance of the centres of the sun and moon be 87° 52′ 12″, the apparent altitude of the sun's centre 13° 5′, and that of the moon's 32° 14′, when a chronometer shewed 7 h. 7 m. 3 s. mean Greenwich time; find the true distance, the error of the chronometer, and the longitude of the place of observation—having given that the distance on the 30th October, at 6 h. was 88° 18′ 28″, and at 9 h. was 86° 45′ 1″, that the sun's declination at the instant of observation was 13° 34′ 19″ S., and the moon's 13° 53′ 24″ N., and also the moon's horizontal parallax 57′ 32″.

(591.) If, on September 25, 1866, at a place in lat. 36° 18′ S., the apparent distance of Aldebaran from the moon's centre be

41° 12′ 54″, the apparent altitude of the moon's centre being 29° 54′, and that of Aldebaran 37° 28′, when a chronometer shewed 20 h. 47 m. 12 s. mean Greenwich time; required the true distance, the error of the chronometer, and the longitude of the place of observation—having given the distance of the moon from Aldebaran at 18 hours 42° 34′ 20″, and at 21 hours 40° 44′ 14″; the moon's R.A. = 1 h. 43 m. 41 s., and dec. 8° 26′ 6″ N.; Aldebaran's R.A. = 4 h. 28 m. 47 s., and dec. 16° 14′ 19″ N.; the R.A. of the mean sun being 12 h. 19 m. 43 s., and also the moon's horizontal parallax 60′ 13″.

(592.) On the 17th September 1866, at a place in lat. 37° 12′ S., the apparent distance of the centres of the sun and moon was 94° 14′ 50″, the apparent altitude of the sun's centre being 10° 40′, and of the moon's 33° 24′, when a chronometer shewed 0 h. 25 m. 12 s.; required the true distance, the error of the chronometer for mean Greenwich time, and the longitude of the place of observation—there being given the true distance of the centres of the sun and moon on the 17th September at noon 93° 54′ 29″, and at 3 h. 95° 17′ 15″, the moon's horizontal parallax 54′ 37″, and the sun's declination 2° 14′ 33″ N., and equation of time 5 m. 32 s. to be subtracted from the apparent time.

(593.) On the 14th September 1866, at a place in lat. 51° 2′ N., the apparent distance of the centres of the sun and moon was 68° 50′ 20″, the sun's apparent altitude being 11° 40′, and that of the moon 32° 19′, the sun being west of the moon, when a chronometer shewed 16 h. 52 m. 32 s. mean Greenwich time; required the true distance of the sun and moon, the error of the chronometer for mean Greenwich time, and the longitude of the place of observation—there being given the moon's horizontal parallax 54′ 11″, the true distance of the centres of the sun and moon at 15 h. 68° 1′ 27″, and at 18 h. 69° 22′ 49″, the sun's declination at the instant of observation 3° 8′ 13″ N., and the equation of time 4 m. 44 s. to be subtracted from the apparent time.

(594.) On the 25th November 1866, at a place in lat. 41° 3′ S., the apparent distance of the centres of the sun and moon was 101° 18′ 30″, the sun's apparent alt. being 14° 28′, and the moon's 23° 5′, the sun being east of the moon, when a chronometer shewed 17 h. 18 m. 50 s. mean Greenwich time; required the true distance of the sun and moon, the error of the chronometer, and the longitude of the place of observation—there being given the moon's horizontal parallax 57′ 42″, the true distance of the centres of the sun and moon at 15 h. being 102° 13′ 21″, and at 18 h. 100° 40′ 36″; the sun's declination at the instant of observation being 21° 16′ 16″ S., and the equation of time 11 m. 58 s. to be subtracted from the apparent time.

(595.) On the 30th November 1866, at a place in lat. 53° 1′ S., the apparent distance of the centres of the sun and moon was 71° 54′ 12″, the sun's apparent altitude being 28° 17′, and that of the moon 35° 2′, the sun being east of the moon, when a chronometer shewed 4 h. 46 m. 12 s. mean Greenwich time; required the true distance of the sun and moon, the error of the chronometer, and the longitude of the place of observation—there being given the moon's horizontal parallax 55′ 42″, the distance of the centres of the sun and moon at 3 h. 72° 28′ 39″, and at 6 h. 71° 2′ 49″; the sun's declination at the instant of observation being 21° 42′ 11″ S., and the equation of time 11 m. 4 s. to be subtracted from the apparent time.

(596.) On the 27th October 1866, at a place in lat. 35° 10′ S., the apparent distance of the moon from Regulus was 60° 12′ 47″, the apparent altitude of the moon's centre being 36° 50′, and that of Regulus 14° 50′ east of the moon, when a chronometer shewed 8 h. 15 m. 48 s. mean Greenwich time; find the true distance, the error of the chronometer, and the longitude of the place—there being given the distance of the moon from Regulus at 6 h. 61° 40′ 32″, and at 9 h. 59° 51′ 19″; the moon's R.A. 5 h. 50 m. 59 s., and declination 18° 14′ 44″ N.; the R.A. of Regulus 10 h. 1 m. 16 s., and declination 12° 37′ N.; the R.A.

of the mean sun 14 h. 23 m. 47 s., and the moon's horizontal parallax 60′ 6″.

(597.) On September 2, 1866, at a place in lat. 48° 5′ N., the apparent distance of the centres of the sun and moon was 76° 14′ 16″, the apparent altitude of the moon being 58° 35′, and that of the sun 6° 40′ east of the moon, when a chronometer shewed 12 h. 58 m. 4 s.; required the true distance, the error of the chronometer, and the longitude of the place of observation—there being given that the true distance of the centres of the sun and moon at 12 h., was 76° 57′ 3″, and at 15 h. 75° 19′ 2″; the sun's declination at the instant of observation being 7° 43′ 20″ N., and the moon's horizontal parallax 59′.

(598.) On September 5, 1866, at a place in lat. 43° S., the apparent distance of the centres of the sun and moon was 40° 12′ 19″, the sun being east of the moon, the moon's altitude being 28° 50′, and that of the sun 16° 5′, when a chronometer shewed 8 h. 7 m. 10 s. mean Greenwich time; required the true distance, the error of the chronometer, and the longitude of the place—having given the true distance of the centres at 6 h. 41° 29′ 4″, and at 9 h. 39° 53′ 59″; the sun's declination at the instant of observation = 6° 40′ 39″ N., the moon's horizontal parallax 58′ 3″, and the equation of time 1 m. 31 s. to be subtracted from the apparent time.

(599.) On November 15, 1866, at a place in latitude 52° 5′ S., the apparent distance of the centres of the sun and moon was 96° 57′ 12″, the sun being west of the moon, when a chronometer shewed 15 h. 30 m. 44 s. mean Greenwich time, the sun's apparent altitude being then 21° 8′, and the moon's 41° 50′; required the true distance, the error of the chronometer, and the longitude of the place of observation—there being given the true distance of their centres at 15 h. 96° 19′ 56″, and at 18 h. 97° 49′ 23″; the sun's declination at the instant of observation being 18° 40′ 47″, the moon's horizontal parallax 56′ 44″, and the equation of time 15 m. 7 s. to be subtracted from the apparent time.

(600.) In order to shew the importance of attending to the height of the thermometer and barometer when they are different from the mean heights for which the Table on page 307 is calculated—namely, 50° and 29·6 inches—resume the solution of Question 592, taking the data the same as there given, with the addition that the thermometer stood at 68°, and the barometer at 28·56 inches. From the difference in the two answers, which is six miles of longitude, it will appear that in order to obtain correct results, the temperature and density of the atmosphere should always be attended to, at least when they are far from the mean.

ANSWERS.

1.

(1.) ·375; ·625; ·4375; ·$\dot{7}\dot{3}$; ·$\dot{2}\dot{1}4285\dot{7}$; and ·34375.

(2.) ·$\dot{5}38461\dot{}$; ·6470588+; ·6315789+; ·$1\dot{8}5\dot{1}8\dot{5}$; ·$\dot{7}16\dot{7}235$+; and ·3183099−.

(3.) 7·75; 4·$\dot{3}\dot{7}\dot{5}$; 19·3125; 186·6; 493·34375; and 714·$\dot{7}14\dot{2}8\dot{5}$.

(4.) £5·875; £11·63125; £118·389$5\dot{8}\dot{3}$; and £218·79375.

(5.) 12°·1875; 17°·$6\dot{7}9\dot{1}6$; 108°·26875; and 85°·$3\dot{2}\dot{5}$.

(6.) 11·5925 days; 28·$7\dot{1}649\dot{3}0\dot{5}$ days; 15·$0\dot{1}24\dot{3}0\dot{5}$; and 113·$2\dot{2}170\dot{1}3\dot{8}$.

(7.) £3, 16*s.* $11\frac{1}{10}d.$; 19*s.* $0\frac{1}{2}d.6544$; 1*s.* 6*d.*; and £11, 12*s.* $7\frac{3}{4}d.2$.

(8.) 7° 51′ 15″·3; 123° 52′ 21″; 64° 22′ 57″; and 57° 17′ 44″·8062.

(9.) 3 weeks 5 days 23 h. 34 m. $37\frac{2}{10}$ s.; 7 days 19 h. 50 m. $31\frac{1}{5}$ s.; 3 h. 6 m. 54 s.; and 11 h. 46 m. 57 s.

(10.) 1618·2585; 4087·3765625; 3·632475; 1·51647830625.

(11.) 9·870−; 3·365+; 89·795; 2·467+.

(12.) 2·39; ·25; 1·13; ·11.

(13.) ·8810891; 908·8376804; ·1108754; ·01745329.

(14.) ·013244; ·699264; 25·13; ·318309.

(15.) 3·71; 2719·89; ·42; 2·21.

(16.) ·0375; ·069444̇; ·373913; ·234375; ·226666̇; ·328.

(17.) 57°·2957795 = 206264″·8, or 206265″ nearly = (18.)

(19.) 3·14159292.

(20.) 1° Fahrenheit = ·5̇ Centigrade; 1° C = 1·8 F.

(21.) ·99726956o day.

(22.) 1 N.M. = 1·15202 I.M.; 1 I.M. = ·86804 N.M.; 1·151515; and ·868421.

(23.) 1 kilomètre = ·621382 miles; 1 mile = 1·609315 kilomètres.

(24.) ·122222̇. (25.) 16·579525.

(26.) 3·10324 days; this is the reason why three leap-years are omitted in four centuries.

(27.) ·18988; ·25; and £4, 4*s.* 4½*d.*

(28.) 80·655 degrees, or 80° 39′ 30″; 89·6166̇ grades.

(29.) 20° C.; 108°·05 F.

2.

(30.) 17; 20. (31.) 32; 330.

(32.) 100; 28. (33.) 119; 800.

(34.) −19; 0. (35.) −2280; 401.

(36.) 112; 7½. (37.) 7889; 29.

(38.) 5546⅝; 19195. (39.) 26.

(40.) 30. (41.) 9.

3.

(42.) (1) ⅘; (2) ⅔; (3) ⅚. (43.) ⅜; ¼; and $\frac{1}{60}$.

(44.) 12; 16 : 12 = 8 : 6; 12 : 6 = 16 : 8.

(45.) $\frac{50}{70}=\frac{5}{7}$; $\frac{16}{20}=\frac{4}{5}$; $\frac{11}{15}$. (46.) 20; 45.

(47.) 9; 90.

(49.) Let the three given terms be represented by a, b, and c, and the unknown term by x; then x may be either $\frac{bc}{a}$, $\frac{ac}{b}$, or $\frac{ab}{c}$.

(50.) 48, 27, and $5\frac{1}{3}$. (51.) 54; and 66.

(52.) $\frac{37}{60}$; $\frac{11}{9}$; $\frac{49}{124}$. (53.) 12; $2\frac{2}{7}$.

(54.) $\frac{bc-ad}{d-c}$; $\frac{be-af}{e-f}$. (55.) 20; ·05.

(56.) $2\frac{3}{16}$; ·051. (57.) $\frac{bm-an}{m+n}$.

4.

(58.) 146·0348. (59.) 14041·28.

(60.) 20551·41. (61.) ·04971256.

(62.) 23·11325. (63.) 12811·38.

(64.) ·002500753. (65.) 1·328502.

(66.) 103·6748. (67.) 1186·798.

(68.) ·09583. (69.) 1021·663.

5.

(70.) ·0709275. (71.) 87·19792.

(72.) 1345·982. (73.) 23·89525.

(74.) 4·45051. (75.) ·6803791.

(76.) ·1073335. (77.) 1·149831.

(78.) 71·4982. (79.) 6·815951.

(80.) 13·92615. (81.) ·0520757.

(82.) 109·6415. (83.) 1·158575.

(84.) 1·333713. (85.) 13·50614.

(86.) ·2168867. (87.) 26·12764.

(88.) 32·36034.

6.

(89.) 56·67833; 9433·264; ·00016384; 1701·562.

(90.) 143713·1; 815·4349; 276·2124; ·00060872.

(91.) 97·41002; ·07516201; 7995214.

7.

(92.) 1·772456; 1·648721; 22·98369.

(93.) ·5604996; ·5213714; ·2298369.

(94.) 2·252261; 4·634259; 20·44621.

(95.) 10·59038; 1·331336; ·6070394.

(96.) 1·414213; 1·73205; 2·236068; 2·449489; 2·645751; 2·828427; 3·162277.

(97.) ·6899832; 2·395349; 19·17029.

(98.) ·01452887; ·00329329; 260120900000.

(99.) ·4887874; ·09993715; 54·67499.

(100.) ·4817537; 1·298318; 2·639197.

8.

(101.) $\angle B = 57° 49'$, perp. = 213·052, and base = 338·539.

(102.) $\angle A = 46° 37'$, perp. = 186·0543, and base = 175·8403.

(103.) $\angle A = 36° 52' 11''·6$, $\angle B = 53° 7' 48''·4$, and BC = 333.

(104.) $\angle A = 40° 8' 46''$, $\angle B = 49° 51' 14''$, and AC = 221·9546.

(105.) $\angle B = 38° 30'$, BC = 792·0185, and AB = 1012·024.

(106.) $\angle A = 41° 3'$, BC = 295·208, and AB = 449·5206.

(107.) $\angle A = 45° 28' 50''·5$, $\angle B = 44° 31' 9''·5$, and BC = 434·2257.

(108.) $\angle A = 54° 34' 39''$, $\angle B = 35° 25' 21''$, and $AB = 426{\cdot}9832$.

(109.) $\angle A = 44° 23'$, $AC = 418{\cdot}9219$, and $AB = 586{\cdot}1703$.

(110.) $\angle B = 2° 49'$, $AC = 47{\cdot}3793$, and $AB = 964{\cdot}1648$.

(111.) $\angle A = 18° 33' 38''{\cdot}7$, $\angle B = 71° 26' 21''{\cdot}3$, and $AC = 336{\cdot}5353$.

(112.) $\angle A = 17° 39' 24''{\cdot}4$, $\angle B = 72° 20' 35''{\cdot}6$, and $AB = 372{\cdot}5506$.

9.

(113.) $\angle C = 52° 58' 4''$, $\angle A = 66° 18' 56''$, and $BC = 1053{\cdot}092$.

(114.) $\angle B = 48° 51' 5''$, $\angle C = 59° 31' 55''$, and $AB = 181{\cdot}6525$.

(115.) $\angle B = 93° 24'$, $AB = 936{\cdot}3958$, and $BC = 774{\cdot}3735$.

(116.) $\angle C = 62° 33'$, $AB = 1119{\cdot}005$, and $BC = 1048{\cdot}462$.

(117.) $\angle C = 45° 7' 6''$, $B = 74° 52' 54''$, and $AC = 599{\cdot}7277$.

(118.) $\angle B = 53° 59' 21''$, $\angle A = 45° 15' 39''$, and $BC = 426{\cdot}7683$.

(119.) $\angle A = 30° 45' 46''$, $\angle B = 89° 56' 14''$, and $AC = 858{\cdot}2866$.

(120.) $\angle B = \begin{cases} 60° 40' 32'' \\ 119° 19' 28'' \end{cases}$ $\angle C = \begin{cases} 78° 49' 28'' \\ 20° 10' 32'' \end{cases}$ $AB = \begin{cases} 551{\cdot}3587 \\ 193{\cdot}888 \end{cases}$.

(121.) $\angle C = 26° 56' 55''$, $\angle A = 90° 3' 5''$, the side $AB = 213{\cdot}624$, and $BC = 471{\cdot}376$, whose sum is 685.

(122.) AB being the greatest side, $\angle C = 54° 50' 19''$, $\angle A = 48° 39' 41''$, $AB = 490{\cdot}1658$, and $BC = 450{\cdot}1658$.

10.

(123.) 20° 33′, or 1233 miles. (124.) 7° 39′, or 459 miles.

(125.) 43° 9′, or 2589 miles. (126.) 24° 26′, or 1466 miles.

(127.) 18° 41′, or 1121 miles. (128.) 33° 27′, or 2007 miles.

(129.) 19° 31′, or 1171 miles. (130.) 49° 25′, or 2965 miles.

11.

(131.) 6° 46′, or 406 miles. (132.) 12° 0′, or 720 miles.
(133.) 99° 18′, or 5958 miles. (134.) 98° 35′, or 5915 miles.
(135.) 114° 19′, or 6859 miles. (136.) 154° 45′, or 9285 miles.
(137.) 154° 26′, or 9266 miles. (138.) 180° 0′, or 10800 miles.

12.

(139.) Lat. in 43° 58′ N., long. in 14° 22′ W.
(140.) Lat. in 37° 32′ S., long. in 51° 57′ E.
(141.) Lat. in 24° 48′ S., long. in 80° 47′ E.
(142.) Lat. in 61° 42′ N., long. in 17° 45′ W.
(143.) Lat. in 32° 14′ N., long. in 23° 20′ W.
(144.) Lat. in 16° 0′ S., long. in 29° 31′ E.
(145.) Lat. in 0° 0′, long. in 62° 15′ W.
(146.) Lat. in 38° 20′ S., long. in 1° 0′ W.
(147.) Lat. in 43° 29′ N., long in 12° 2′ W.
(148.) Lat. in 51° 50′ S., long. in 176° 15′ W.

13.

(149.) 88·2 miles. (150.) 161·4 miles.
(151.) 231·7 miles. (152.) 300·3 miles.
(153.) 239·7 miles. (154.) 272·6 miles.
(155.) 289·1 miles. (156.) 203·4 miles.
(157.) 332·1 miles. (158.) 122·54 miles.
(159.) 195·3 miles.

14.

(160.) S. $7\frac{3}{4}$ W., or W. $\frac{1}{4}$ S. (161.) S. $5\frac{3}{4}$ E., or SE. by E. $\frac{3}{4}$ E.
(162.) N. 6 W., or WNW. (163.) E. $\frac{3}{4}$ S., or E. by S. $\frac{1}{4}$ E.
(164.) N. $\frac{1}{4}$ E. (165.) S. $\frac{3}{4}$ E.

(166.) SW. (167.) SW. $\frac{1}{2}$ S.
(168.) SSW. $\frac{3}{4}$ W. (169.) N. 51° W.

15.

(170.) S. 6 E., or ESE. (171.) S. $3\frac{1}{4}$ W., or SW. $\frac{3}{4}$ S.
(172.) S. $1\frac{3}{4}$ W., or S. by W. $\frac{3}{4}$ W. (173.) N. $1\frac{1}{4}$ E., or N. by E. $\frac{1}{4}$ E.
(174.) N. $6\frac{1}{2}$ W., or WNW. $\frac{1}{2}$ W. (175.) S. $3\frac{1}{4}$ E., or SE. $\frac{3}{4}$ S.
(176.) NW. by N. (177.) S. by E. $\frac{1}{2}$ E.
(178.) S. 57° E. (179.) N. 5° W.

16.

(180.) S. by W. $\frac{1}{4}$ W. = S. $1\frac{1}{4}$ W. (181.) SE. by E. $\frac{1}{2}$ E. = S. $5\frac{1}{2}$ E.
(182.) W. $\frac{1}{2}$ N. = N. $7\frac{1}{2}$ W. (183.) SE. by E. = S. 5 E.
(184.) NW. $\frac{1}{4}$ W. = N. $4\frac{1}{4}$ W. (185.) SE. = S. 4 pts. E.
(186.) ESE. $\frac{3}{4}$ E. = S. $6\frac{3}{4}$ pts. E. (187.) SE. = S. 4 pts. E.
(188.) S. (189.) NW. $\frac{1}{2}$ N. = N. $3\frac{1}{2}$ pts. W.
(190.) SSW. $\frac{1}{4}$ S. = S. $1\frac{3}{4}$ pts. W. (191.) SE. by E. = S. 5 pts. E.
(192.) N. 40° E. (193.) S. 49° W.
(194.) S. 79° W. (195.) N. 78° W.

17.

(196.) Dif. lat. 234·7, dept. 97·2, lat. in 46° 13′ N.
(197.) Dif. lat. 241·1, dept. 161·1, lat. in 38° 18′ S.
(198.) Dif. lat. 325·1, dept. 194·8, lat. in 23° 53′ N.
(199.) Dif. lat. 257, dept. 92, lat. in 2° 42′ N.
(200.) Dif. lat. 209·1, dept. 169·3, lat. in 43° 20′ N.
(201.) Dif. lat. 79·5, dept. 222·2, lat. in 54° 37′ S.
(202.) Dif. lat. 371·5, dept. 346·5, lat. in 6° 5′ N.
(203.) Dif. lat. 474·3, dept. 145, lat. in 5° 37′ S.

18.

(204.) Dif. lat. 176, dist. 277·4, dept. 214·5.
(205.) Dist. 520, dept. 497·6, lat. in 36° 49′ S.
(206.) Dist. 235·8, dept. 79·4, lat. in 24° 35′ N.
(207.) Dist. 241·3, dept. 124, lat. in 1° 9′ S.
(208.) Dist. 357·6, dept. 287·2, lat. in 50° 51′ N.
(209.) Dist. 305·3, dept. 193·6, lat. in 42° 48′ N.
(210.) Dist. 320·4, dept. 266·4, lat. in 56° 15′ N.
(211.) Dist. 178, dept. 83·9, lat. in 41° 48′ S.

19.

(212.) Dist. 273·9, dif. lat. 104·7, lat. in 57° 43′ N.
(213.) Dist. 496·7, dif. lat. 276, lat. in 30° 1′ S.
(214.) Dist. 246, dif. lat. 116, lat. in 19° 11′ N.
(215.) Dist. 798·4, dif. lat. 774·5, lat. in 8° 39′ N.
(216.) Dist. 329·5, dif. lat. 310·3, lat. in 54° 24′ N.
(217.) Dist. 227·8, dif. lat. 66·1, lat. in 50° 18′ N.
(218.) Dist. 237, dif. lat. 90·7, lat. in 39° 10′ S.
(219.) Dist. 497, dif. lat. 487·5, lat. in 6° 1′ N.

20.

(220.) Course S. 43° W., dept. 191 miles.
(221.) Course S. 38° E., dept. 252·4 miles.
(222.) Course S. 29° 27′ W., dept. 175 miles.
(223.) Course S. 53° 24′ E., dept. 175 miles.
(224.) Course N. 52° W., dept. 231·7 miles.
(225.) Lat. in 5° 25′ N., course N. 48° W., dept. 431 miles.
(226.) Lat. in 48° 27′ N., course N. 48° E., dept. 211·1 miles.
(227.) Lat. in 30° 17′ N., course S. 33° W., dept. 243 miles.

21.

(228.) Course S. 32° W., dif. lat. 241·7, lat. in 43° 49′ N.

(229.) Course N. 55° W., dif. lat. 298·2, lat. in 24° 20′ S.

(230.) Course S. 58° E., dif. lat. 188·6, lat. in 28° 38′ S.

(231.) Course N. 48° W., dif. lat. 286·4, lat. in 44° 5′ N.

(232.) Course S. 58° W., dif. lat. 143·1, lat. in 56° 15′ N.

(233.) Course ESE. $\frac{1}{4}$ E., dif. lat. 195·4, lat. in 16° 26′ S.

(234.) Course ENE. $\frac{1}{2}$ E., dif. lat. 121·4, lat. in 39° 58′ N.

(235.) Course S. 20° 23′ W., dif. lat. 587, lat. in 23° 57′ S.

22.

(236.) Course SSE. $\frac{1}{2}$ E., dist. 297, lat. in 49° 57′ N.

(237.) Course S. 51° W., dist. 278, lat. in 31° 8′ S.

(238.) Course N. 58° E., dist. 283, lat. in 13° 49′ N.

(239.) Course N. 22° E., dist. 275, lat. in 46° 1′ S.

(240.) Course SW. $\frac{1}{2}$ S., dist. 216, lat. in 22° 6′ N.

(241.) Course S. 35° E., dist. 354, lat. in 55° 21′ N.

(242.) Course N. 38° W., dist. 453, lat. in 47° 15′ N.

(243.) Course S. 46° E., dist. 285, lat. in 34° 35′ S.

23.

(244.) Course SSE. $\frac{1}{4}$ E., dif. lat. 244·1, dept. 115·4, lat. in 33° 14′ N.

(245.) Course SSE. $\frac{3}{4}$ E., dif. lat. 187, dept. 112·1, lat. in 55° 7′ N.

(246.) Course E. $\frac{3}{4}$ S., dif. lat. 49·3, dept. 332·4, lat. in 28° 45′ S.

(247.) Course WSW. $\frac{1}{2}$ W., dif. lat. 84·8, dept. 279·4, lat. in 44° 52′ N.

(248.) Course W. by S., dif. lat. 44, dept. 221·2, dist. 225·5.

(249.) Course SW. by W. $\frac{3}{4}$ W., dist. 283, time 47 hours 10 minutes.

(250.) Course S. 49° W. nearly, dist. 300, dept. 274·1 miles.

(251.) Course N. 26° W., dist. 296, dept. 129·8 miles.

(252.) Course SSW., dept. 105·6, lat. in 25° 14′ N.

(253.) Course S. 65° W., dept. 258·3, lat. in 56° 12′ N.

(254.) Course W. by S., dept. 412, lat. in 35° 16′ S.

(255.) Course SSW. $\frac{3}{4}$ W., dept. 173·8, variation $1\frac{1}{4}$ W.

(256.) True course SW. $\frac{1}{2}$ S., app. course SW. $\frac{3}{4}$ W., dept. 151 miles.

(257.) True course N. 24° W., app. course N., dist. 162, dept. 65·9.

(258.) True course NE. $\frac{1}{2}$ N. dept. 159·2, lat. in 54° 29′ N.

(259.) Dif. lat. 114, dist. 298, dept. 275·3 miles.

(260.) Compass course N. by W. $\frac{1}{4}$ W., dist. 246, lat. in 35° 11′ S.

(261.) Compass course N. 29° E., dist. 203, lat. in 32° 15′ N.

(262.) Compass course S. 30° W., dist. 257, lat. in 51° 31′ N.

(263.) Compass course N. 40° W., dept. 258, lat. in 43° 52′ N.

(264.) True course NE. $\frac{1}{4}$ N., app. course NE. by E. $\frac{1}{4}$ E. lat. in 51° 4′ N.

24.

(265.) Course S. 61° 48′ W., or SW. by W. $\frac{1}{2}$ W. nearly, distance 180·9, and lat. in 49° 59$\frac{1}{2}$′ N.

(266.) Course N. 4° 13′ W., dist. 150·9, lat. in 40° 47$\frac{1}{2}$′ S.

(267.) Course N. 6° 51′ E., dist. 120·5 miles, lat. in 39° 58′ N.

(268.) Course N. 78° 10′ W., dist. 21·5 miles, lat. in 43° 14′ S.

(269.) Course SW., dist. 71·7, dif. lat. 50·7, and dept. 50·7 miles.

(270.) Dif. lat. 86·2 S., dept. 42 E., course S. 25° 59′ E., dist. 95·89.

(271.) Dif. lat. 130·7 N., dept. 189·2 W., course N. 55° 22′ W., dist. 229·97, and lat. in 48° 26′ N.

(272.) The sum of the northing and southing is each 209·9, and the sum of the easting and westing is each 262·4; hence, the ship has returned to the place she sailed from, and is therefore again in latitude 40° 29′ N.

(273.) Dif. lat. 77·7 S., dept. 14·8, course made good S. 10° 47′ E., dist. 79·1 miles, lat. in 4° 36′ S.

(274.) Course N. 80° 20′ W., dist. 147·1 miles, lat. in 34° 36′ N., and the dept. 145·1 W.

(275.) Dif. lat. 99·1, dept. 77, course S. 37° 51′ W., dist. 125·5 miles, and the lat. in 48° 57′ S.

(276.) Dif. lat. 17·4 N., dept. 157·9 E., course N. 83° 43′ E., dist. 159·0 miles, and lat. in 56° 15′ N.

(277.) Dif. lat. 96·1 S., dept. 62·8 W., course S. 33° 10′ W., dist. 114·8 miles, and lat. in 39° 50′ S.

25.

(278.) 37·528 miles.

(279.) 125·475 miles.

(280.) 587·705 miles.

(281.) Long. 5° 52′ E.

(282.) Long. in 3° 40′ E.

(283.) 90 miles.

(284.) Long. in 2° 9′ E.

(285.) Dif. lat. 88° 3′.

(286.) Dif. long. 4° 57′.

(287.) Dist. 113·7 miles.

(288.) Dist. 296·6 miles.

(289.) Lat. 50° 55′, long. 6° 30′ W.

(290.) Lat. 38° 26′.

(291.) Lat. 70° 31′ 43″.

(292.) Dist. 210·1 miles.

(293.) Lat. 40° 22′, dist. 313·6 ms.

(294.) Lat. 36° 59′ nearly.

(295.) Lat. 43° 8′.

(296.) 371·7 and 444·2 miles.

(297.) 34° 55′ + 45° 34′ = 80° 29′.

26.

(298.) Bearing S. 16° 17′ 20″ W., dist. 1175·1 miles.

(299.) Bearing S. 71° 50′ E., dist. 381·67 miles.

(300.) Bearing S. 48° 11′ 40″ W., dist. 754·6 miles.

[illegible] course E. dist. 300. dep. 274.

[illegible] course E. by N. dist. 210. dep. 193 miles.

[illegible] course E. by S. dist. 300. dep. 296 lat in 55° 24′ N.

[illegible] course E. by S. dep. 300. lat in 55° 12′

[illegible] course E. by S. dist. 210. lat in 55° 16′ S.

[illegible] course E. by S. dep. 1 minute 1½

[illegible] course E. by S. dep. course SW. [illegible]

[illegible] course [illegible] E. dep. course N., [illegible]

[illegible] course E. by S. dep. 291 lat in 54°

[illegible] course [illegible] dist. 210. dep. 203 miles.

[illegible] course [illegible] dep. [illegible] ½ W., dist. 2

[illegible] course N. 20° E. dist. 200. lat in 3[illegible]

[illegible] course S. 30° W. dist. 250. lat in 51

[illegible] course N. 12° W. dep. 258. lat in [illegible]

The course NE. ½ N. dep. course NE. by [illegible] in 50° 4 N.

24.

(25) Course S. 61° 43′ W., or SW. by W. ½ W. ne[illegible] 180·5, and lat. in 49° 59½′ N.

(26) Course N. 4° 13′ W., dist. 150·9, lat. in 40° [illegible]

(27) Course N. 6° 51′ E., dist. 120·5 miles, lat. [illegible]

(28) Course E. [illegible] 251·5 miles, lat. [illegible]

(29) C[illegible] 80·

S., long. in

86° 35′ W.,

25° 26′ E.,

3° 49′ W.,

89° 22′ E.,

, and AB

, and AB

and AB

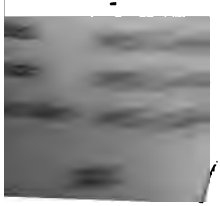
and AB

and AB

and AB

and AB

and AB



(301.) Bearing S. 54° 58′ 26″ E., dist. 1864·2 miles.

(302.) Course S. 52° 48′ E., dist. 593·78 miles, long. in 3° 20′ W.

(303.) Course N. 69° 22′ 24″ E., dist. 190·2 miles, long. in 2° 12′ E.

(304.) Course N. 68° 2′ 22″ W., dist. 668·5 miles, long. in 7° 0′ W.

(305.) Course N. 50° 50′ 30″ W., dist. 771·22 miles, long. in 25° 17′ W.

(306.) Lat. in 0° 49′ S., long. in 0° 24′ W.

(307.) Lat. in 24° 23′ S., long. in 25° 59′ E.

(308.) Lat. in 7° 52′ S., long. in 20° 41′ W.

(309.) Lat. in 47° 4′ S., long. in 57° 45′ W.

(310.) Dist. 556·7 miles, dept. 472·1, long. in 23° 14′ W.

(311.) Dist. 491·97 miles, dept. 282·2, long. in 10° 44′ E.

(312.) Dist. 225·1 miles, dept. 142·8, long. in 11° 25′ W.

(313.) Dist. 792·2 miles, dept. 656·8, long. in 161° 21′ E.

(314.) Course steered N. 38° 26′ E., long. in 21° 11′ E.

(315.) Course steered N. 69° 39′ W., long. in 23° 1′ W.

(316.) Course steered S. 59° 33′ E., long. in 51° 26′ W.

(317.) Course steered N. 59° 13′ 50″ E., long. in 63° 51′ W.

(318.) Dist. 521·5 miles, lat. in 28° 54′ S., long. in 80° 35′ W.

(319.) Dist. 666 miles, lat. in 23° 58′ N., long. in 24° 3′ W.

(320.) Dist. 415·4 miles, lat. in 49° 54′ N., long. in 12° 35′ W.

(321.) Dist. 313·6 miles, lat. in 27° 47′ N., long. in 14° 39′ W.

(322.) Course steered N. $5\frac{3}{4}$ E., lat. in 40° 54′ N., long. in 54° 56′ W.

(323.) Course steered N. 36° W., lat. in 45° 50′ N., long. in 16° 27′ W.

(324.) Course steered S. 32° E., lat. in 42° 58′ S., long. in 25° 11′ E.

(325.) Course steered N. 55° E., lat. in 16° 42′ S., long. in 44° 7′ E.

(326.) Lat. in 43° 7′ N., long. in 19° 29′ W., course S. 86° 35′ W., dist. 2433 miles.

(327.) Lat. in 39° 40′ S., long. in 58° 41′ W., course N. 25° 26′ E., dist. 6052·6 miles.

(328.) Lat. in 41° 39′ S., long. in 8° 7′ E., course S. 73° 49′ W., dist. 3067·7 miles.

(329.) Lat. in 36° 59′ S., long. in 157° 35′ E., course S. 89° 22′ E., dist. 814·2 miles.

27.

(330.) Angle A = 82° 48′ 51″·75, B = 48° 29′ 8″·25, and AB = 401·322.

(331.) Angle A = 63° 43′ 32″·48, B = 46° 16′ 27″·52, and AB = 637·161.

(332.) Angle A = 42° 55′ 16″·4, B = 32° 46′ 43″·6, and AB = 140·359.

(333.) Angle A = 48° 14′ 28″·8, B = 37° 15′ 31″·2, and AB = 461·068.

(334.) Angle A = 64° 11′ 53″, B = 43° 58′ 7″, and AB = 848·518.

(335.) Angle A = 71° 55′ 4″·15, B = 42° 34′ 55″·85, and AB = 77·0575.

(336.) Angle A = 55° 33′ 29″·3, B = 38° 12′ 30″·7, and AB = 1451·93.

(337.) Angle A = 79° 15′ 35″·23, B = 53° 14′ 24″·77, and AB = 447·252.

28.

(338.) Angle A = 34° 29′ 22″·48, B = 48° 2′ 32″·08, and C = 97° 28′ 5″·44.

(339.) Angle A = 44° 22′ 42″·76, B = 103° 36′ 21″·88, and C = 32° 0′ 55″·36.

(340.) Angle A = 41° 19′ 35″, B = 73° 35′ 7″·2, and C = 65° 5′ 17″·8.

(341.) Angle A = 41° 24′ 34″·8, B = 55° 46′ 16″, and C = 82° 49′ 9″·2.

(342.) Angle A = 49° 52′ 6″·2, B = 57° 14′ 53″·8, and C = 72° 53′.

(343.) Angle A = 47° 37′ 48″·8, B = 63° 25′ 30″·2, and C = 68° 56′ 41″.

(344.) Angle A = 53° 47′ 52″·6, B = 59° 35′ 13″·2, and C = 66° 36′ 54″·2.

(345.) Angle A = 31° 49′ 15″·6, B = 21° 53′ 14″, and C = 126° 17′ 30″·4.

29.

(346.) True course, NW. by W.; and rate per hour, $5\frac{1}{2}$ miles.

(347.) True course, S. 88° 3′ E.; and rate per hour, 10·247 miles.

(348.) Setting of the current, N. 46° 10′ W.; rate per hour, 1·444 miles.

(349.) Setting of the current, S. 64° 47′ W.; drift, 14·012 miles nearly.

(350.) Course, N. 44° 2′ W.; rate of approach to the port, 11·115 miles.

(351.) Course, N. 55° 19′ E.; the cape will be reached in 3 h. 4 m. nearly.

(352.) True course, N. 80° 43′ W.; hourly rate of sailing, 12·068 miles.

(353.) True course, N. 82° 41′ E.; distance from the Lizard, 44·077 miles, of which she is carried 12·39 by the current, and sails 31·69 miles.

(354.) The direction is S. 58° 33′ W., and the time 35 m. 23 s.

(355.) The direction is S. 36° 3′ W., and the time 26 m. 26 s.

30.

(356.) Port tack east, starboard tack NW.; dist. on each, 36·5 miles.

(357.) Dist. on each tack, 104·5 miles; time, 32 h. 9 m. 14 s.

(358.) Starboard tack ENE. ½ E. 12·884 miles, port tack S. by W. ½ W. 33·028 miles.

(359.) Starboard tack N. by E. 24·99 miles, port tack SSE. 15 miles.

(360.) Starboard tack NW. 13·36 miles, port tack east 23·58 miles.

(361.) Starboard tack apparent course, S. 16° 52′ E.; real course, S. 30° 56′ E.; port tack apparent course, N. 73° 8′ W.; real course, N. 59° 4′ W.

(362.) Starboard tack SE. ½ S. 38·245 miles, port tack west 12·016 miles.

(363.) A will reach the river about 30 m. 43 s. before B.

(364.) The first will take 40 h. 18 m.; the second, 37 h. 14 m.

(365.) In 39 h. 20 m. nearly.

(366.) Each port tack 32 miles, the starboard tack 90·5 miles, and the time of reaching the port 30 h. 54 m.

31.

(367.) $x = 15° 39′ 37″$, $A = 49° 59′ 24″$; courses 42° 29′ and 87° 50′.

(368.) $x = 34° 55′$, $A = 50° 58′ 40″$; courses 50° 2′ 10″, and 60° 1′ 4″.

(369.) $x = 25° 36′$, $A = 57° 49′ 36″$; courses 40° 14′ 23″, and 72° 3′ 56″.

(370.) $x = 10°\ 34'$, $A = 57°\ 19'\ 25''$; course on leaving St Helena, S. 34° 13′ 28″ W.; course for L = 16°, S. 36° 3′ W.; for L = 21°, S. 38° 15′ W.; for L = 26°, S. 40° 53′ W.; for L = 31°, S. 43° 52′ W.; for L = 36°, S. 47° 7′ W.; for L = 41°, S. 50° 36′ W.; for L = 46°, S. 54° 15′ W.; for L = 51°, S. 58° 2′ W.; for L = 56°, S. 61° 57′ W.; for L = 61°, S. 65° 56′ W.; for L = 66°, S. 70° W.; for L = 72° 12′, S. 75° 6′ W.

32.

(371.) 1 h. 21 m. 11 s.; 1 h. 57 m. $13\frac{1}{15}$ s.; 2 h. 48 m. 45 s.; 3 h. 25 m. 19 s.

(372.) 1 h. 9 m. 14 s.; 3 h. 19 m. $36\frac{11}{15}$ s.; 5 h. 57 m. 4 s.; 5 h. 0 m. 50 s.

(373.) 3 h. 52 m. 45 s.; 4 h. 20 m. 55 s.; 4 h. 37 m. 56 s.; 6 h. 13 m. 11 s.

(374.) 7 h. 20 m. 50 s.; 8 h. 28 m. $21\frac{2}{15}$ s.; 8 h. 41 m. 16 s.; 7 h. 31 m. 55 s.

(375.) 9 h. 56 m. 45 s.; 11 h. 52 m. 10 s.; 11 h. 21 m. 4 s.; 10 h. 27 m. 6 s.

33.

(376.) 48° 6′; 79° 4′ 45″; 110° 3′ 45″; 7° 27′.

(377.) 67° 16′ 15″; 89° 46′; 90° 49′ 30″; 12° 30′ 45″.

(378.) 123° 29′; 142° 3′; 155° 1′ 30″; 14° 49′.

(379.) 168° 55′ 45″; 164° 33′ 30″; 145° 47′; 14° 29′.

(380.) 153° 10′; 174° 47′; 179° 50′; 12° 42′ 30″.

34.

(381.) Jan. 19, 5 h. 17 m. 4 s.; Jan. 26, 15 h. 7 m. 15 s.

(382.) Feb. 14, 22 h. 14 m. 23 s.; Aug. 13, 11 h. 16 m. 12 s.

(383.) May 10, 19 h. 35 m. 49 s.; Sept. 12, 8 h. 24 m. 16 s.

(384.) June 25, 0 h. 0 m. 0 s.; July 13, 16 h. 12 m. 56 s.

(385.) Mar. 13, 10 h. 12 m. 17 s.; Mar. 26, 23 h. 43 m. 16 s.
(386.) Oct. 31, 19 h. 24 m. 13 s.; Nov. 19, 10 h. 12 m. 26 s.

35.

(387.) Jan. 22, 4 h. 14 m. 3 s. A.M.; Feb. 9, 11 h. 12 m. 14 s. P.M.
(388.) May 14, 7 h. 11 m. 15 s. P.M.; Mar. 13, 11 h. 7 m. 15 s. A.M.
(389.) June 28, 4 h. 11 m. 6 s. A.M.; July 23, 5 h. 17 m. 33 s. P.M.
(390.) April 1, 8 h. 5 m. 12 s. A.M.; Mar. 21, 7 h. 12 m. 24 s. P.M.
(391.) Sept. 12, 5 h. 7 m. 30 s. P.M.; Oct. 21, 5 h. 5 m. 28 s. A.M.
(392.) Oct. 12, 3 h. 13 m. 27 s. A.M.; Dec. 11, 5 h. 12 m. 16 s. P.M.

36.

(393.) Oct. 16, 19 h. 15 m. 52 s. (394.) Feb. 4, 8 h. 8 m. 6 s.
(395.) May 30, 9 h. 56 m. 56 s. (396.) July 26, noon.
(397.) Dec. 27, 7 h. 39 m. 3 s. (398.) July 31, 21 h. 27 m. 52 s.
(399.) July 17, 20 h. 5 m. 20 s. (400.) Jun. 18, 19 h. 10 m. 10 s.
(401.) July 17, 22 h. 46 m. 40 s. (402.) Jun. 18, 20 h. 10 m. 13 s.

37.

(403.) Dec. 6° 45′ 41″·3 S., R.A. 13 h. 3 m. 26·28 s.
(404.) Dec. 3° 3′ 9″·1 N., R.A. 0 h. 28 m. 14·46 s.
(405.) Dec. 4° 2′ 24″·6 S., R.A. 23 h. 22 m. 40·58 s.
(406.) Dec. 0° 4′ 19″·6 S., R.A. 23 h. 59 m. 20·53 s.
(407.) Dec. 6° 4′ 6″·6 S., R.A. 23 h. 3 m. 15·69 s.
(408.) Dec. 6° 47′ 12″·7 S., R.A. 13 h. 3 m. 41·13 s.
(409.) Dec. 10° 0′ 8″·7 S., R.A. 13 h. 35 m. 56·65 s.
(410.) Dec. 3° 59′ 8″·1 N., R.A. 0 h. 36 m. 58·16 s.
(411.) Dec. 0° 23′ 19″·8 N., R.A. 0 h. 3 m. 35·13 s.
(412.) Dec. 11° 50′ 39″·6 S., R.A. 13 h. 53 m. 37·61 s.

38.

(413.) Dec. 17° 12′ 49″·8 S., R.A. 19 h. 21 m. 29·15 s.
(414.) Dec. 11° 34′ 18″·4 S., R.A. 9 h. 27 m. 18·98 s.
(415.) Dec. 12° 12′ 27″·0 S., R.A. 21 h. 15 m. 23·37 s.
(416.) Dec. 17° 32′ 41″·5 N., R.A. 4 h. 47 m. 4·08 s.
(417.) Dec. 3° 44′ 21″·2 N., R.A. 11 h. 0 m. 35·54 s.
(418.) Dec. 3° 44′ 9″·0 S., R.A. 12 h. 31 m. 45·17 s.
(419.) Semidiameter 15′ 12″·1, Hor. par. 55′ 41″·9.
(420.) Semidiameter 16′ 17″·8, Hor. par. 59′ 42″·4.
(421.) Semidiameter 16′ 5″·5, Hor. par. 58′ 57″·6.
(422.) Semidiameter 16′ 28″·6, Hor. par. 60′ 22″·1.

39.

(423.) 14° 22′ 24″.	(424.) 25° 42′ 17″.	(425.) 40° 30′ 14″.
(426.) 26° 54′ 51″.	(427.) 58° 0′ 41″.	(428.) 46° 24′ 12″.
(429.) 60° 28′ 24″.	(430.) 59° 34′ 28″.	(431.) 43° 9′ 7″.
(432.) 19° 35′ 36″.	(433.) 59° 12′ 26″.	(434.) 65° 7′ 25″.

40.

(435.) 42° 53′ 34″.	(436.) 31° 23′ 21″.	(437.) 41° 8′ 39″.
(438.) 20° 17′ 46″.	(439.) 54° 9′ 38″.	(440.) 49° 49′ 32″.
(441.) 57° 20′ 58″.	(442.) 64° 22′ 8″.	(443.) 20° 32′ 37″.
(444.) 7° 8′ 10″.	(445.) 49° 45′ 43″.	(446.) 56° 49′ 50″.

41.

(447.) 19 h. 6 m. 17 s.	(448.) 23 h. 51 m. 43 s.
(449.) 16 h. 4 m. 0 s.	(450.) 7 h. 18 m. 58 s.
(451.) 15 h. 46 m. 49 s.	(452.) 22 h. 28 m. 55 s.
(453.) 16 h. 25 m. 29 s.	(454.) 6 h. 26 m. 12 s.
(455.) 12 h. 25·6 m.	(456.) 11 h. 43 m.
(457.) 12 h. 19·2 m.	(458.) 11 h. 34·7 m.

42.

(459.) 14° 10′ 35″ N. (460.) 22° 24′ 31″ N.
(461.) 39° 0′ 25″ N. (462.) 24° 52′ 48″ S.
(463.) 40° 48′ 2″ S. (464.) 65° 14′ 42″ N.
(465.) 36° 44′ 16″ S. (466.) 20° 7′ 31″ N.
(467.) 40° 46′ 1″ S. (468.) 26° 41′ 29″ S.
(469.) 33° 46′ 12″ N. (470.) 41° 0′ 47″ N.
(471.) 55° 7′ 33″ S. (472.) 32° 37′ 56″ N.
(473.) 28° 52′ 16″ S. (474.) 22° 55′ 31″ N.
(475.) 29° 45′ 17″ S. (476.) 0° 36′ 27″ S.
(477.) 29° 18′ 38″ N. (478.) 18° 13′ 37″ S.
(479.) 26° 14′ 14″ S. (480.) 0° 35′ 54″ N.
(481.) 46° 25′ 20″ S. (482.) 43° 24′ 16″ N.
(483.) 29° 10′ 33″ S. (484.) 58° 59′ 16″ N.
(485.) 1° 22′ 8″ S. (486.) 44° 40′ 39″ N.
(487.) 22° 19′ 49″ N. (488.) 0° 8′ 11″ S.
(489.) 27° 2′ 24″ S.

43.

(490.) Time 4 h. 44 m. 45 s. A.M., and azimuth 66° 24′ 42″.
(491.) Time 3 h. 20 m. 14 s. A.M., and azimuth 44° 41′ 48″.
(492.) Time 4 h. 34 m. 16·6 s. A.M., and ampli. 26° 3′ 57″ N.
(493.) Time 8 h. 8 m. 26·4 s. A.M., and ampli. 39° 0′ 24″ S.
(494.) Time 4 h. 34 m. 0·5 s. A.M., and lat. 44° 48′ 32″ N.
(495.) Time 8 h. 39 m. 45·7 s. A.M., and ampli. 45° 18′ 8″ S.
(496.) $\angle A = 67° 25′ 30″$, $\angle B = 42° 24′ 14″$, and $c = 62° 55′ 10″$.
(497.) $c = 91° 41′ 21″$, $a = 73° 11′ 2″$, and $b = 95° 50′ 55″$.
(498.) $\angle A = 4° 15′ 47″$, $a = 3° 16′ 17″$, and $c = 50° 9′ 41″$.
(499.) $\angle A = 54° 26′ 29″$, $\angle B = 88° 51′ 28″$, and $b = 88°$ ३५′ ४५″.

(500.) $\angle A = 78^\circ\,51'\,38''$, $c = 80^\circ\,20'\,49''$, and $b = 48^\circ\,38'\,23''$.

(501.) $\angle B = 131^\circ\,53'\,5''$, $a = 48^\circ\,12'\,4''$, and $b = 140^\circ\,15'\,47''$.

44.

(502.) $\angle A = 48^\circ\,31'\,35''$, $\angle B = 62^\circ\,55'\,44''$, and $\angle C = 125^\circ\,18'\,56''$.

(503.) $\angle A = 138^\circ\,15'\,45''$, $\angle B = 31^\circ\,11'\,14''$, and $\angle C = 35^\circ\,49'\,58''$.

(504.) $\angle A = 70^\circ\,36'\,30''$, $\angle B = 75^\circ\,50'\,22''$, and $\angle C = 90^\circ\,5'\,10''$.

(505.) $\angle B = 135^\circ\,5'\,29''$, $\angle C = 50^\circ\,30'\,9''$, and $a = 69^\circ\,34'\,56''$.

(506.) $\angle B = 95^\circ\,38'\,4''$, $\angle C = 97^\circ\,26'\,29''$, and $a = 64^\circ\,23'\,19''$.

(507.) $\angle A = 30^\circ\,28'\,11''$, $\angle B = 130^\circ\,3'\,11''$, and $c = 40^\circ\,0'\,0''$.

(508.) $a = 43^\circ\,37'\,37''$, $b = 37^\circ\,10'\,1''$, and $C = 120^\circ\,59'\,46''$.

(509.) $c = 120^\circ\,48'\,0''$, $B = 59^\circ\,15'\,57''$, and $C = 65^\circ\,44'\,53''$.

(510.) Dist. $51^\circ\,6'\,49''$.

(511.) Dist. $135^\circ\,21'\,26''$.

(512.) Dist. $87^\circ\,46'\,2''$.

(513.) Altitude $41^\circ\,26'\,50''$.

45.

(514.) Lat. $13^\circ\,8'\,8''$ of the same name as the elevated pole.

(515.) Lat. $44^\circ\,21'\,24''$.

(516.) Lat. $48^\circ\,33'\,9''$.

(517.) Lat. $48^\circ\,54'\,56''$.

(518.) Lat. $44^\circ\,52'\,26''$.

(519.) Lat. $30^\circ\,4'\,29''$.

(520.) Lat. $33^\circ\,10'\,41''$.

(521.) Lat. $36^\circ\,37'\,47''$.

(522.) Lat. $24^\circ\,23'\,44''$.

(523.) Lat. $15^\circ\,11'\,2''$.

(524.) Lat. $7^\circ\,39'\,46''$.

(525.) Lat. $50^\circ\,41'\,48''$.

(526.) Lat. $42^\circ\,26'\,22''$.

46.

(527.) Ap. time 7 h. 23 m. 59 s. A.M.

(528.) Ap. time 7 h. 22 m. 24·8 s. A.M.

(529.) Ap. time 5 h. 40 m. 55 s. P.M.

(530.) Ap. time 3 h. 5 m. 2·5 s. P.M.

(531.) Ap. time 9 h. 17 m. 24·7 s. A.M.

(532.) Ap. time 8 h. 29 m. 36 s. A.M.

(533.) Ap. time 6 h. 21 m. 49 s. P.M.

(534.) Ap. time 6 h. 24 m. 4·5 s. A.M.

(535.) Mean time 5 h. 26 m. 44·8 s., long. in 32° 21′ 12″ E., variation 17° 24′ W.

(536.) Mean time 7 h. 24 m. 17·2 s. A.M., long. in 102° 44′ 42″ W., variation 19° 56′ 58″ W.

(537.) Mean time 6 h. 10 m. 38 s. P.M., long. in 45° 31′ 46″·5 W., variation 9° 3′ 8″ W.

(538.) Mean time 6 h. 45 m. 26 s. A.M., long. in 159° 28′ 3″ E., variation 15° 18′ E.

(539.) Mean time 4 h. 34 m. 23 s. P.M., long. in 0° 42′ 15″ W., variation 16° 6′ 18″ W.

(540.) Mean time 5 h. 9 m. 11·6 s. P.M., long. in 75° 45′ 36″ W., variation 19° 48′ 20″ W.

(541.) Mean time 6 h. 9 m. 20·3 s. A.M., long. in 15° 49′ 55½″ W., variation 12° 6′ 56″ E.

(542.) Mean time 6 h. 18 m. 51 s. A.M., long. in 85° 17′ 15″ W., variation 31° 9′ 38″ W.

47.

(543.) Lat. in 46° 27′ 36″ N., app. time at greater altitude 11 h. 27 m. 23 s. A.M.

(544.) Lat. in 47° 20′ 2″ N., app. time at greater altitude 1 h. 10 m. P.M.

(545.) Lat. in 50° 0′ 5″ N., app. time at greater altitude 11 h. 29 m. 58 s. A.M.

(546.) Lat. in 60° 0′ 27″ N., watch 9 s. slow for app. time at the place.

(547.) Lat. in 7° 7′ 52″ N., app. time at greater altitude 8 h. 58 m. 53 s. A.M.

(548.) Lat. in 48° 55′ 7″ N., watch 1 s. fast for app. time at the place.

(549.) Lat. in 52° 8′ 22″ N., watch 10 m. 55 s. slow for apparent time.

(550.) Lat. in 39° 26′ 40″ N., watch 18 m. 54 s. fast for apparent time.

(551.) Lat. in 50° 3′ 36″ N., watch 12 m. 37 s. slow for app. time at the place.

48.

(552.) + 13·47.	(553.) + 13·32.	(554.) − 3·80.
(555.) + 16·57.	(556.) − 11·35.	(557.) + 10·66.
(558.) − 2·35.	(559.) − 19·64.	(560.) + 10·21.

49.

(561.) Eq. eq. alts. + 12·32; chron. slow on app. T. 50 m. 18 s., slow on M.T. at the place 53 m. 39 s., and slow on M. Gr. T. 1 h. 44 m. 35 s.

(562.) Eq. eq. alts. − 8·46; chron. fast on app. T. 37 m. 52 s., fast on M.T. at the place 41 m. 43 s., and fast on M. Gr. T. 2 h. 4 m. 43 s.

(563.) Eq. eq. alts. − 1″; chron. fast for app. T. 1 h. 46 m. 44 s., fast for mean time at the place 1 h. 49 m. 53 s., and slow on M. Gr. T. 39 m. 27 s.

(564.) Eq. eq. alts. + 1″; chron. fast for app. T. 3 h. 8 m. 23 s., for M.T. at the place 3 h. 6 m. 19 s., and fast for M. Gr. T. 17 m. 7 s.

(565.) Eq. eq. alts. − 10″; chron. slow for app. T. at the place 1 h. 20 m. 23 s., for M.T. at the place 1 h. 26 m. 23 s., and for M. Gr. T. 28 m.

(566.) Eq. eq. alts. 12·85, or + 13″; chron. fast on app. T. at the place 43 m. 56 s., on M.T. at the place 39 m. 8 s., and on M.T. at Gr. slow 10 m. 28 s.

(567.) Eq. eq. alts. + 10·43, or + 10 s.; chron. slow on app. T. at the place 7 m. 19 s., slow on M.T. at the place 6 m. 51 s., and fast on M.T. at Gr. 14 m. 21 s.

(568.) Eq. eq. alts. − 5·29, or − 5 s.; chron. slow on app. T. at the place 4 h. 2 m. 32 s., slow on M.T. at the place 3 h. 50 m. 15 s., and fast on Gr. M.T. 1 h. 30 m. 47 s.

50.

(569.) True alt. 40° 22′ 48″, app. alt. $\underline{\odot}$ 40° 7′ 41″.

(570.) True alt. 37° 3′ 42″, app. alt. $\underline{\odot}$ 36° 49′ 4″.

(571.) True alt. 13° 47′ 36″, app. alt. $\overline{\odot}$ 14° 6′ 58″.

(572.) True alt. 61° 53′ 59″, app. alt. $\overline{\odot}$ 62° 10′ 12″.

(573.) True alt. 29° 57′ 26″, app. alt. $\underline{\odot}$ 29° 43′ 4″.

(574.) True alt. 17° 51′ 29″, app. alt. 17° 54′ 28″.

(575.) True alt. 2° 36′ 17″, app. alt. 2° 51′ 22″.

(576.) True alt. 20° 8′ 6″, app. alt. 20° 10′ 40″.

(577.) True alt. of the moon's centre above the rational horizon 5′ 14″, below the sensible horizon 21′ 25″, and upper limb 5′ 8″ under the sensible horizon.

(578.) Moon's centre is 2° 2′ 5″ below the rational horizon.

51.

(579.) D = 50° 41′ 13″.

(580.) D = 42° 59′ 25″.

(581.) D = 46° 4′ 27″.

(582.) D = 69° 49′ 10″.

(583.) D = 80° 9′ 32″.

(584.) D = 120° 1′ 24″.

(585.) D = 28° 8′ 25″.

(586.) D = 48° 45′ 31″.

(587.) D = 18° 8′ 15″.

(588.) D = 59° 48′ 11″.

52.

(589.) D = 58° 1′ 22″; chron. fast for M. Gr. T. 20 m. 49 s.; long. 118° 31′ 15″ W.

(590.) D = 87° 42′ 50″; chron. slow for M. Gr. T. 1 m. 24 s.; long. in 26° 7′ W.

(591.) D = 40° 52′ 43″; chron. fast 1 m. 4 s. for M. Gr. T.; long. 69° 8′ 45″ W.

(592.) D = 94° 5′ 38″; chron. fast 57 s. for M. Gr. T.; long. 67° 27′ 0″ E.

(593.) D = 68° 52′ 10″; chron. fast 20 s. for M. Gr. T.; long. 178° 48′ 15″ W.

(594.) D = 101° 1′ 42″; chron. slow 13 s. for M. Gr. T.; long. 1° 53′ 15″ W.

(595.) D = 71° 38′ 20″; chron. fast 41 s. for M. Gr. T; long. 144° 56′ 15″ W.

(596.) D = 60° 17′ 50″; chron. slow 30 s. for M. Gr. T.; long in 120° 37′ 30″ E.

(597.) D = 76° 25′ 58″; chron. fast 59 s. for M. Gr. T.; long. in 74° 11′ 0″ E.

(598.) D = 40° 22′ 3″; chron. fast 19 s. for M. Gr. T.; long. in 177° 12′ 15″ E.

(599.) D = 96° 34′ 22″; chron. fast 1 m. 41 s. for M. Gr. T.; long. in 156° 39′ 30″ W.

(600.) D = 94° 5′ 26″; chron. fast 1 m. 23 s. for M. Gr. T.; long. in 67° 33′ E.

APPENDIX.

ON WEATHER FORECASTS AND STORMS.

In the previous chapters of this work, there have been given such rules and directions as will be found sufficient to enable the mariner to find his place at sea, and to determine on what course he should steer, so as to reach any desired port whose latitude and longitude are known; and there the principles on which these rules and directions are founded, have also been given and demonstrated mathematically.

But in conducting a ship through the wide and pathless ocean, it is often found that the wind and waves are so directly opposed to the course desired to be steered, that the treatise would be defective unless it contained directions to the mariner, by which he may at least so far anticipate the kind of weather he is likely to encounter, and be prepared to adopt the wisest measures when actually overtaken by a storm.

In this chapter, then, our plan must be somewhat different from that followed in the previous chapters, for we can no longer be entirely guided by the unfailing principles of mathematical science, but must also call to our aid the established facts of meteorology to guide us in forecasting the

weather, and the experience of seafaring men, to enable us to ascertain, as far as possible, the laws which storms at sea are found in general to follow, and the best means of avoiding their disastrous consequences.

THE BAROMETER.

The principal instrument used in weather-casts is the barometer, which must also be accompanied by the thermometer; for the weight of the atmospheric air, which is shewn by the height of the barometer, depends in a great degree on its temperature, which is shewn by the thermometer, since heat expands and cold contracts the volume of the atmospheric air, as well as that of solid bodies. In predicting changes of weather, therefore, from the height of the barometer, and its rising and falling, the height of the thermometer must not be neglected, for, by such neglect, the results obtained would in most cases be erroneous, and might in some cases be the very opposite of those at which a skilful observer would arrive, by a proper balancing of the indications of the two instruments.

The barometer, in its simplest and best form, is a hollow tube of glass of uniform bore, and upwards of thirty inches in length, hermetically sealed at one end and open at the other. This tube being filled with mercury, and placed in a vertical position, with the closed end uppermost, and the lower open end inserted in a cup or cistern filled with mercury, the communication between the tube and cistern being made free, the mercury falls in the tube till its weight is equal to that of a column of atmospheric air, of the same diameter, reaching from the surface of the earth to the utmost limits of the atmosphere. The chief difference between this common barometer and the marine one is, that in the latter there is at one part of the glass tube a narrow contraction, through which the mercury can

only pass slowly, to prevent the oscillation of the mercury in the tube, which would otherwise arise from the heaving of the vessel.

Public attention has frequently been called to the invaluable use of the barometer in indicating a coming storm. A notable instance of the advantage arising at sea from attending to its indications, is given by Dr Arnott in his admirable treatise on Physics, where he says: The mercury in the tube had been observed by the captain to be falling with great rapidity, on which he gave orders to prepare with the utmost haste for an approaching storm. The extent of the preparations, and the promptness with which they were ordered to be executed, astonished the crew, the most skilful of whom had not as yet perceived in the sky or clouds the most distant threatening of a storm. But before the preparations were completed, though executed with the utmost expedition, one of the most tremendous storms ever experienced by the oldest seaman on board had burst on the noble vessel; the noise of the thunder, the wind, and the waves, was such that no voice could be heard on board, and even the trumpet sounded in vain. When the storm had passed, the sea was strewed with wrecks; but our noble ship, though sadly buffeted by the storm, still survived. Had it not been for the timeous warning given by that small tube of mercury, neither the strength of the ship nor the skill of her commander would have been sufficient to have saved one man alive to tell the sad tale.

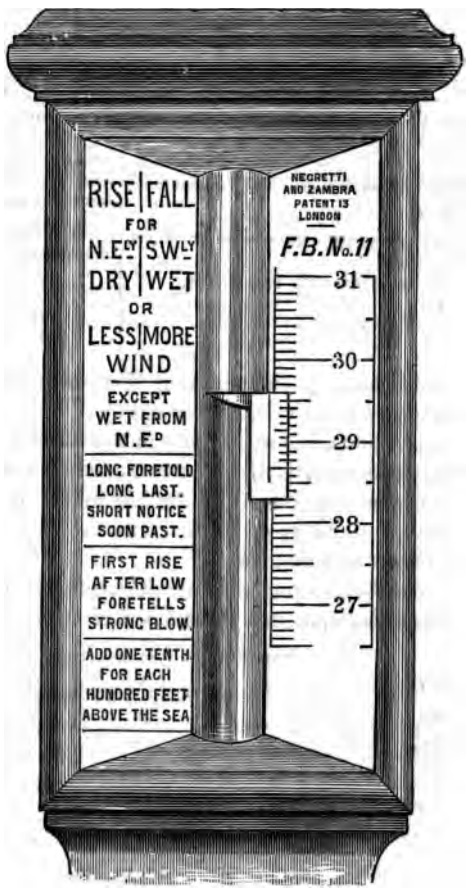
The barometer is, then, an instrument of the highest importance to the mariner; and its indications, combined with the appearance of the clouds, direction of the wind, and height of the thermometer, ought to be made by him a study of first importance, in order that he may be able to take full advantage of its timeous warnings, and thereby avoid many disastrous consequences of the fierce storms which he frequently encounters at sea. But to do so successfully, it is necessary, in the first place, that he have an instrument whose indications may be depended upon. The instruments generally used are those

constructed by Messrs Negretti and Zambra, instrument-makers to Her Majesty, the Royal Observatory, the Admiralty, and the British Meteorological Society, 1 Hatton Garden, London.

In order to render their barometers permanently useful, and as little as possible liable to derangement, they have introduced the following changes in the regular instruments, which may fairly come under the head of important improvements. In the first place, those intended for public stations and general use are mounted in a frame of solid oak, firmly screwed together; and, in all, the brass or ivory scales that barometers are generally furnished with, are here replaced by a substantial plate of porcelain, on which the degrees and figures are legibly engraved and permanently blackened in, so that, as far as the divisions and figures are concerned, there will be no danger of their becoming faded or obliterated. This is a very important improvement, especially for an instrument that has to be placed in an exposed position, where the mariner may be able to consult it at any time, even in the middle of the night, should he require to do so.

The mercurial tube of the barometer is of large diameter, so as to render the mercury easily visible, and shew the slightest variation, and is so constructed, that the liability of air entering it, is, we may say, entirely obviated. Should air by any means find its way up the tube of the barometer to the vacuum at the top, let the quantity be ever so minute, the indications of the instrument will be erroneous, and can no longer be relied on. To prevent the admission of air to the vacuum above the mercury, a trap is laid at the bottom of the tube, near the part called the cistern, so that if any air should find its way into the tube, it cannot possibly pass the trap, but is there detained, and the instrument is in no way deteriorated or injured by its presence. These tubes, after being filled, are, moreover, boiled. The operation of boiling a barometer tube consists in placing the tube, after it is filled with mercury, over a charcoal fire, and retaining it there till the mercury boils. It is an opera-

tion attended with considerable risk to the tube and operator and for this reason is seldom carried into practice; and in the majority of barometers made, it is altogether neglected. The diameter of the mercurial column averages four-tenths of an inch, so that, altogether, instruments of the greatest efficiency will be obtained. A great change has also been effected in the old system of making the scales with 'Fair,' 'Change,' 'Rain,' &c., words which in reality have often a tendency to mislead, and to throw discredit on barometrical indications.



The Fitzroy Barometer.

The plan to be adopted in these barometers is shewn in the accompanying engraving. This has been arranged systematically by the late Admiral Fitzroy, F.R.S., who devoted his life to studying the indications of the

barometer, and their true explanation. The meanings of the contractions marked on the dial of these barometers is explained by the Admiral as follows:

THE BAROMETER RISES *

For north-easterly wind (including from north-west, by the north, to the eastward)—for dry, or less wet weather—for less wind—or for more than one of these changes.

EXCEPT on a few occasions when rain (or snow) comes from the north-eastward with *strong* wind.

For change of wind towards *any* of the above directions,

A THERMOMETER FALLS.

Long foretold—long last.
Short notice—soon past.

THE BAROMETER FALLS *

For south-westerly wind (including from south-east, by the south, to the westward)—for wet weather—for stronger wind—or for more than one of these changes.

EXCEPT on a few occasions when *moderate* wind with rain (or snow) comes from the north-eastward.

For change of wind towards the *upper* directions only,

A THERMOMETER RISES.

First rise after low,
Indicates a stronger blow.

Moisture or dampness in the air (shewn by a hygrometer) increases *before* rain, fog, or dew.

Add one-tenth of an inch to the observed height of the barometer, for each hundred feet it is situated above the mean sea-level.

The *average* height of the barometer in England, at the sea-level, is about 29·94 inches; and the *average* temperature of air is nearly fifty degrees.

The thermometer falls about one degree for each three hundred feet of elevation, more than fifty feet, above the level of the ground.

DIRECTIONS FOR POSITION AND USE.

A barometer, for a coast weather-glass, should be placed where it may be seen at any time, in a good light, by fishermen, boatmen, seafaring persons, &c., and should be set regularly by a duly authorised person about sunrise, noon, and sunset.

The barometer at a fixed station should be suspended against a post or wall, in a well-lighted situation, at such a height that the middle of the scale shall be level with the eye of an observer, or about five feet above the ground. When possible, the barometer should be fixed, so that light may be seen *through*

* These rules apply to northern latitudes. For southern hemisphere, substitute S. for N., allowing E. and W. to remain unchanged.

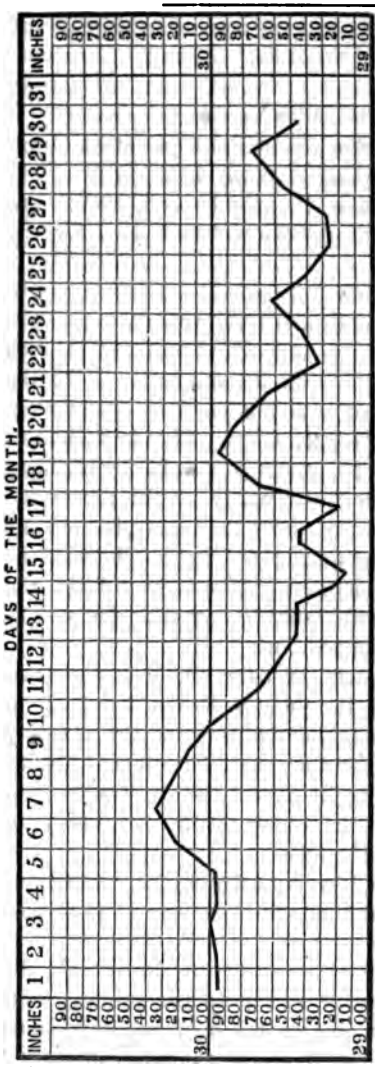
the tube, or a piece of white paper, or of ivory, should be placed behind the glass, at the back of the scale. When the barometer is suspended by a strong nail or screw, take the milled-head key—that used for moving the index or vernier—and with it unscrew, to its fullest extent, the square brass pin at the lower end of the instrument, by turning it gently from right to left. The mercury will then descend in the tube to its true level, and the true height or length of the column will then be shewn by the scale and vernier.

To Read the Barometer.—With the milled-head key move the index at the side of the tube, until it is level with the top of the mercury; then note the number of inches and tenths of an inch on the scale, and the hundredths on the vernier at which the column stands; this, compared with the previous observation, will shew if the mercury has risen or fallen. If it is desired to remove the barometer, it may be rendered portable by inclining it gradually until the mercury flows nearly to the top of the tube; then, with the key screw up slowly the brass pin until the mercury fills the tube, avoiding forcing the screw, otherwise the tube or the cistern may be injured. When *screwed up*, the barometer may be carried with safety either with the cistern end upwards, or in a horizontal position.

The Marine Barometer, instead of being fixed as described above, should be suspended in gimbols like a compass, to prevent, as much as possible, the tendency of the mercury to oscillate by the motion of the ship; in other respects, the directions given above apply equally to it, except that unscrewing at the bottom will not so soon take effect on account of the contraction in the tube.

The barometer should be set regularly by an authorised person twice each day, say at nine o'clock in the morning and three o'clock in the afternoon.

On taking the reading every morning, a dot should be placed on a ruled scale, corresponding to the reading, and a line drawn from the last dot to the new one, and thus successively day by day from one point to the next, indicating



Specimen of a Barometer Chart or Diagram.

to the eye the changes of the preceding days, as shewn in this chart:

The annexed chart is for the month of November 1860. An inspection will shew that, till the 5th day, the deviations from a horizontal line are very small; then there is an ascending line to the 7th, when the highest point in the month is reached; from this time till the 12th, the barometer reading was constantly decreasing; on the 13th day, there was scarcely any change; on the 14th, two points are laid down, as the reading fell from 29·46 inches in the morning to 29·28 inches by the evening; on the 15th, the lowest reading in the month took place; on the 16th, the reading was steady all day; it then decreased during the night to 29·20 inches on the following morning; there was a rise of half an inch between the 17th and 18th, and the increase continued till the 19th; there was then a decrease to the 21st, and alternately an increase and

decrease about the point 29·5 inches till the end of the month.

OF WINDS AND THEIR CAUSES.

Wind is air in motion relatively to the surface of the earth, and is generally caused by the varying temperature of the air in different places, produced chiefly by the heating power of the sun's rays. When the air at any place becomes heated, its volume expands, and it becomes specifically lighter, and ascends (the barometer falls). The colder and denser air then rushes in from the neighbourhood to restore the equilibrium, thus producing wind. If the partial vacuum, formed as stated above, be of large extent, the air rushes in from various quarters, and the currents, striking each other obliquely, cause a rotation of the air round the point of least density, which appears to be the manner of formation of those circular storms frequently encountered at sea, and now generally known by the name of *Cyclones.*

Since the earth revolves on its axis, and the perpendicular distance of the earth's surface from this axis is nothing at the poles, and increases from there to the equator, if a wind blow in the direction of a parallel of latitude, all the parts of the earth over which it passes will have the same rotatory motion, and consequently will have no tendency to cause the wind to change its direction; it will therefore blow directly east or directly west. But if a wind proceed from the neighbourhood of the north or south pole towards the equator, it will pass over portions of the earth's surface, whose velocity towards the east is becoming greater at each advance; hence, as it advances towards the equator, it will gradually veer more and more towards the east, until it arrives in the vicinity of the equator; it will then blow directly from the east, and in this manner is formed the two belts of easterly wind, one on each side of the equator, called the *Trade-winds.*

The sun being always vertical to some portion of the torrid zone, the air there becomes much heated and rarefied, which causes it to ascend; and as the air from the north and south is

colder and denser, it fills its place, while the warm air, which thus ascends, proceeds toward the poles, carrying a great portion of its heat with it. Gradually, however, it mixes with the colder currents below, and raises their temperature. This heated air, proceeding from the equator, has the easterly motion of that portion of the earth; and as it advances toward the poles, the easterly motion of the surface over which it passes is less than that of the current of air, hence the wind appears to blow from the south-west. The air at the poles is the coldest, that at the equator the hottest, and, as shewn above, the wind from the north polar region comes to us from the north-east, while the wind from the equatorial regions comes to us from the south-west; whence the reason that north-easterly wind is proverbially cold, while south-westerly wind is mild and warm.

Now, inasmuch as the atmosphere expands by heat and contracts by cold, varying to a corresponding degree its density or specific gravity, so it follows that the height of the barometric column will be influenced by winds. In Europe it will generally be found that a fall of the barometer corresponds with a rise in the thermometer; the same rule also applies to the tropics—nevertheless, it is subject to many variations. The barometer may rise and fall without any corresponding change in the thermometer, or both may rise and fall together; but sudden changes in either are indications of changeable weather.

The application of the barometer as a weather-glass is altogether collateral and secondary; nevertheless, its indications in this respect are, for the most part, worthy of confidence; generally, the barometric column sinks the day before rain occurs, and rises during its prevalence. The barometric column is much agitated both before and during the existence of a storm, arising from the conflict which then takes place between atmospheric currents having tendencies in different and frequently in opposite directions.

There exist other fluctuating causes of the barometric column, which, though scarcely perceptible in the *temperate*, are very conspicuous in the *torrid* zone; there, every day it twice attains

a maximum, and as often a minimum. The two periods of maximum elevation occur about 9 h. 37 m. A.M., and 10 h. 11 m. P.M. on an average; while those of minimum elevation occur about 3 h. 45 m. A.M., and 4 h. 5 m. P.M., but each varies to about an hour earlier or later than the averages stated above.

The aneroid, or metallic barometer, indicates by the motion of its hand the height at which the mercury will stand in the other instrument, and, if used at sea, for which, in several respects, it is well adapted, it should be tested as often as opportunity occurs with the mercurial column in a trustworthy instrument, to see if its hand is set at the right pitch.

In general, wind affects the barometer more than rain; and temperature is affected by the direction of the prevailing wind more than by time of day or night, or even by the state of sky while exposed to radiation. Hence, in estimating the height of the barometer, special attention should be paid to the direction of the wind.

FORECASTS OF WEATHER BY THE BAROMETER.*

The column of mercury in the barometer ranges about 3 inches in temperate climates, towards the higher latitudes—namely, from 30·9 to less than 28 inches on extraordinary occasions; but the ordinary range is from 30·5 to about 29 inches. In the vicinity of the equator, the ordinary range is only a few tenths, but in storms it sometimes falls as low as 27 inches. A fall of from one-twentieth to one-tenth of an inch in an hour, is a *sure warning* of a *coming storm.*

If the height of the barometer is about 30 inches at the level of the sea, and is steady or rising, while the thermometer falls and dampness becomes less, then north-westerly, northerly, or north-easterly wind may be expected, or less rain or snow.

If the barometer fall, while the thermometer rises with increased dampness, expect wind and rain from the south-eastward, southward, or south-westward; or in winter, when the thermometer is low, expect snow.

* These remarks on forecasts apply to northern latitudes. For southern latitudes, substitute S. for N., leaving E. and W. unchanged.

An important exception to the two preceding rules occurs when a *northerly* wind, with rain, hail, or snow, is impending; for then the barometer often *rises* on account of the direction of the coming wind, and thus deceives persons who from the rising of the barometer alone, expect fair weather.

With a rather low barometer, such as 29·5 inches, at the level of the sea, a rise foretells less wind, or a change in its direction towards the northward, or less wet; when, however, it has been very low, about 29 inches, the first rising usually precedes strong wind, sometimes heavy squalls, from the north-westward, northward, or north-eastward; after which, if the thermometer falls, and the barometer continues to rise gradually, better weather may be expected. But if the warmth continue, the wind will probably shift back against the sun's course, and more southerly or south-westerly wind will follow, especially if the barometrical rise has been sudden.

The heaviest northerly gales, and the most dangerous shifts of wind, frequently happen soon after the barometer first rises from a very low point, or at some time afterward if the wind veers gradually, though with a rising glass.

Approaching changes of weather, and the direction and force of winds, are indicated less by the *height* of the barometer than by its *rising* or *falling*. A height of more than 30 inches at the level of the sea, however, indicates fine weather and moderate winds, except from north to east, whence it may occasionally blow strongly.

A rapid and considerable fall of the barometer is a sign of stormy weather, with rain or snow; whilst alternate rising and falling, or oscillation of the mercury, indicates unsettled and threatening weather.

The greatest depressions of the barometer are experienced during gales from the SW., S., or SE., or during a thaw which follows a long frost; for the thaw loads the warm wind with vapour, and the air is expanded by the warmth of the wind, thus causing the barometric column to descend.

The barometer rises the highest with wind from NE., N., or

NW., or with a calm; and as frost winds generally blow from some of these directions, it rises highest of all during a continuous frost, which greatly condenses the air, and so increases its pressure.

With the barometer above 30 inches, the air must be very dry or very cold, and is frequently both, hence no rain may be expected; for if the air is very dry it will absorb moisture instead of parting with it in rain, and if very cold it is so much condensed that it has already parted with all the moisture that it can spare.

If, contrary to the general law, the barometer rise with a southerly wind, or sink with a northerly wind, the southerly wind is uncommonly dry, and consequently accompanied with fine weather; but the northerly wind is violent, and accompanied by rain, snow, or hail, and also frequently with lightning. When the barometer *stands* very low indeed, there will never be much rain, but a fine day will seldom occur at such times; for the air must then be very warm or very moist, or both, and so there will only be short heavy showers, accompanied with sudden squalls of wind from the west. When, however, the barometer *sinks* considerably, much rain or snow will follow, with or without lightning. The wind will be from the northward if the thermometer is low for the season; from the southward, if the thermometer is high. A low glass is occasionally followed or accompanied only by lightning, while a storm is raging beyond the horizon.

When the wind is from the west, a *sudden* fall of the barometer is sometimes followed by a violent storm from NNE. or NW.

During summer, after a long continuance of fair weather, the barometer will continue to fall gradually for two or three days before rain comes; if, however, the fall be very sudden, a thunder-storm is at hand.

The barometer is sometimes *low* when the sky is cloudless, seeming to promise fine weather; but in this case the sky will soon be overcast and the appearances changed.

With a high barometer dense clouds will frequently pass over

without rain; but with a low one, it will often rain with almost no appearance of clouds.

The barometer appears to be affected by at least three causes: *First*, The direction of the wind; the NE. wind, in the northern hemisphere, tending most to raise it, and the SW. to lower it, the amount of change due to these directions being about half an inch, while proportional changes take place for intermediate directions, the winds from NW. and SE. giving the mean height. NE. and SW. may therefore, in reference to the barometer, be considered as the winds' poles. These changes are of importance in weather forecasts, though they are too often overlooked.

Secondly, The amount of vapour, moisture, wet, rain, or snow in the current of air whose direction and strength remain the same, seems to cause a change amounting, in an extreme case, to about half an inch.

Thirdly, The strength or force of the wind from whatever quarter, moisture and direction remaining the same, is accompanied by a fall or rise, according as its force increases or diminishes, ranging in extreme cases, as from a hurricane to a calm, to more than two inches.

When these three causes all act together so as to raise the barometer, it will ascend to about 31 inches; and when they all act so as to depress it, it will descend at times as low as 27 inches; but such extreme changes seldom occur even in tropical latitudes. In general, the three causes operate in different directions, and less strongly, so that ordinary varieties of weather are much more common than extreme changes.

It is also worthy of attention that the wind usually appears to veer or go round with the sun, in N. latitudes, *right-handed*, or from left to right; in S. latitudes, *left-handed*, or from right to left. If in the opposite directions, more wind or bad weather may be expected, rather than improvement, after a short interval.

Before the conclusion of a gale the barometer begins to rise considerably, sometimes even at its commencement. Although

it falls lowest before high winds, it often sinks very much before heavy rain.

In the northern hemisphere, before thunder, the barometer generally falls, but not always; when the thermometer is very high, and the barometer falls, thunder may be expected. Thunder-clouds rising from north-eastward, against the direction of the surface wind, do not usually cause a fall in the barometer, being borne onwards by a polar current.

The fall of the barometer denotes, in hot weather, high winds; in frosty weather it denotes a thaw.

If wet weather happen *soon* after the fall of the barometer, there will be but little of it.

In wet weather, if the glass falls, expect much wet.

In fair weather, if the mercury falls much and remains low, expect much wet in a few days, and likewise wind.

The barometer sinks lowest of all for wind and rain together; next for wind, except it blow from the north or north-east.

Our storms are generally indicated by a tendency of the wind towards east and south, and a fall of the barometer; the return of fine weather by a rising glass, and a pretty strong west wind, which becomes apparent in the motion of the clouds before it is felt below.

The barometer is highest in May and August; next to these, in June, March, September, and April. It is lowest in November and February; then in October, July, December, and January.

It should always be remembered, in endeavouring to forecast weather, that the barometric column stands highest with wind from the polar region, and lowest with wind from the direction of the equator; so that in the northern hemisphere the mercury stands highest with the wind from N. or NE., and lowest with wind from the S. or SW.; while in the southern hemisphere, it is highest with the wind from the S. or SE., and lowest with the wind from the N. or NW.

The barometer is subject to regular fluctuation, of small amount at certain times of the day, arising from variations

of temperature and the amount of vapour in the air. In Britain, the *maximum* height is at about ten o'clock in the morning and evening; and the minimum at about four o'clock in the morning and evening. Hence, should it fall between *four* and *ten*, when it usually rises, the fall is much more ominous than a similar or even greater fall between ten and four, when it generally falls slightly from ordinary causes.

Navigators should be aware that there is a belt round the earth, from five to ten degrees north latitude, where the range of the barometer is so small, that an instrument may be verified there as satisfactorily as by removal to the shore for comparison with a standard. In long voyages, where meteorological observations have been taken by untested instruments, attention to this fact may be of the utmost importance.

It should also be observed that the tides are affected by the pressure of the atmosphere, so that a *rise* of one inch in the mercury will cause a *fall* of about one foot in the height of the tide, and other changes proportionally. When, therefore, a vessel is about to enter a dock or harbour where the depth of water is little more than her draught, the state of the barometer should also be attended to, or serious consequences may ensue.

Winds of diurnal periodicity are, in many countries near the sea, known as land and sea breezes; there an aërial current begins to flow about eight or nine in the morning from the sea towards the land, and continues till about three P.M.; then a reverse current commences from the land to the sea, and continues during the night until sunrise next morning, when a calm ensues, and continues until the completion of a period of twenty-four hours from the occurrence of the preceding land-breeze. These currents in opposite directions can be easily accounted for when we consider the heating agency of the sun's rays. Land becomes hotter than water under an equal power of heating rays; whence it follows, that the surface of the earth becoming heated after sunrise, causes the ascent of an

atmospheric current, while the colder air from the surface of the sea flows into the land and supplies its place. But when the sun sets, the surface of the ground becomes, by radiation, cooler than that of the sea, and then the current is reversed.

Where currents of air on the earth's surface proceed constantly in the same direction as the trade-winds, it is evident there must be currents in different directions in the upper regions of the atmosphere, in order to restore the equilibrium; that there are such currents is proved both by observing the course of the clouds in the upper strata, and the direction of the wind on the summit of very high mountains, both of which are frequently found to proceed in a direction altogether different from the lower currents or surface winds: thus, at the top of the Peak of Teneriffe, the wind at the summit usually blows from SW., while that at its base is most frequently from the NE.

OF STORMS AT SEA.

It seems to be now generally admitted by the best authorities, that all storms at sea are more or less circular; and that, while they revolve round a centre, they also progress forward in a certain direction, or bear towards a fixed quarter of the compass. These storms have been called by the names of *Tornadoes*, *Trovadoes*, *Hurricanes*, *Typhoons*, &c., but are now most commonly called *Cyclones*. As these storms are very dangerous, it is of the utmost importance that seafaring men should be acquainted with their nature, with the signs of their approach, and the indications by which they may know that they have actually been overtaken by one.

Bad weather is often preceded by what fishermen call a 'ground-swell.' The reason of this is, that in a cyclone the action of the wind on the surface of the sea raises in every direction a swell, tangential to the circumference of the cyclone. The lateral swells, which run obliquely to the storm's track, continually pass over new regions of the ocean, and gradually subside in each locality; but that which runs directly ahead of the cyclone, being continually reinforced by the advancing

whirlwind, becomes stronger, and remains more persistent in direction than the others, and thus becomes a well-marked and trustworthy indication of the coming storm. Admiral Fitzroy, in his *Weather Book*, says that he has been warned telegraphically, from Valentia, in Ireland, of a coming gale by the heavy swell on the shore there, a day before the wind reached even that projecting point.

The indications of a coming storm are a threatening appearance of the clouds, accompanied by a falling barometer; shifting and unsteady winds, and a cross irregular sea, with the waves often running at right angles to the direction of the wind.

An intense red colour both of the sky and sea is often seen just before the coming on of severe cyclones, such as the Chinese typhoons, the Mauritius and Jamaica hurricanes, &c.; several striking and instructive examples of which are given in Piddington's *Sailor's Hornbook.*

When a ship has actually entered into a cyclone or hurricane, the wind increases, and the barometer falls rapidly; and in most directions of sailing, the wind will soon veer round to different points of the compass. Now, the skilful mariner will take advantage of this circumstance to enable him to sail clear of the centre or most dangerous part of the storm, and if practicable, to get altogether beyond its limits.

All cyclones or hurricanes in the *northern hemisphere* are what sailors call left-handed—that is, their rotatory motion is contrary to that of the hands of a watch when placed on its back with its face upwards, or the revolution of the wind is N. W. S. E. N.; while their progressive motion is first from east to west, curving towards the north, and then round towards the east, which is called their *recurve.* Now, when one of these storms is encountered, the first and most important thing to be determined is whether the centre of the storm is proceeding westward or northward, or if it has already commenced its recurve.

In order to this, let it be observed that the direction of the

wind, at whatever point of the circular storm he may be situated, will always be a tangent to that circle; consequently, if the sailor turn his face in the direction from which the wind blows, the centre of the storm will always be at right angles to that direction, and on the *right-hand side* of it: thus, if the wind is blowing from the NE., the centre of the storm will be SE. from the observer; or if the wind be from the south, the centre of the storm will be directly west of the observer. In the same manner, the direction of the centre may be determined at any other point in the circle of the cyclone. Now, if the centre of the cyclone were either fixed or were known to move always in the same direction, there would be no difficulty in sailing out of it, for it is evident, that sailing at right angles to the wind on the starboard tack, the ship would run, in either case, in a short time quite out of the seat or track of the storm. But in many instances this might be running away from the port to which she was bound, which is not advisable. The centre of the storm, however, is neither stationary nor does it progress always at the same rate nor in the same direction; it becomes, therefore, the first matter of importance to determine in what part of the cyclone circle the vessel is situated, and towards what point the centre of the storm is progressing.

In order to make this point clear, let us suppose a line drawn through the centre of the storm in the direction of its progress; conceive next a person situated on that line, and looking along it in the direction towards which the storm is moving; it is plain that part of that line will be a diameter of the circle of the cyclone or hurricane, and our first object should be to ascertain *whether the ship is situated in the right-hand or left-hand semicircle thus formed*, and second, to find *in what direction or towards what point the storm is moving.* In order to effect these objects, he may scud before the wind either on the port or starboard tack, till he perceives the direction of the wind to veer round, which will enable him to find a new bearing of the centre of the storm; or he may lie to with close-reefed sails, and allow the storm to pass—in this

case also the wind will soon veer round, and enable him again to determine the bearing of the storm's centre from his position; or lastly, he may beat up against the storm, close hauled, either on the port or starboard tack, till in this case also the wind is found to have veered round at least one point of the compass.

Suppose, then, that a ship has got within the limits of one of those circular storms in the northern hemisphere, with the wind blowing strong from SW., and that the method of scudding before the wind is adopted; and that after running N. by E. on the port tack for a distance of 30 miles, the wind is found to have veered round to SW. by S., and to be blowing with increased fury. It is plain that the centre of the storm now bears NW. by W. from the ship; and as the force of the wind has increased, it is in all probability at less distance than before—say in the former case 100 miles, and in the latter, 80. It will now become evident, by projecting a plan of the ship's course and distance run, with the bearing and supposed distance of the storm's centre, that the ship is in the right-hand semicircle, and the storm is progressing towards the E. by N. $\frac{1}{2}$ N. nearly, and has therefore in all probability commenced its recurve. Were the ship then to continue on the same course, she might perhaps cross the storm's track, and get to the left-hand semicircle before the centre of the storm came up; but it appears fully as likely that she would run into the very centre of the evil which it is most desirable to avoid. Under these circumstances, it appears that the safest plan would be to change the course, and to beat up against the wind on the starboard tack, or on the same tack to run at right angles to the wind, by either of which plans she would soon be run beyond the circuit of the storm, and might then resume her course to the desired port.

The above may be wrought very simply, by means of the Traverse Table, as follows: Since the centre of the storm at the first position bore NW., distant 100 miles, it was then N. 70·7 miles and W. 70·7 miles from that of the ship's position; the ship then sailed, by the same Table, north 29·4 miles, and east 5·9 miles, and the centre, from this second position, lay north

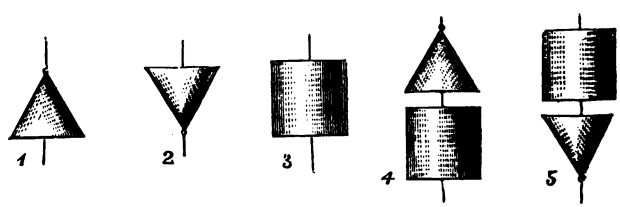
44·4 miles and west 66·5 miles, so that it was north of the first station 73·8 miles, and west of the same 60·6; the centre had therefore moved 3·1 miles towards the north and 10·1 miles towards the east, or N. 73° E. a distance of about 10½ miles. In this manner, both the rate and direction of the motion of the storm's centre may be estimated; but their accuracy depends, to a great degree, on the correctness of the two estimated distances of the storm's centre from the first and second positions of the ship.

The cyclones or hurricanes in the *southern hemisphere* are, on the contrary, what sailors call right-handed, or they revolve in the direction of the hands of a watch placed on its back with its face upwards, or the revolution is N. E. S. W. N.; while their progressive motion is first from east to west, then curving towards the south, and finally veering round to the east, which is called their *recurve*. Here, as in the northern hemisphere, the first thing to be determined is, In what semicircle of the cyclone is the ship situated? and the second, In what direction is the body of the storm moving? This will be determined here in the same manner as was formerly shewn in the northern hemisphere. It is of importance, however, to remark here, that since the storm now revolves in the opposite direction, when the mariner looks towards the direction from which the wind blows, the centre of the storm will still be at right angles to that direction, but now on the *left-hand* side instead of on the *right*, as was shewn to be the case in the northern hemisphere. With the above difference, the method of determining the two principal problems is the same in both hemispheres.

Suppose now that the mariner has reason to believe that he has got into the limits of a cyclone or hurricane in the southern hemisphere, and he wishes to determine in what direction the storm is progressing, and in which semicircle his vessel is situated. Let the wind be from the NNE., then it is plain that the centre of the storm bears WNW.; suppose he determines to beat up against the wind on the port tack, and that after he has run E. by. N 40 miles, the wind has somewhat

abated and is blowing from N. by E., from the force of the wind he assumes that the centre of the storm was at the first position distant 100 miles, and at the second 120 miles. By the Traverse Table, the centre was, at his first position, N. 38·3 miles and W. 92·4 miles; he has since run N. 7·8 miles and E. 39·2 miles, while the centre of the storm is still W. by N. distant 120 miles, and therefore N. 23·4 miles and W. 117·7 miles, and therefore is from the ship's first position N. 31·2 miles and W. 78·5 miles; it has therefore moved S. 7·1 miles, and E. 13·9 miles; it has therefore moved S. 63° E. a distance of about 15½ miles. The ship is therefore in the left-hand semicircle, at a great distance from the centre, which appears to be moving slowly; the ship may therefore easily escape from the fury of this storm.

The principles stated above are so simple, and at the same time so universally applicable, that it appears quite unnecessary to give more examples of their application. Those who choose to solve the problem by projection, will find no difficulty in doing so from the directions given above, or they may find several illustrative examples in Captain James Sedgwick's *True Principles of the Law of Storms.* On the other subjects treated in this chapter, additional information may be found in a compact and cheap *Manual of Weather-casts and Storm Prognostics*, by Andrew Steinmetz (Routledge, London), a work to which we are indebted for much of the information given above.



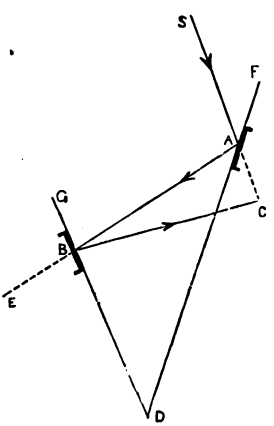
Storm Signals.

Explanation of Storm Signals.—1. Probable gale from the north; 2. Probable gale from the south; 3. Dangerous winds from nearly opposite quarters; 4. A dangerous wind expected, its first direction from the north; 5. A dangerous wind expected, its first direction from the south.

SEXTANT.

THE sextant is a reflecting astronomical instrument for measuring angles, of very extensive use in Navigation, since it does not require to be fixed, but may be used to measure angles in any plane. It derives its name from the length of its graduated arc being the sixth part of the circumference of a circle; but this arc being graduated doubly fine, enables the instrument to measure angles up to 120°.

A similar instrument, whose graduated arc is the eighth part of a circle, is also extensively applied for the same purpose, and should on the same principle be called an octant; but the name by which it is universally known is 'Hadley's Quadrant,' because it was invented by Hadley, and measures the fourth part of the circumference, or an angle of 90°.



The principle on which the construction of both these instruments depends is a theorem in optics, that *if a ray of light suffer double reflection, the angle between the original direction of the ray, and its direction after the second reflection, is double of the angle between the position of the reflecting surfaces.*

Thus, let A and B be two mirrors perpendicular to the same plane, and inclined to each other by the amount of ∠ D, and let SA be a ray of light, which, falling on mirror

A, is reflected to mirror B, and thence in the direction BC; produce SA to meet BC in C, and produce AB to E; then ACB is the angle between the original and second reflected rays, and ADB is the angle of inclination of the two mirrors; it is now our object to prove that the angle ACB is double of the angle ADB.

Now, as the angle of reflection is equal to the angle of incidence, their complements are also equal; hence

$$\angle BAD = \angle SAF, \text{ and } \angle DBC = \angle GBA.$$

But since the exterior angle of a triangle is equal to the sum of the two interior angles,

$$\begin{aligned} \angle EBC &= \angle BAC + \angle ACB \\ &= \angle BAD + \angle DAC + \angle ACB \\ &= \angle BAD + \angle SAF + \angle ACB \\ &= 2 \angle BAD + \angle ACB. \end{aligned} \quad (1)$$

Again $$\begin{aligned} \angle EBC &= \angle EBD + \angle DBC = \angle GBA + \angle DBC \\ &= 2 \angle EBD = 2 \angle BDA + 2 \angle BAD. \end{aligned} \quad (2)$$

Equating the values of the $\angle EBC$ in (1) and (2), we have

$$2 \angle BAD + \angle ACB = 2 \angle BAD + 2 \angle BDA;$$

and taking $2 \angle BAD$ from each of these equals, there remains

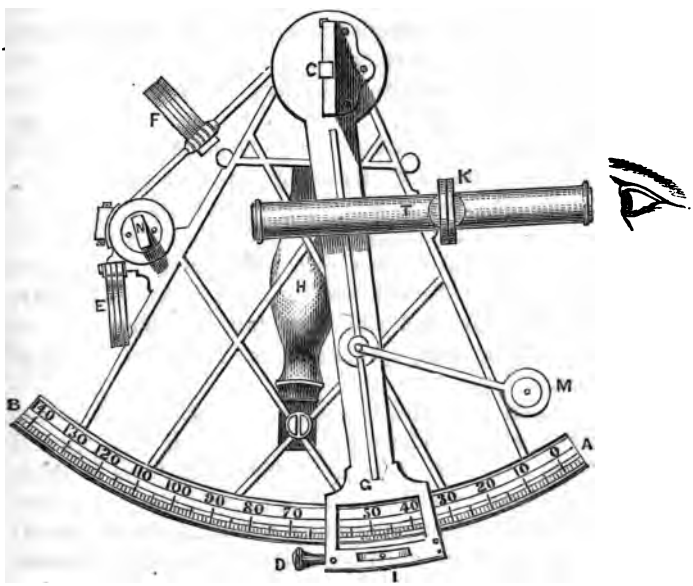
$$\angle ACB = 2 \angle BDA.$$

Now, $\angle BDA$ is the angle measured on the graduated arc of the sextant, and hence every half degree in it measures a whole degree in the angle sought, which is the reason why the arcs of the sextant and quadrant are graduated double, or one degree for every half on the arc: by this means the true angle is read at once.

The adjoining figure represents a sextant, the frame being made of brass or other hard metal, while the handle, H, at its back, is made of wood. In taking an observation, the instrument is to be held with one hand by the handle, while the other hand moves the *index-arm*, G. The arc AB is divided into 120 or more degrees, numbered from A towards B, and each degree, in the best instruments, is again subdivided into six

equal parts of 10′ each; while the vernier, which has 59 of the equal parts on the arc divided into 60 equal parts, makes the instrument capable of measuring angles to 10″.

The divisions are also continued a short distance on the other side of zero, towards A, forming what is called the arc of excess.



Sextant.

The microscope, M, is movable about a centre, and may be adjusted to read off the divisions on the graduated limb and vernier. When the index is to be moved any considerable distance, the clamp-screw, I, at the back of the index, used for fixing the index to the arc, must be loosened; and when by the hand the index is moved so as to make the contact required nearly, the screw, I, is again to be fixed; then, the contact is to be made perfect by means of the tangent screw, D, which is used for making the contact more accurate than can be made by the

hand. The upper end of the index, G, terminates in a circle, the centre of which is over the centre of motion, across which is fixed the silvered index-glass C, over the centre of motion, and *perpendicular* to the plane of the instrument. To the frame, at N, is attached a second glass, called the horizon-glass, the lower half of which only is silvered. The horizon-glass must also be perpendicular to the plane of the instrument, and in such a position that its plane shall be parallel to the plane of the index-glass, C, when the zero of the vernier is set to the zero on the limb, AB.

The telescope, T, is carried by a ring, K, attached to a stem, which can be elevated or depressed by turning a milled screw, whose use is to place the telescope so that its field of view may be bisected by the line on the horizon-glass that separates the silvered from the unsilvered part, or opposite the unsilvered part of the horizon-glass. In the telescope are placed two wires, parallel to each other, and also other two at right angles to the former, each pair being equidistant from the centre or axis of the telescope.

Four dark glasses of different depths of shade and colour are placed at F, which can be turned down between the index and horizon glasses; also three more at E, any one or more of which can be turned down to moderate the intensity of the light before reaching the eye when observing a bright object, such as the sun.

Having described the various parts of the sextant, we shall next give some directions for detecting the defects in its construction, if any exist, and for which it ought to be rejected.

In selecting an instrument, observe if all the joints of the framework be perfect, close, and tight; if all the screws act well and remain steady though the instrument be shaken. Test the accuracy of the centring, by observing the distance between two stars, and compare the results with the same distance observed with a standard sextant, or with the distance obtained by calculation.

The graduations on the *limb* and *vernier* should be perfect: this may be tested by making the zero of the vernier coincide

accurately with a line on the arc, then also the last line on the vernier will coincide with a line on the arc; if this condition be fulfilled, whatever part of the arc is taken, the graduations are correctly made, or uniform throughout; and if it also stand the previous test, they are also of the proper magnitude, and may be relied on. It should also be examined carefully to see if the inlaid rim on which the graduations are made be quite level with the surface of the instrument, and if the arc of the vernier coincides uniformly throughout the whole length with the graduated arc on the limb. If in any of the above tests the instrument appear defective, it should be rejected.

All the glasses used in the sextant should be perfect. The index and horizontal glasses should be free from veins, and each should have its two faces ground and polished parallel to one another. To test whether this is the case, look with a small telescope into each reflector separately, in a very oblique direction, and observe the image of some well-defined distant object; this image should appear clear and distinct in every part of the reflector, and well defined, not misty, nor notched about the edges. In like manner, the glass of the coloured shades should be of the best quality, and each shade have its two faces ground and polished parallel to one another. Each set of shades should also work with all the faces parallel. To examine the perfection of the shades, having fitted the dark glass to the eye-end of the telescope, make a contact of the reflected and direct images of the sun, without using any of the shades; fix the index with the contact perfect, and removing the telescope shade, turn down the others singly, and also in combination: if in all these cases the contact continue perfect, the shades are good; but if it do not, the shades should be replaced by perfect ones; or if this cannot be done, the errors of each shade should be recorded, and also that of each combination, and applied to the observations made while they are used with their proper sign, under the name of *shade errors*.

There are *four* adjustments of the sextant to be attended to, in order to obtain correct results in making observations with it.

1*st*, *To make the index-glass perpendicular to the plane of the sextant.*—Move the index forward to about the middle of the limb; then, hold the instrument with the divided limb from the observer, and the index-glass to the eye, and look obliquely down the glass, so as to see the circular arc by direct vision and by reflection in the index-glass at the same time; and if they appear as one continued arc of a circle, the index-glass is adjusted. If the arc appear broken where the reflected and direct parts of the limb meet, it requires to be corrected. But as this glass is, in the first instance, set right by the maker, and firmly fixed in its place, its position cannot be altered, except by violence or accident, and therefore no direct means are supplied for its adjustment.

2*d*, *To set the horizon-glass perpendicular to the plane of the sextant.*—Screw in the telescope, T, and point it towards the sun or a star. Move the index-arm backwards and forwards past the zero of the limb, and if the two images of the sun or the star do not exactly coincide in passing one another, turn a screw at the top or bottom of the horizon-glass, N, until this coincidence is effected.

3*d*, *To set the axis of the telescope parallel to the plane of the sextant.*—There are two parallel wires on opposite sides, and equidistant from the centre of the field of view of the telescope, and these are usually crossed by two others. Turn either pair round until they are parallel to the plane of the instrument. Select two stars distant from each other from 90° to 120°, and bring them into contact just at the wire of the telescope which is nearest to the plane of the sextant. Fix the index, and alter the position of the instrument so as to make the objects appear on the other wire. If the contact still remains perfect, the axis of the telescope is in proper adjustment; if not, it must be altered by moving the two screws which fasten, to the up-and-down piece, the collar into which the telescope screws. It is usual to employ the sun and moon, or the moon and a star, in a similar manner in testing this adjustment. This adjustment is not very liable to be deranged.

4th, To find the index error of the sextant.—When the zero of the vernier is set to the zero on the limb, the index and horizon-glasses should be parallel, and if the telescope be then directed to the sun or a star, the reflected and direct images should exactly coincide. If the two images do not coincide, this deviation constitutes what is called the *index error*. The amount of the index error may be found in the following manner. Turn the tangent screw of the vernier until the images of the sun or star do coincide; then the minutes and seconds read will be the index error, marked *plus* when the reading is on the arc of excess, and *minus* when it is on the arc: or, clamp the index at about 32 minutes to the left of zero, and, with the plane of the instrument so situated as to observe the sun's *horizontal* diameter, look toward the sun, the two images will appear either nearly in contact, or overlapping each other. Then perfect the contact by moving the tangent screw, and call the minutes and seconds denoted by the vernier, the reading *on* the arc. Next place the index about the same distance to the right of zero, or on the arc of excess, and make the contact of the two images perfect as before, and call the minutes and seconds on the arc of excess, the reading *off* the arc. Half the difference of these numbers is the index error; *additive* when the reading on the arc of excess is *greater* than that on the limb, and *subtractive* when the contrary is the case.

Example I.		**Example II.**	
Reading *on* the arc	32′ 12″	Reading *on* the arc	30′ 52″
Reading *off* the arc	31′ 48″	Reading *off* the arc	33′ 18″
Difference	0′ 24″	Difference	2′ 26″
Index error	− 0′ 12″	Index error	+ 1′ 13″

NOTE.—Should the index error be so great that both the readings are *on* the arc, or both *off* the arc, that is, both on the arc of excess; then in the former case, half the sum of the readings is the index error *subtractive;* and in the latter case, half the sum of the readings is the index error *additive.*

Example I.		**Example II.**	
1st reading *on* the arc	65′ 24″	1st reading *off* the arc	1′ 28″
2d reading *on* the arc	1′ 32″	2d reading *off* the arc	64′ 54″
Sum	66′ 56″	Sum	66′ 22″
Index error	− 33′ 28″	Index error	+ 33′ 11″

When the index error is large, the adjusting screw, if one is fitted, should be turned to reduce or destroy the error.

To take an altitude of the sun by means of the natural horizon.—If the observer be at sea, the natural horizon must be employed. Direct the sight to that part of the horizon beneath the sun, and, interposing the necessary index shades, move the index till you bring the reflected image of his lower limb to touch the horizon directly underneath him; but as this point cannot be exactly ascertained, the observer should move the instrument round to the right and left a little, making the axis of the telescope the centre of motion. By this means the sun will appear to sweep the horizon, and must be made to touch it at the lowest point of the arc. The upper limb can be used when the lower limb is obscured by cloud. Then the reading on the arc corrected for *index error*, *dip*, *refraction*, *parallax*, and *semidiameter*, will give the true altitude of the sun's centre. In the same manner the altitude of any other celestial object is to be taken.

To measure the altitude of the sun by reflection from mercury, oil, treacle, or any reflecting liquid—that is, by means of an artificial horizon.—Set the index near zero. Hold the sextant with the right hand in the vertical plane of the sun, towards which the telescope should be pointed. Two images will be seen in the field of view, one of which—namely, that formed by reflection—will appear to move downward when the index is pushed forward. Follow the reflected image as it travels downward, until it appear to be as far below the horizon as it was at first above it, and the image of the sun, reflected from the mercury, also appears in the field of view. Then fasten the index, and by means of the tangent screw, bring the upper or lower limb of the sun's image reflected from the index-glass, into contact with the opposite limb of the image reflected from the mercury, taking care that the limbs in contact shall be midway between the parallel wires. The angle shewn on the instrument, when corrected for the index error, will be double the altitude of the sun's limb above the horizontal plane; to the half of which, if the refraction, parallax, and

semidiameter be applied, the result will be the true altitude of the sun's centre. In the same manner may be observed the altitude of any other celestial body, by means of an artificial horizon.

To find the distance between the moon and sun, or between the moon and a star, by the sextant.—Hold the sextant so that its plane may pass through the sun and moon. If the sun be to the right of the moon, the sextant is to be held with its face upward; if to the left, the face is to be held downward. With the instrument in this position, look directly at the moon through the telescope, and move the index forward till the sun's image is brought nearly into contact with the moon's nearest limb. Fix the index by the screw, I, under the sextant, and make the contact perfect by means of the tangent-screw, D. At the same time, move the sextant slowly, making the axis of the telescope the centre of motion; by which means the images will pass each other, and the contact be more accurately made; observing that the point of contact of the limbs must always be in the middle of the field of view between the parallel wires. The index will then point out, after correcting for the index error, the apparent distance of the nearest limbs of the sun and moon. In a similar manner may the distance between the moon and a star be measured; but in doing so, the contact must be made with the moon's *enlightened* limb, whether that be the nearer or the more remote. The true distance can then be found as directed in Arts. 372 and 373.

FURTHER DIRECTIONS FOR USING THE SEXTANT.

When the brilliancy of two objects under observation is very unequal, the telescope should be brought opposite the unsilvered part of the horizon-glass by the large milled-headed screw attached to the telescope ring for that purpose, as the brilliancy is rendered more nearly equal by this means.

In testing the adjustments, the great heat of the sun will affect the sextant, if it be kept too long in the sunshine.

The various powers of the telescope should be practised with, as a good observation often depends on using a suitable power.

In observing an angle between objects in motion, the best contact is got by setting the sextant a little in advance, and waiting the moment when the contact occurs.

In measuring lunar distances, it is sometimes an advantage to bring the objects nearly in contact without using a telescope, and to perfect the contact after the telescope is screwed in.

To insure accuracy in observing, the observer must place himself in a comfortable position, and grasp his sextant firmly, but not too tightly.

The screws, tubes, &c. should be turned, pushed out and in, &c. as little as possible, and the sextant should always be held by the handle, or, in getting it out of or into its box, it should be held as near its centre of gravity as possible.

OF A SHIP'S JOURNAL OR LOG-BOOK.

A SHIP'S JOURNAL is a regular and exact register of the various transactions which take place on board, whether at sea or in a harbour, and more particularly those which relate to the ship's track—from which her place at noon, or at any other time, may be truly ascertained.

The account of the ship's way, &c. kept at sea, is called *sea-work;* and the remarks taken down while the ship is in port, are called *harbour-work.*

The computations made to determine the ship's place, from the course and distance run in 24 hours, or from one noon to the following, are called a *day's work;* and the latitude and longitude of the ship thus deduced, are called the latitude and longitude *by account,* or by *dead reckoning* (indicated by D.R.), in order to distinguish them from the latitude and longitude deduced from observation. The latter is always taken as the true place of the ship, from which by D.R. her place is calculated, till another observation can be favourably obtained.

When a ship is bound to a port so situated that, in running to it, she will be out of sight of land, the bearing and distance of the port must be previously found. This may be done by Mercator's or Middle-latitude Sailing, or, if the shortest course be sought, by Great-circle Sailing; it may also be found by construction on the chart. But as all these methods—the chart sometimes excepted—give the true bearing, and not that by the compass, the variation and deviation must be applied to the *right* if they be *westerly,* but to the *left* if they be *easterly.* If islands, capes,

or headlands intervene, it will be necessary to find the several courses and distances between each successively.

When the ship is about to leave sight of land, the bearing and distance are taken of some prominent object whose latitude and longitude are known; this can be done most accurately by taking two bearings of the same object, and the distance and direction the ship has sailed between the observations: by this means a triangle is formed whose three angles are known, and also the distance between the two positions of the ship, whence her distance from the object at either position can easily be calculated. This is called taking a departure; and the bearing and distance of the object are then set down in the column of remarks on the log-board, on which board the other courses and distances made during the day are also entered in their proper columns. In the column of remarks should also be inserted the setting and drift of currents, the estimated effect of the swell of the sea, the kind of weather experienced, and particularly the variation of the compass, with any other occurrence that may be deemed of importance. The leeway is generally inserted in a column kept for the purpose, unless, as is sometimes the case, the officer of the watch corrects the course for leeway before entering it on the log-board. The particulars on the log-board are transcribed every day at noon into the *log-book*, which is ruled and divided in the same manner as the log-*board*.

An easy and accurate method of determining the quantity of the leeway is to have a semicircle described on the taffrail, with its diameter at right angles to the ship's keel, and its circumference divided into points and quarter points, and having a low crutch or swivel in its centre. Then, after the log has been heaved, and is about to be drawn in, let the line be slipped into the crutch, then the points and parts of a point between the direction of the log-line and the fore-and-aft line of the semicircle will be the leeway. The leeway may also be found by observing the direction of the ship's wake on the same semicircle.

The following rules are generally given by writers on navigation for inferring the quantity of leeway, independently of observation, and may be useful for finding it when the more accurate method of observing it has been neglected at the time.

1. When a ship is close-hauled, has all her sails set, with the water smooth, and only a light breeze of wind, she is then supposed to make little or no leeway.

2. Allow one point when the top-gallant sails are handed.

3. " two points when under close-reefed top-sails.

4. " two points and a half when one top-sail is handed.

5. " three points and a half when both top-sails are handed.

6. Allow four points when the fore-course is handed.

7. " five points when under the main-sail only.

8. " six points when under the balanced mizzen.

9. " seven points when under bare poles.

As these rules refer only to the quantity of sail set, without having any regard to the form of the ship, or the nature of the cargo on board, it is hence evident they are only approximate conjectures; yet they may be of some use when, through negligence or otherwise, the quantity of the leeway has not been ascertained from observation.

The leeway, when determined by any of the above methods, is then to be applied to the *right* of the apparent course, when the ship is on the *port* tack; and to the *left*, when she is on the *starboard* tack.

In a violent gale, with a head-wind and heavy sea, when it would be dangerous to carry sail, it is usual to *lie-to* under sufficient sail to prevent the vessel from rolling so much as to endanger the masts and rigging. When a ship is lying-to, the tiller is put over to leeward, and when the ship has headway, the rudder acts upon her to bring her to the wind; the ship then loses her way in the water, which ceasing to act on the rudder, her head falls off from the wind, and the sail which is set fills and gives her fresh way through the water, which, acting on the rudder, brings her head again to the wind. Thus the ship is

kept continually falling off and coming to. In this case, you must observe the points on which she comes up and falls off, and take the middle point between the two for the apparent course, to which apply the variation and leeway, and you will obtain the true course. Allow the drift of the vessel as a distance.

As this is a case which generally presents considerable difficulty to pupils, we will illustrate it by the following example:

A ship lying-to under her mainsail, on the starboard tack, comes up E. by S., and falls off NE. by E., there being one point westerly variation, and she makes 5 points leeway; what course does she make good?

The middle between E. by S. and NE. by E., is E. by N.; and by allowing 6 points to the left hand (namely, 5 points for leeway and 1 for westerly variation), the true course will be obtained N. by E.

Notwithstanding all the care that may be taken in keeping a sea-journal, the place of the ship as deduced from it can only at the best be considered as *approximately* determined. It is often extremely difficult to arrive at any close certainty respecting the precise course and distance which a ship has actually made. Different rates of sailing between the times of heaving the log, want of care in steering, and sometimes also the great difficulty in steering steadily; sudden squalls, incorrect allowances for leeway and variation, and many other circumstances, such as currents and drift of the sea, conspire to render the ship's place, as deduced from the common reckoning, very uncertain. Hence no opportunity should be lost of determining her place by celestial observations; and the sea reckoning should be chiefly considered as a means of estimating her situation *nearly* in the interval between such observations.

The log is dated according to the civil reckoning of time, and the place of the ship therefore, computed at noon, is for the middle of the civil day; the courses and distances in the afternoon, or P.M., of the preceding day, being connected with those made in the morning, or A.M., of the current day, in computing the change of her place from the last noon.

When the place of the ship has been determined by observation, her place by D.R. should be disregarded, and the future reckoning should be carried forward from her place found by observation, till other observations give a new point of reference. Some mariners, however, keep separate accounts of the ship's place by D.R. and by observation, during the whole voyage; which they consider enables them to judge of the total effect of currents and other generally operating causes of error on the reckoning.

The rules necessary for working a day's work have all been given already in this treatise; but for convenience of reference we here collect them into one section.

RULES FOR WORKING A DAY'S WORK.

1. Correct the several courses sailed for variation and leeway, and enter the corrected results in a Traverse Table, and opposite each course place the distance run on that course, found by summing up the knots and tenths sailed by the ship on that course. Find in the 1st or 2d tables of Difference of Latitude and Departure, the dif. lat. and dept. corresponding to each course and distance, and place them in their respective columns; then the difference between the sums of the northings and southings will be the difference of latitude made good of the same name with the greater; and the difference between the sums of the eastings and westings will be the departure made good of the same name with the greater quantity.

NOTE.—The setting and drift of a current (if there be any) is to be entered in the traverse as a course and distance; and on the first day after sight of land, the bearing and distance of the land is to be used as the first course and distance, with the opposite point of the compass, corrected for variation, as a course, and the real or estimated distance as a distance.

2. Seek in the same tables until the above dif. lat. and dept. are found together in their respective columns; opposite to these, in the dist. column, will be the distance made good, and at the top or bottom of the page, according as the dept. is less or greater than the dif. lat., will be found the course.

3. If the latitude from which the ship's departure is taken, or

yesterday's latitude, be of the same name as the dif. lat. found, add them together; but if of different names, take their difference; the sum or remainder will be the present latitude, of the same name as the greater.

4. Find the middle latitude between the latitude of yesterday and this day, which take as a course in Table II., and seek for the dept. in the column of Dif. Lat.; then will the corresponding distance be the *difference of longitude*, of the same name as the departure found above.

5. If the longitude of yesterday be of the same name as the dif. long., add them together; but if of different names, take their difference; the sum or remainder will be the long. in of the same name as the greater.

6. If an observation for longitude were taken at any time of the day, you must find, by the above method, the difference of longitude made since taking the observation, and thence the longitude in at noon by that observation, and enter it in the journal as the longitude by observation.

7. On a general chart, find the point corresponding to the latitude and longitude by observation, and that place will represent the position of the ship, whence the bearing and distance of the intended port may be found. The same may be obtained by Middle-latitude Sailing by inspection of Table II., thus: Find the *true* middle latitude between the place of the ship and the place bound to, and seek for that latitude as a course in Table II., and find in the corresponding page of the table the difference of longitude (between the ship and the proposed place) in the Distance column, opposite to which, in the Latitude column, will be the departure. Seek in the same table or the preceding for this departure and the difference of latitude (between the ship and the proposed place) till they are found together; corresponding thereto will be the bearing and distance required. If the magnetic bearing be required, the variation must be allowed on the true bearing; to the right if the variation be westerly, but to the left hand if easterly.

We shall now proceed to exemplify the above rules by a

few examples of separate days' work, arranged in the usual form.

Example I.

Yesterday, at noon, we were, by observation, in lat. 14° 6′ 12″ N., long. 22° 55′ 30″ W.; we have since sailed, as contained in the following log-book: required the present latitude and longitude of the ship by D. R., and the course and distance made good.

LOG-BOOK.

Hours.	Knots.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
1	4	5	S. by W.	East.	0	P.M. Light, moderate wind;
2	4	5				clear weather.
3	4	5	South.	..	½	
4	5	0	..	..	..	4 h. Steady, light breeze—
5	5	0				all possible sails set.
6	5	0				
7	5	0				
8	4	5	..	..	..	8 h. Steady, light breeze; clear
9	5	0				atmosphere.
10	5	0				
11	5	0				
12	5	0				
1	5	0	South.	NNE.	0	Midnight. Steady, light
2	5	0				breeze; fine weather.
3	5	0				
4	5	0	..	..	..	4 h. A.M. steady weather—
5	5	0				all sail set.
6	5	0				
7	5	0				
8	5	0	..	..	..	8 h. All sail set, pleasant wea-
9	5	5	S. ¼ W.	NE.	0	ther. Bar. 29·87; ther. 88°.
10	5	5	..	..	..	Variation of compass, 1¾ W.
11	6	0				
12	6	0	..	..	..	Noon. Steady trade-wind.

Courses.	Dist.	N.	S.	E.	W.
S. ¾ E.	9	..	8·9	1·3	..
S. by E. ¼ E.	49	..	47·5	11·9	..
SSE. ¼ S.	40	..	37·7	13·5	..
S. by E. ½ E.	23	..	22·0	6·7	..
			116·1	33·4	

Lat. from 14° 6′ 12″ N., mer. pts. = 854·9
Dif. lat. 116·1, 1 56 6 S.

Lat. in 12 10 6 mer. pts. = 733·7

Sum of Lats. 26 16 18
Middle lat. 13 8 9
Meridian difference of latitude = 121·2

By inspection in table (page 375), the dif. lat. 116·1 and dept. 33·4 are found to correspond to a course of 16°, and a distance of 121 miles nearly; and on the same page, opposite to the meridian difference of latitude in the Dif. Lat. column, is

found in the Dept. column by Mercator's Sailing, the dif. long. = 34·7.

∴ Dif. longitude	=	0° 34′ 42″ E.
Long. yesterday at noon	=	22° 55′ 30″ W.
Hence longitude in	=	22° 20′ 48″ W.

Again, with the middle latitude as a course, and the departure as a dif. lat., we find in the Distance column 34·3 for the difference of longitude, which is less than that found by Mercator's Sailing by 24″.

The first course inserted in the above traverse table is got by applying the variation 1¾ W., given in the remarks, to the left hand of the apparent course, since it was westerly, which gives S. ¾ E., the leeway being 0. And since she sailed for two hours on that course, the distances sailed during these two hours are added together, which gives the 9 miles opposite it in the Distance column.

In the second course the ship sailed south, and made ½ point of leeway, which, since the wind is from the east, must be allowed to the right-hand side of the course, the ship being on the port tack; while the variation 1¾ W. must be applied to the left-hand side, making together 1¼ points to be applied to the *left* of south, which gives S. by E. ¼ E.; and as she sailed for 10 hours on that course, the sailings in these 10 hours are added together, which gives the distance placed opposite this course 49 miles.

Since there was no leeway during the third course, the variation 1¾ points W. is applied to the *left* of the south, which gives SSE. ¼ S., and the distances sailed during the 8 hours that she continued on that course, gives the whole distance sailed 40 miles.

In the fourth course, the variation is applied to the *left*, making 1½ points to the left of south, which gives S. by E. ½ E.; and the distances sailed in the four hours on this course being added, gives the whole distance 23 miles.

Example II.

Yesterday, at noon, we were in latitude 35° 46′ N., and

longitude 17° 42′ W., and have sailed till this noon as in the following log-board: required the latitude and longitude in, and the bearing and distance to Cape St Vincent, in latitude 37° 3′ N., and longitude 9° 1′ W.

LOG-BOARD.

H.	K.	10ths.	Courses.	Winds.	Leeway.	Remarks.
1	6	6	S. by E. ½ E.	SW. ½ W.	1½	
2	6	6	..	..	..	2 h. P.M. Moderate gales and clear weather.
3	5	8				
4	5	8				
5	5	8				
6	5	8				
7	5	8				
8	5	8	..	..	..	8 P.M. Pleasant favourable breeze.
9	5		SSE.	SW.	1½	
10	5					
11	5	2				
12	5	2	..	..	..	Midnight. Clear weather and moderate gale.
1	5	3				
2	5	3				
3	5	5	SSE. ½ E.	SW. by S. ½ W.	1½	
4	5	5				
5	5	5				
6	5	5				
7	5	5				
8	5	5	..	..	..	
9	5	6	SE. by S.	SW. by S.	1½	8 A.M. Sighted a ship to windward steering east.
10	5	6				
11	5	4				
12	5	4	..	..	..	Variation 1½ points westerly.

Courses.	Dist.	N.	S.	E.	W.
SE. ½ E.....	48	..	30·5	37·1	..
ESE. by E....	31	..	17·2	25·8	..
SE. by E.½E.	33	..	15·6	29·1	..
ESE.....	22	..	8·4	20·3	..
			71·7	112·3	

The courses being corrected for leeway and variation, and the distances sailed on the several courses summed up, will stand as in the adjoining Traverse Table.

Hence, the difference of latitude made good is 71·7 or 72′ S., and the departure 112·3 E.; consequently the course made good is S. 57½ E., and the distance run 133 miles.

Latitude left	=	35° 46′ N.
Dif. lat. 72′	=	1° 12′ S.
Latitude in	=	34° 34′ N.
Sum of lats.	=	70° 20′
∴ Mid. lat.	=	35° 10

With the mid. lat. 35°, and dep. = 112·3, the dif. long. is found to be 137 miles.

Longitude left	=	17° 42′ W.
Dif. long. 137′	=	2° 17′ E.
∴ The longitude in	=	15° 25′ W.

To find the bearing and distance of Cape St Vincent.

Latitude in	34° 34′	Mer. parts =	2212·7	Long. in	15° 25′ W.
Cape St Vincent's lat.	37° 3′	Mer. parts =	2396·4	C. St Vincent long.	9° 1′ W.
Dif. lat. =	2° 29′	=	140		
	Mer. dif. of latitude =		183·7	Dif. long. =	6′ 24′ = 384′

Hence, by inspection, the course to Cape St Vincent is N. 64½° W., and distance 346 miles nearly. But, by calculation, it is found to be N. 64° 26′ W., and distant 345·26 miles.

Example III.

Yesterday, at noon, we were in latitude 49° 10′ N. and longitude 24° 23′ W., and have sailed till this day at noon as in the following log-board; required the course and distance made good, with the latitude and longitude in.

LOG-BOARD.

H.	K.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
2	6	..	SW. by W. ¾ W.	N.	0	2 h. P.M. Moderate gale, cloudy weather.
4	5	5				
6	5	..	..	NW.	1½	4 h. P.M. Spoke the ship *Washington* from New
8	5					
10	3	6	SW. ¾ W.			York, bound to Cork.
12	3	4				
2	3	5				
4	4	5				
6	4	5	..	..	..	6 h. A.M. Stowed the anchors and unbent the cables, and coiled them between decks. Variation 2 points westerly.
8	5	..	SW. ¼ S.	WNW.	¾	
10	4	5				
12	4	..	..	..	..	

TRAVERSE TABLE.

Courses.	Dist.	N.	S.	E.	W.
SW. ¼ S....	23	..	17·0	..	15·4
SSW. ¼ W..	20	..	17·6	..	9·4
S. by W. ¼ W.	39	..	37·3	..	11·3
S. by W.....	27	..	26·5	..	5·3
			98·4		41·4

In this example, since the rate of sailing per hour is only inserted in the log-board each alternate hour, the distance found by adding for each course is *doubled* before it is put in the Traverse Table as the distance sailed.

The first course has 2 points applied to the left to correct it for westerly variation; the second and third have 3¼ points applied to the left to correct for variation and leeway; and the fourth course has 2¾ points, the sum of the variation and leeway, applied to the left.

The difference of latitude being 98·4, and departure 41·4 miles; by inspection the course is S. 23° W., and distance made good 107 miles nearly.

Difference of lat. 98′	=	1° 38′ S.	With the middle latitude as a course, and the departure 41·4 as a dif. lat., we find		
Latitude from	=	49° 10′ N.	Dif. longitude 62′	=	1° 2′ W.
∴ Latitude in	=	47° 32′ N.	Longitude from	=	24° 23′ W.
Sum of latitudes	=	96° 42′	∴ Longitude in	=	25° 25′ W.
∴ Middle latitude	=	48° 21′			

Example IV.

Yesterday, at noon, we were in lat. 37° 2′ N., longitude 16° 24′ W., and have since sailed as in the following log-book; required the latitude and longitude in, and the course and distance made good.

LOG-BOOK.

H.	K.	10ths.	Courses.	Winds.	Leeway.	Remarks.
1	9	5	ENE.	Southerly.	..	P.M. Moderate breeze and fine weather.
2	8	5				
3	9	..	SW. by W. ½ W.	NW. by N.		
4	9	5				
5	9					
6	8	5	..	..	..	6 P.M. In lower and main-topmast studding-sails, and braced up.
7	8	5				
8	8	5				
9	7	..	SW. by W. ½ W.	NW.	½	
10	6					
11	5					
12	5					
1	4	..	..	..	..	A.M. Set starboard studding-sails; shifted fore-top and top-gallant sails.
2	5	..	..	Variable.		
3	5					
4	5					
5	6					
6	6					
7	7	..	..	NNE.	..	Lat. by double alt. 35° 15′ N.
8	7	..	..	..	..	Long. by chron. reduced up to noon 17° 59′ W.
9	7					
10	7					
11	7	..	..	..	..	Variation by azimuth, 2 points W.
12	7					

Course.	Dist.	Dif. Lat.	Dept.	Lat. D. R.	Lat. Obs.	Long. D. R.	Long. by Obs.
S. 34° W.	130	108·9 S.	72·2 W.	35° 15′ N.	35° 15′ N.	18° 3′ W.	17° 59′ W.

The above courses, corrected for variation and leeway, will be

found as in the following Traverse Table, with their corresponding distances:

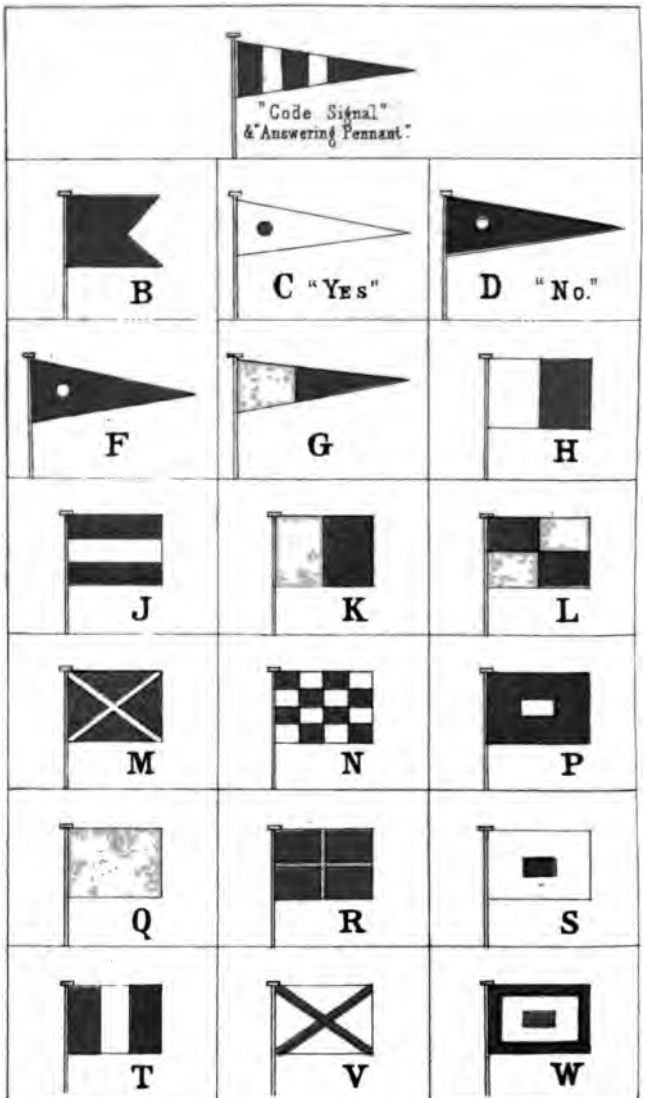
True Courses.	Dist.	N.	S.	E.	W.
NE.	18	12·7	..	12·7	..
SW. by S. ½ W.	53	..	39·3	..	35·6
SSW. ½ W. ...	96	..	82·3	..	49·3
		12·7	121·6	12·7	84·9
			12·7		12·7
			60)108·9		72·2
Difference of lat.		=	1° 49′ S.		
Lat. left by obs.		=	37° 2′ N.		
Lat. in		=	35° 15′ N.		
Sum of lats.		=	72° 19′		
Middle latitude		=	36° 9′		

With the middle latitude as a course, and the departure in the Dif. Lat. col., will be found in the Distance column the

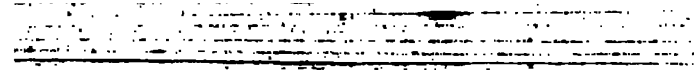
Difference of long. 89′ = 1° 29′ W.
Long. from obs. = 16° 24′ W.
Long. in by D. R. = 18° 3′ W.

Lastly, with the dif. lat. 108·9, and departure 72·2, by inspection we find the course S. 34 W., and distance 130 miles nearly.

COMMERCIAL CODE OF SIGNALS.



W.H.M.ª Farlane, Lith.ª Edin.ª



THE COMMERCIAL CODE OF SIGNALS FOR THE USE OF ALL NATIONS.

To enable a vessel to hold a conversation with another, or with the shore, when outside of speaking-distance, various schemes have been invented, usually consisting of flag-signals of different shapes and colours. The code which will be described here was formed by a committee appointed by the Board of Trade, and published in July 1857, for the use of the British merchant-service. It has already been adopted by the French government for exclusive use in the imperial and merchant service of France; several other European countries are considering it favourably, and will probably order its use in their navies; and it has been approved of by the United States government; so that it may shortly become what the Board of Trade designed it to be—a mode of universal communication, enabling vessels of all nations to converse with each other, even when entirely ignorant of each other's language.

The universal series of the commercial code of signals is composed of flags of three shapes—the burgee or fish-tail flag, the pennant or pointed flag, and the square flag. They are coloured in various ways so as to form nineteen different flags. Whenever this particular code is used, one of the pennants, called the 'code-pennant,' is hoisted under the national colours as a *code-signal*—that is, to shew that this code is used; and the same pennant is shewn, where it can be best seen, as an *answering-pennant*—that is, to indicate that any signal which has been

hoisted is understood. The remaining eighteen flags are called by eighteen letters of the alphabet, omitting the vowels, the letters not being used as letters, but merely as names for the flags. The nineteen flags, consisting of one burgee, five pennants, and thirteen square flags, are given in the accompanying coloured chart.

In signalling, they are to be hoisted at the mast-head in combinations of two, three, or four, thus composing 78,642 separate and distinct signals, of which about 20,000 are used as conversational signals, and about 50,000 for the names of vessels in the royal navy and merchant service. Accompanying the flags, the Board of Trade has issued an official signal-book, which must be obtained by every one desiring to use this code, giving the meanings which have been attached to as many of the 78,642 combinations as have been appropriated. The meanings are quite arbitrary, but are arranged on such an excellent system, that, while each hoist has its own meaning which never varies, the *nature of the signal* (although, of course, not its precise meaning) may be seen from the *form of the hoist*—that is, from the number of flags and the name of the uppermost.

The signal-book consists principally of Part I., Part II., and a Supplement. Part I. is specially adapted for international purposes, and, in the case of any foreign nation adopting this code, translating the meanings of the signals into its own language, but restricting itself to Part I., the British seaman must confine himself to the signals of Part I. in conversing with vessels of that nation. Information as to whether foreign nations have adopted Part I. only, or both Part I. and Part II., is inserted in the signal-book. When other nations adopt this code, it is not necessary, although it is desirable, that they should use the same signal-flags. If they adopt other flags, they supply the Board of Trade with a plate of their flags for insertion in the British signal-book, to which plate the British seaman refers in reading a signal hoisted by one of that nation, while his own answering signals are invariably taken from the British plate of flags, of which the foreigner has a copy.

The arrangement of Part I., Part II., and the Supplement in the British signal-book is as follows:

PART I.

First are given the hoists consisting of two flags together, arranged in alphabetical order, as BC, BD, BF, &c.; CB, CD, CF, &c.; DB, DC, DF, &c. with the meanings printed on the right hand. The first-mentioned flag is always to be hoisted uppermost.

The hoists of two flags with the B or burgee uppermost are used to represent attention or demand signals: the hoists of two flags with the C, D, F, or G pennant uppermost, represent compass signals: the hoists of two flags with the H or any other square flag uppermost, represent danger, distress, or very important signals. As:

BD.....What ship is that?
BF.....Vessels just arrived, shew your distinguishing signals.

DB.....East.
GS.....N. by W. ½ W.

HB.....Want immediate assistance.
LR.....Weigh, cut, or slip; wait for nothing.

Next are given the hoists combining three flags together, in alphabetical order, as BCD, BCF, BCG, &c. *The three-flag signals are called general signals*, and represent questions and replies on damage to the ship, dispatches, telegraphing, crew, passengers, quarantine, latitude, longitude, time, weather, repairs, equipment, supplies, engines, towing, anchorage, land-marks, pilots, making the land, working ship, commercial transactions, auxiliary phrases, British and Foreign weights and measures, numbers, address, and the alphabet. As:

BSJ.....I will forward your letters on.
FGH.....What is your present longitude?
NRS.....Where can I get any freight?
QMS.....Wrong; incorrect.
VFB.....Franc.
VWL.....Five.
WST.....A.

Next are arranged the hoists of four flags together, by far the most numerous of the combinations, also in alphabetical order, as BCJD, BCJF, BCJG, &c.

All hoists of four flags with B or burgee uppermost represent geographical signals. As—

BDPQ.....London.
BFML.....Pentland Skerries, Lights.
BQGL.....New York.

This completes Part I., the universal or international portion of the signal-book, also called the condensed code.

PART II.

Part II. continues the four-flag signals from where Part I. ends, the arrangement of the signal-flags being still alphabetical, as CBDF, CBDG, &c. The first batch of these represents an alphabetical spelling-table. Thus, to spell Allan:

CBDV.....Al
CDLW.....lan } Completing Allan in two separate hoists.

The remaining portion of Part II. serves two purposes. It is a national vocabulary, and it is an index to Part I. The four-flag signals are continued alphabetically from CHBD to FGMD, and the meanings of these signals are also arranged alphabetically, thus constituting a vocabulary. As:

CHBD.....A 1.
CHBF.....ABACK.
CHBG.....ABANDONED.

And the whole of the signals and their meanings already given in Part I. are here repeated, but they are now indexed or classed *under the principal word in the meaning of the signal*, the signal letters being printed in small type on the right hand, thus constituting an index to Part I. For instance, under the word 'Aback,' the arrangement is as follows:

CHBF.....ABACK—
Taken aback.....NCP.
You will be taken aback.....NCQ.
Heave all aback.....MWT.
Heave all aback. Go astern.....KB.

A similar arrangement holds good throughout the whole of this part; so that, if a British seaman wants to find what signal to hoist for any purpose, he must first look for the *principal word* of his intended signal in the vocabulary, Part II., which we are now describing, and under that principal word he will find the signal and the signal-flags which he wants. Thus, if he wishes to signal a ship to leeward that a squall is coming, he must look up the word 'Squall' in Part II. of the signal-book. Under the word 'Squall,' he will find

Severe squall not far off; look sharp.....HND.

and he therefore hoists HND at his mast-head, at the same time running up his ensign with the code-pennant under it at the peak. The officer of the ship signalled looks up HND in Part I. of his signal-book, and he finds the meaning to be as just stated—

HND.....Severe squall not far off; look sharp.

after which he hoists his answering-pennant where it can be best seen, to indicate that he understands the other's signal.

The geographical signals are indexed separately at the end of Part II.

SUPPLEMENT.

The remaining combinations of four flags, beginning with the G pennant uppermost, and continuing with the H, J, K, and other square flags uppermost, are given in the Supplement, still alphabetically, and constitute the 50,000 signals intended to denote ships' names.

Those signals of four flags with the G pennant uppermost represent vessels in H.M. Royal Navy or H.M. Indian Marine. As

GVDQ.....*Trafalgar*, screw, 72 guns.
GVLR.....*Warrior*, iron, screw, *armoured*, 32 guns.
GWBK.....*Marie*, surveying-vessel, 2 guns.

Those signals of four flags with the H, J, K, or any other square flag uppermost, represent vessels belonging to the merchant-service. As:

NJFK.....*Great Eastern*, of London. 2600 H.P.; offl. no. 21,891.
TQDB.....*Scotia*, of Glasgow. 1000 H.P.; offl. no. 43,711.

Foreign nations using this code have their own special supplements containing the list of ships belonging to their own marine, and the four flags which represent them.

In using the universal code of signals, British vessels must hoist their ensign at the peak, and the code-pennant underneath it. Foreign vessels hoist their national colours, and the same code-pennant underneath.

In sighting ships or stations, the Board of Trade recommend commanders of ships first to shew the national colours with the code-pennant underneath; second, to hoist the four flags representing the vessel's name; third, to hoist the geographical signal of four flags denoting the port bound from; fourth, the four flags denoting the port bound to; and fifth, the three flags from the numeral table in Part I. denoting the number of days out; and also to keep the four flags denoting the ship's name flying, up or down Channel, if not blowing hard.

PART III.

A third part has been added to the signal-book, giving a set of 'Distant Signals,' to be hoisted at the mast-head when the colours of the flags cannot be distinguished.

The symbols employed are two black balls, two pennants of any colour, and two square flags of any colour. These are combined in eighteen different ways to represent the eighteen letters of the universal code, each of the eighteen combinations containing at least one black ball, which thus becomes the distinguishing feature of the hoists.

A system of signals, consisting of a vertical post with three movable projecting arms, is extensively used at signal stations on the French coast, and is known as the 'Semaphore.' The movable arm of the semaphore is equivalent to the ball of the distant signals when horizontal, to the pennant when oblique with the end pointing down, and to the square flag when oblique with the end pointing up.

For an explanation of the meaning of these, and of the 'Boat Signals,' the reader must refer to Part III. of the signal-book itself.

INSTRUCTIONS

FOR THE GUIDANCE OF MASTERS AND SEAMEN WHEN USING THE MORTAR AND ROCKET APPARATUS FOR SAVING LIFE.

Issued by the Board of Trade, January 1865.

IN the event of your vessel stranding on the coasts of the United Kingdom, and the lives of the crew being placed in danger, assistance will, if possible, be rendered from the shore in the following manner—namely:

1. A rocket or shot, with a thin line attached, will be fired across your vessel. Get hold of this line as soon as you can; and when you have secured it, let one of the crew be separated from the rest, and, if in the daytime, wave his hat or his hand, or a flag or handkerchief; or, if at night, let a rocket, a blue light, or a gun be fired, or let a light be displayed over the side of the ship, and be again concealed, as a signal to those on shore.

2. When you see one of the men on shore separated from the rest, wave a RED flag, or (if at night) shew a RED light, and then conceal it; you are to haul upon the rocket-line until you get a tailed block, with an endless fall rove through it.

3. Make the tail of the block fast to the *mast about* 15 *feet above the deck*, or if your masts are gone, to the *highest secure* part of the vessel. When the tail block is made fast, and the rocket-line unbent from the whip, let one of the crew, separated from the rest, make the signal required by Art. 1 above.

4. As soon as the signal is seen on shore, a hawser will be bent to the whip-line, and will be hauled off to the ship by those on shore.

5. When the hawser is got on board, the crew should at once make it fast to the same part of the ship as the tailed block is made fast to, *only about* 18 *inches higher, taking care that there are no turns of the whip-line round the hawser.*

6. When the hawser has been made fast on board, the signal directed to be made in Art. 1 above is to be repeated.

7. The men on shore will then pull the hawser taut, and by means of the whip-line will haul off to the ship a sling life-buoy, into which the person to be hauled ashore is to get and be made fast. When he is in, and secure, one of the crew must be separated from the rest, and again signal to the shore as directed in Art. 1 above. The people on shore will then haul the person in the sling to the shore, and, when he has landed, will haul back the empty sling to the ship for others. This operation will be repeated until all persons are hauled ashore from the wrecked vessel.

8. It may sometimes happen that the state of the weather and the condition of the ship will not admit of a hawser being set up; in such cases a sling life-buoy will be hauled off instead, and the shipwrecked persons will be hauled through the surf, instead of along a hawser.

Masters and crews of stranded vessels should bear in mind that SUCCESS in landing them in a great measure DEPENDS UPON THEIR COOLNESS, AND ATTENTION TO THE RULES HERE LAID DOWN; and that, by attending to them, many lives are annually saved by the mortar and rocket apparatus on the coasts of the United Kingdom.

The system of signalling must be strictly adhered to, and all women, children, passengers, and helpless persons should be landed before the crew of the ship.

REGULATIONS FOR PREVENTING COLLISIONS AT SEA,

APPENDED TO THE ORDER IN COUNCIL,

Dated 9th January 1863.

1. By virtue of the 'Merchant Shipping Act Amendment Act, 1862,' and of an Order in Council dated 9th January 1863, the following regulations, containing certain verbal amendments, are substituted for the regulations contained in the schedule to the act.

2. The following regulations came into operation on the 1st of June 1863.

3. The following regulations apply to all ships, whatever their nationality, within the limits of British jurisdiction, and to British and French ships, whether within British jurisdiction or not.

4. They also apply to ships belonging to the following countries and places, whether within British jurisdiction or not—viz.:

Country or Place.	Date of Gazette containing Order in Council.	Country or Place.	Date of Gazette containing Order in Council.
Austria	May 1, 1863.	Netherlands	Sept. 15, 1863.
Argentine Republic	Sept. 15, 1863.	Norway	Sept. 15, 1863.
Belgium	May 1, 1863.	Oldenburg	May 1, 1863.
Brazil	July 28, 1863.	Peru	July 28, 1863.
Bremen	July 28, 1863.	Portugal	May 1, 1363.
Chili	Nov. 20, 1863.	Prussia	May 1, 1863.
Denmark Proper	Feb. 5, 1864.	Roman States	Nov. 20, 1863.
Equator (Republic of the)	July 28, 1863.	Russia	July 28, 1863.
Hamburg	July 28, 1863.	Schleswig	Feb. 5, 1864.
Hanover	May 1, 1863.	Spain	July 28, 1863.
Hayti	May 1, 1863.	Sweden	July 28, 1863.
Italy	May 1, 1863.	Turkey	Nov. 20, 1863.
Lubeck	July 28, 1863.	United States	Aug. 30, 1864.
Mecklenburg-Schwerin	May 1, 1863.	Uruguay	July 28, 1863.
Morocco	July 28, 1863.		

The governments of the countries and places named above have already notified their acquiescence. Other governments are expected to follow.

BOARD OF TRADE, *Sept.* 2, 1864.

Preliminary.

Art. 1. In the following Rules every steam-ship which is under sail and not under steam is to be considered a sailing-ship; and every steam-ship which is under steam, whether under sail or not, is to be considered a ship under steam.

Rules concerning Lights.

Art. 2. *Lights.*—The lights mentioned in the following articles, numbered 3, 4, 5, 6, 7, 8, and 9, and no others, shall be carried in all weathers, from sunset to sunrise.

Art. 3. *Lights for Steam-ships.*—Sea-going steam-ships when under-way shall carry:

(*a.*) *At the foremast head,* a bright white light, so fixed as to shew a uniform and unbroken light over an arc of the horizon of 20 points of the compass; so fixed as to throw the light 10 points on each side of the ship—viz., from right ahead to 2 points abaft the beam on either side; and of such a character as to be visible on a dark night, with a clear atmosphere, at a distance of at least five miles.

(*b.*) *On the starboard side,* a green light so constructed as to shew a uniform and unbroken light over an arc of the horizon of 10 points of the compass; so fixed as to throw the light from right ahead to 2 points abaft the beam on the starboard side; and of such a character as to be visible on a dark night, with a clear atmosphere, at a distance of at least two miles.

(*c.*) *On the port side,* a red light, so constructed as to shew a uniform and unbroken light over an arc of the horizon of 10 points of the compass; so fixed as to throw the light from right ahead to 2 points abaft the beam on the port side; and of such

a character as to be visible on a dark night, with a clear atmosphere, at a distance of at least two miles.

(*d.*) The said green and red side-lights shall be fitted with inboard screens, projecting at least three feet forward from the light, so as to prevent these lights from being seen across the bow.

Art. 4. *Lights for Steam-tugs.*—Steam-ships, when towing other ships, shall carry two bright white mast-head lights vertically, in addition to their side-lights, so as to distinguish them from other steam-ships. Each of these mast-head lights shall be of the same construction and character as the mast-head lights which other steam-ships are required to carry.

Art. 5. *Lights for Sailing-ships.*—Sailing-ships under-way, or being towed, shall carry the same lights as steam-ships under-way, with the exception of the white mast-head lights, which they shall never carry.

Art. 6. *Exceptional Lights for small Sailing-vessels.*—Whenever, as in the case of small vessels during bad weather, the green and red lights cannot be fixed, these lights shall be kept on deck, on their respective sides of the vessel, ready for instant exhibition; and shall, on the approach of or to other vessels, be exhibited on their respective sides in sufficient time to prevent collision, in such manner as to make them most visible, and so that the green light shall not be seen on the port side, nor the red light on the starboard side.

To make the use of these portable lights more certain and easy, the lanterns containing them shall each be painted outside with the colour of the light they respectively contain, and shall be provided with suitable screens.

Art. 7. *Lights for Ships at anchor.*—Ships, whether steam-ships or sailing-ships, when at anchor in roadsteads or fairways shall exhibit, where it can best be seen, but at a height not exceeding twenty feet above the hull, a white light, in a globular lantern of eight inches in diameter, and so constructed as to shew a clear, uniform, and unbroken light visible all round the horizon, and at a distance of at least one mile.

Art. 8. *Lights for Pilot-vessels.*—Sailing pilot-vessels shall not carry the lights required for other sailing-vessels, but shall carry a white light at the mast-head, visible all round the horizon—and shall also exhibit a flare-up light every fifteen minutes.

Art. 9. *Lights for Fishing-vessels and Boats.*—Open fishing-boats and other open boats shall not be required to carry the side-lights required for other vessels; but shall, if they do not carry such lights, carry a lantern having a green slide on the one side and a red slide on the other side; and on the approach of or to other vessels, such lantern shall be exhibited in sufficient time to prevent collision, so that the green light shall not be seen on the port side, nor the red light on the starboard side.

Fishing-vessels and open boats, when at anchor, or attached to their nets and stationary, shall exhibit a bright white light.

Fishing-vessels and open boats shall, however, not be prevented from using a flare-up in addition, if considered expedient.

Rules concerning Fog Signals.

Art. 10. *Fog Signals.*—Whenever there is fog, whether by day or night, the fog signals described below shall be carried and used, and shall be sounded at least every five minutes—viz.:

(*a.*) Steam-ships under-way shall use a steam-whistle placed before the funnel, not less than eight feet from the deck.

(*b.*) Sailing-ships under-way shall use a fog horn.

(*c.*) Steam-ships and Sailing-ships, when not under-way, shall use a bell.

Steering and Sailing Rules.

Art. 11. *Two Sailing-ships meeting.*—If two sailing-ships are meeting end on or nearly end on, so as to involve risk of collision, the helms of both shall be put to port, so that each may pass on the port side of the other.

Art. 12. *Two Sailing-ships crossing.*—When two sailing-ships are crossing so as to involve risk of collision, then, if they have the wind on different sides, the ship with the wind on the port

side shall keep out of the way of the ship with the wind on the starboard side; except in the case in which the ship with the wind on the port side is close-hauled and the other ship free, in which case the latter ship shall keep out of the way; but if they have the wind on the same side, or if one of them has the wind aft, the ship which is to windward shall keep out of the way of the ship which is to leeward.

Art. 13. *Two Ships under steam meeting.*—If two ships under steam are meeting end on or nearly end on, so as to involve risk of collision, the helms of both shall be put to port, so that each may pass on the port side of the other.

Art. 14. *Two Ships under steam crossing.*—If two ships under steam are crossing so as to involve risk of collision, the ship which has the other on her own starboard side shall keep out of the way of the other.*

Art. 15. *Sailing-ship and Ship under steam.*—If two ships, one of which is a sailing-ship, and the other a steam-ship, are proceeding in such directions as to involve risk of collision, the steam-ship shall keep out of the way of the sailing-ship.

Art. 16. *Ships under steam to slacken speed.*—Every steam-ship, when approaching another ship so as to involve risk of collision, shall slacken her speed, or, if necessary, stop and reverse; and every steam-ship shall, when in a fog, go at a moderate speed.

Art. 17. *Vessels overtaking other vessels.*—Every vessel overtaking any other vessel shall keep out of the way of the said last-mentioned vessel.

Art. 18. *Construction of Articles* 12, 14, 15, *and* 17.—Where by the above Rules one of two ships is to keep out of the way, the other shall keep her course, subject to the qualifications contained in the following Article:

Art. 19. *Proviso to save special cases.*—In obeying and construing these Rules, due regard must be had to all dangers of

* That is, if two steamers A and B are crossing each other, if steamer A sees steamer B on its starboard bow, A ports its helm and goes under B's stern; and if steamer A sees steamer B on its port bow, B ports its helm, and goes under A's stern.

navigation; and due regard must also be had to any special circumstances which may exist in any particular case, rendering a departure from the above Rules necessary in order to avoid immediate danger.

Art. 20. *No Ship, under any circumstances, to neglect proper precautions.*—Nothing in these Rules shall exonerate any ship, or the owner, or master, or crew thereof, from the consequences of any neglect to carry lights or signals, or of any neglect to keep a proper look-out, or of the neglect of any precaution which may be required by the ordinary practice of seamen, or by the special circumstances of the case.

DIAGRAMS

TO ILLUSTRATE THE USE OF THE LIGHTS CARRIED BY VESSELS AT SEA, AND THE MANNER IN WHICH THEY INDICATE TO THE VESSEL WHICH SEES THEM THE POSITION AND DESCRIPTION OF THE VESSEL THAT CARRIES THEM.

When both Red and Green Lights are seen:

A sees a red and green light ahead; A knows that a vessel is approaching her on a course directly opposite to her own, as B.

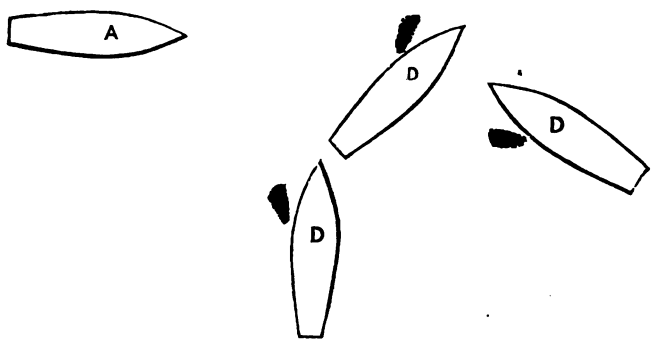
If A sees a white mast-head light above the other two, she knows that B is a steam-vessel.

When the Red, and not the Green Light, is seen:

A sees a red light ahead or on the bow; A knows that either,

1. A vessel is approaching her on her port bow, as B;

or, 2. A vessel is crossing in some direction to port, as D, D, D.



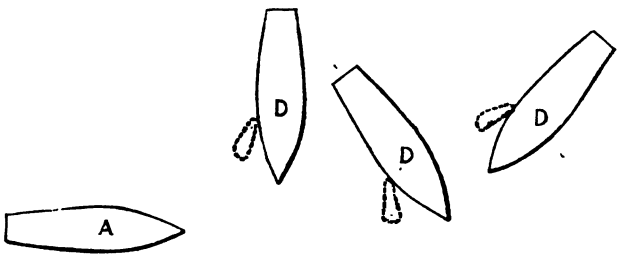
If A sees a white mast-head light above the red light, A knows that the vessel is a steam-vessel, and is either approaching her in the same direction, as B, or is crossing to port in some direction, as D, D, D.

When the Green, and not the Red Light, is seen:

A sees a green light ahead or on the bow; A knows that either,

1. A vessel is approaching her on her starboard bow, as B;

or, 2. A vessel is crossing in some direction to starboard, as D, D, D.



If A sees a white mast-head light above the green light, A knows that the vessel is a steam-vessel, and is either approaching her in the same direction, as B, or is crossing to starboard in some direction, as D, D, D.

ORDINARY EXAMINATION.

EXAMINATION PAPER, No. I.

For Second Mate.

(1.) Multiply 489 by 324, by common logarithms.

(2.) Divide 956 by 49, by common logarithms.

(3.) In latitude 53°, the departure made was 118 miles; required the difference of longitude by Parallel Sailing.

(4.) In the following day's work, correct the courses for variation and leeway; find the course and distance made good, and the latitude and longitude in, by inspection:

H.	K.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
1	5	8	SW.	NW.	0	Noon. I take my departure from Cape Clear, in lat. 51° 25′ N., long. 9° 29′ W., bearing by compass NNE. 10 m.
2	6	4				
3	6	4				
4	6	4				
5	6	8	SW. ½ W.	NW. by N.	0	
6	7	2				
7	6	8				
8	5	2				
9	5	6	SSW.	WNW.	¼	
10	5	4				
11	5	8				
12	6	2				
1	6	4				
2	6	6				
3	6	4	SW. by S.	West.	¾	A current set by compass NE. by N.; estimated drift during the day, 16 miles.
4	5	6				
5	5	2				
6	4	8				
7	5	0				
8	5	4	SSW.	W. by S.	1½	
9	4	8				
10	4	8				
11	5	0	..	..	..	Variation, 2¾ pts. W.
12	5	0				

(5.) October 22, 1866—when the sun's declination was 11° 12′ 16″ S., the observed meridian altitude of his lower limb was 23° 16′ 40″ south of the observer; index error − 1′ 12″, and height of the eye, 18 feet; required the latitude.

(6.) Required the course and distance from lat. 51° 12′ N., long. 11° 6′ W., to lat. 48° 23′ N., long. 15° 35′ W., by Mercator's Sailing.

Additional for Only Mate.

(7.) On December 8, 1866, at 4 h. 30 m. 40 s. P.M. apparent time at ship, in lat. 42° N., long. 8° 10′ W., the sun's setting amplitude was observed to be W. ¼ N.; required the variation of the compass.

(8.) On November 13, 1866—P.M. at ship, in lat. 47° 50′ S.; the observed altitude of the sun's lower limb was 20° 46′ 20″; index error, 2′ 15″ to add; height of the eye, 22 feet; time by chronometer, November 13 d. 7 h. 12 m. 20 s., which was fast for Greenwich *mean* time 4 m. 33 s. on the 1st October at noon, and losing 4·5 s. daily; required the longitude.

Additional for Chief Mate.

(9.) On February 9, 1866—in lat. 44° 13′ S., long. 58° 34′ 45″ E., at 5 h. 14 m. 14 s. P.M., mean time at ship; the observed altitude of the sun's lower limb was 20° 45′ 10″; index error, 2′ 30″ to subtract; height of the eye, 18 feet; sun's bearing by azimuth compass W. 12° N.; required the variation of the compass.

(10.) On October 23, 1866—P.M. at ship, latitude by account 41° 45′, longitude 94° 24′ E.; observed altitude of the sun's lower limb, south of the observer, was 36° 15′ 20″; index error + 1′ 15″; height of the eye, 20 feet; time by watch, 23 d. 2 h. 12 m. 15 s., which had been found to be 1 h. 40 m. 36 s. fast on apparent time at the ship; the difference of longitude made to

the *east*, since the error of the watch on apparent time at the ship was determined, was 45′·4; required the latitude by reduction to the meridian.

Additional for Master Ordinary.

(11.) On October 24, 1866, the observed meridian altitude of α Canis Majoris (Sirius) was 38° 15′ 35″, south of the observer, index error + 1° 25″; height of the eye, 22 feet; required the latitude.

TABLE OF DEVIATIONS.

Ship's Head.	Deviation.	Ship's Head.	Deviation.
North	0° 22′ W.	South	0° 16′ E.
N. by E.	1° 46′ E.	S. by W.	1° 50′ W.
NNE.	3° 20′ E.	SSW.	3° 16′ W.
NE. by N.	5° 16′ E.	SW. by S.	4° 48′ W.
NE.	7° 14′ E.	SW.	6° 16′ W.
NE. by E.	8° 54′ E.	SW. by W.	7° 40′ W.
ENE.	10° 44′ E.	WSW.	9° 18′ W.
E. by N.	11° 40′ E.	W. by S.	10° 34′ W.
East	10° 44′ E.	West	11° 50′ W.
E. by S.	9° 54′ E.	W. by N.	11° 10′ W.
ESE.	9° 8′ E.	WNW.	10° 16′ W.
SE. by E.	7° 20′ E.	NW. by W.	9° 18′ W.
SE.	6° 18′ E.	NW.	7° 52′ W.
SE. by S.	5° 0′ E.	NW. by N.	6° 18′ W.
SSE.	3° 24′ E.	NNW.	5° 2′ W.
S. by E.	1° 42′ E.	N. by W.	3° 10′ W.

(12.) Correct the following courses for local attraction (by the accompanying Table of Deviations): NW. $\frac{1}{2}$ W.; E. by N. $\frac{1}{2}$ N.; NNW. $\frac{1}{2}$ W.; NE. $\frac{1}{2}$ E.; W. by S. $\frac{1}{2}$ S.; and ESE. $\frac{1}{2}$ S.

Answers.

(1.) Log. 5·1998539 = nat. number, 158436. (The product.)

(2.) Log. 1·2902618 = nat. number, 19·5102. (The quotient.)

(3.) Dif. long. 3° 16′.

(4.) True courses—S. $\frac{3}{4}$ E. 10 miles; S. $1\frac{1}{4}$ W. 25 miles; S. $1\frac{3}{4}$ W. 26 miles; S. 1 E. 36 miles; S. $\frac{1}{2}$ E. 27 miles; S. $2\frac{1}{4}$ E. 25 miles; N. $\frac{1}{4}$ E. 16 miles; dif. lat. 127·5 S., dept. 7·7 E;

course, S. $3\frac{1}{2}°$ E.; dist. 128, by inspection; course, S. 3° 28′ E. and distance 127·73, by calculation; lat. 49° $17\frac{1}{2}'$ N.; long. 9° 17′ W.

(5.) Lat. in 55° 22′ 20″ N.

(6.) Mer. dif. lat. = 261·9, dif. long. 269′; course, S. 45° 5′ W.; dist. 190·12 miles.

(7.) App. Gr. time, 5 h. 3 m. 55 s.; sun's dec. 22° 45′ 44″ S.; true amplitude, W. 31° 22′ 30″ S.; variation, 33° 11′ 15″ W.

(8.) Mean Gr. time, Nov. 13, 7 h. 11 m. 2 s.; sun's dec. 18° 4′ 33″ S.; mean time at ship, 4 h. 58 m. 15 s.; long. 33° 11′ 45″ W.

(9.) Gr. mean time, Feb. 9, 1 h. 19 m. 55 s. P.M.; sun's dec. 14° 37′ 28″ S.; true azimuth, S. 89° 39′ 29″ W.; variation, 12° 20′ 31″ W.

(10.) App. time past noon at observation, 34 m. 40 s.; mean Gr. time of obs. Oct. 22, 19 h. 4 m. 26 s.; sun's dec. 11° 21′ 38″ S.; arc 1 = 101° 29′ 18″; arc 2 = 53° 4′ 33″; lat. in 41° 35′ 15″ N.

(11.) Trué dec. 16° 32′ 8″·61 S.; true alt. 38° 11′ 16″ S.; lat. 35° 16′ 35″ N.

(12.) Corrected courses, N. 59° $12\frac{1}{2}'$ W.; N. 84° $19\frac{1}{2}'$ E.; N. 33° $42\frac{1}{2}'$ W.; N. 58° $41\frac{1}{2}'$ E.; S. 63° $11\frac{1}{2}'$ W.; S. 53° $38\frac{1}{2}'$ E.

EXAMINATION PAPER, No. II.

For Second Mate.

(1.) Multiply 1866 by 429, by common logarithms.

(2.) Divide 34650 by 185, by common logarithms.

(3.) In latitude 56° the departure made was 208 miles; required the difference of longitude by Parallel Sailing.

(4.) In the following day's work, correct the courses for variation and leeway; find the course and distance made good, and the latitude and longitude in, by inspection:

H.	K.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
1	6	6	NW. by W.	NE. by N.	1½	Noon. I take my departure from Seal Rocks, South Shetland Islands, in lat. 61° 1′ S., long. 55° 32′ W., bearing by compass SE. by S. 9 miles.
2	6	6				
3	5	8				
4	6	4	WNW.	NE. by E.	1½	
5	6	6				
6	7	2				
7	7	8				
8	8	0	WNW. ½ W.	NNE.	2	A current setting by compass SE. by E. 1½ miles an hour during the whole 24 hours.
9	8	4				
10	8	6				
11	7	8				
12	7	2				
1	7	5	W. by N.	North.	2	
2	7	5				
3	8	2				
4	8	8				
5	8	2	West.	N. by W.	1¾	
6	7	8				
7	7	6				
8	7	4				
9	7	0	NNE. ¼ E.	NW.	1¼	
10	6	6				
11	6	4	..	..	..	Variation 2½ points E.
12	6	0				

(5.) On October 22, 1866—longitude 71° 30′ W.; observed meridian altitude of sun's lower limb was 60° 18′ 40″; observer north of the sun; index error, 1′ 12″ to subtract; height of the eye, 22 feet; required the latitude.

(6.) Find the course and distance from lat. 48° 12′ N., long. 12° 30′ W., to lat. 23° 34′ N., long. 53° 27′ W., by Mercator's Sailing.

Additional for Only Mate.

(7.) On October 22, 1866, at 5 h. 23 m. 25 s. A.M. apparent time at ship; lat. 39° 16′ S., long. 22° 45′ W.; the sun's rising amplitude was observed to be E. 12° 34′ N.; required the variation.

(8.) On October 3, 1866—A.M. at ship, lat. 36° 17′ S.; observed altitude of sun's lower limb, 26° 54′; index error, 55″ to add; height of eye, 19 feet; time by chronometer, October 2, 15 h. 34 m. 20 s., which had been found 1 m. 21 s. slow on mean

Gr. time at noon, September 1, and losing 1·8 s. daily; required the longitude.

Additional for Chief Mate.

(9.) On October 20, 1866, at 4 h. 8 m. 9 s. P.M. mean time at ship; lat. 30° 35′ N., long. 137° 8′ 15″ E.; the observed altitude of the sun's lower limb was 20° 35′; bearing by azimuth compass, W. 2½ pts. S.; index error, 1′ 13″ to add; height of eye, 18 feet; required the variation.

(10.) On October 20, 1866—A.M. at ship; lat. by account, 21° 28′ N., long. 87° 12′ W.; observed altitude of sun's lower limb was 57° 12′ 10″ S.; index error, 4′ 2″ to be added; height of eye, 25 feet; time by watch, October 20, 0 h. 1 m. 16 s., which had been found 23 m. 14 s. fast on apparent time at ship, but the ship had made 20½′ dif. long. W. since the error for time had been determined; required the true latitude.

Additional for Master Ordinary.

(11.) Correct the following courses for local attraction (by Table, page 402): NE. ½ N.; E. by N.; W. by N. ½ N.; S. by W.; SSW. ½ W.; NE. ½ E.

(12.) October 12, 1866—long. 56° 12′ W.; the observed altitude of α Canis Minor (Procyon) when on the meridian, south of the observer, was 64° 18′ 10″; index error, 2′ 25″ to subtract; height of eye, 17 feet; required the latitude.

Answers.

(1.) Log. 5·9033689 = log. of 800514. (The product.)

(2.) Log. 2·2725315 = nat. number, 178·297. (The quotient.)

(3.) Dif. of long. 6° 12′.

(4.) True courses—S. ½ E. 9 miles; N. 3¾ W. 19 miles; N. 5 W. 28 miles; N. 6 W. 40 miles; N. 6½ W. 32 miles; N. 7¼ W. 31 miles; N. 6¼ E. 26 miles; S. 2½ E. 36 miles; dif. lat. 26′·9 N.; dept. 92 miles W.—course N. 74° W., dist. 98 miles, by inspection—course N. 73° 42′ W., dist. 95·84 miles, by calculation; lat. in 60° 34′ 6″ S.; long. in 58° 40′ 48″ W.

(5.) Sun's decl. 11° 8′ 45″·5 S.; lat. 18° 22′ 40″·5 N.

(6.) Mer. dif. lat. 1853·9, dif. long. 2457′—course S. 52° 58′ W.; dist. 2454 miles.

(7.) App. Gr. time, 6 h. 54 m. 25 s. A.M.; sun's decl. 11° 0′ 0″·9 S.; true amplitude, E. 14° 16′ S.; variation, 26° 50′ W.

(8.) Mean Gr. time, Oct. 2, 15 h. 36 m. 38 s.; sun's decl. 3° 50′ 34″ S.; mean time at ship, Oct. 2, 19 h. 53 m. 34 s.; longitude in 64° 14′ E.

(9.) Mean time at Gr., Oct. 19, 18 h. 59 m. 36 s.; sun's decl. 10° 17′ 33″ S.; true azimuth, N. 116° 30′ 14″ W.; variation, 1° 37′ 16″ E.

(10.) App. time of obs. before noon, 24 m. 20 s.; app. Gr. time of obs., Oct. 20, 5 h. 24 m. 28 s.; sun's decl. 10° 26′ 37″ S.; arc 1 = 100° 30′ 5″; arc 2 = 32° 3′ 24″; lat. 21° 33′ 19″ N.

(11.) N. 45° 37′ 30″ E.; E. 0° 25′ S.; N. 83° 50′ 30″ W.; S. 9° 25′ W.; S. 24° 5′ 30″ W.; N. 58° 41′ 30″ E.

(12.) True decl. 5° 33′ 51″ N.; tr. alt. 64° 11′ 18″; lat. 31° 22′ 33″ N.

EXAMINATION PAPER, No. III.

For Second Mate.

(1.) Multiply 348·25 by 7·125, by common logarithms.

(2.) Divide 3895 by 93·15, by common logarithms.

(3.) In lat. 57°, the departure made was 128 miles; required the difference of longitude by parallel sailing.

(4.) On October 20, 1866—long. 83° 45′ E.; the observed meridian altitude of the sun's upper limb was 35° 12′ 10″, the observer being north of the sun; index error, 2′ 5″ to add; height of the eye, 24 feet; required the latitude.

(5.) In the following day's work, correct the courses for variation and leeway; find the course and distance made good, and the latitude and longitude in, by inspection:

H.	K.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
1	6	4	SSE.	E.	1½	Noon. I take my departure from Annabona, bearing by compass N. by E. ½ E. in lat. 1° 25′ S., long. 5° 42′ 48″ E.; distant 14 miles.
2	6	6				
3	5	8				
4	5	2				
5	5	2	SSW.	SE.	1½	
6	4	8				
7	4	8				
8	5	2				
9	6	0	SW. by W.	E. by S.	0	
10	6	4				
11	6	6				
12	7	0				
1	7	2	W. by N.	SSE.	½	Variation 2 points west.
2	6	8				
3	6	5				
4	6	5				
5	6	0	ESE.	SSW.	1½	
6	6	8				
7	6	8	..	..	..	A current set the ship by compass N. by E. 20 miles, from the time of departure to the end of the day.
8	6	4				
9	5	8	E. by N.	SSE.	1½	
10	5	2				
11	5	2				
12	4	8				

(6.) Required the course and distance from Cape Clear to St Michael:

Cape Clear, lat. 51° 24′ 55″ N., long. 9° 29′ 0″ W.
St Michael, lat. 37° 48′ 15″ N., long. 25° 13′ 15″ W.

Additional for Only Mate.

(7.) On October 24, 1866, at 4 h. 55 m. 36 s. P.M. apparent time at ship, in lat. 52° 32′ N., long. 157° 14′ W., the sun's magnetic setting amplitude was WSW. ¼ W; required the variation.

(8.) On October 24, 1866—A.M. at ship, in lat. 28° 50′ N., the observed altitude of the sun's lower limb was 25° 16′ 50″; index error, 3′ 17″ to add; height of the eye, 22 feet; time by chronometer, October 24, 1 h. 5 m. 20 s., which was 3 m. 12 s. fast on Gr. mean time at noon, October 8, and gaining 1·4 s. daily; required the longitude.

Additional for First Mate.

(9.) On October 24, 1866, at 2 h. 45 m. 12 s. P.M. mean time at ship; lat. 24° 16 N., long. 155° 33′ E.; observed altitude of sun's lower limb, 35° 47′ 20″, bearing by azimuth compass W. by S.; index error, 1′ 50″ to add; height of eye, 25 feet; required the variation.

(10.) On October 12, 1866—A.M. at ship; lat. by account 20° 10′ S., long. 75° 5′ W.; observed altitude of the sun's lower limb, 76° 52′ 35″, the sun north of the observer; index error, 2′ 15″ to subtract; height of the eye, 22 feet; time by watch, October 12, 0 h. 3 m. 12 s., which had been found 12 m. 50 s. fast on apparent time at ship, but the ship had made 22′ dif. long. to west since the error for time had been determined; required the latitude.

Additional for Master Ordinary.

(11.) Correct the following courses for local deviation (by Table, page 402): NW. ½ W.; ENE. ½ E.; SE. by E. ½ E.; WNW.; NE. ½ E.; west.

(12.) On July 1, 1866—in long. 176° W.; the observed meridian altitude of α Tauri (Aldebaran), north of the observer, was 61° 12′ 40″; height of the eye, 20 feet; index error, 1′ 27″ to be added; required the latitude.

Answers.

(1.) Log. 3·3946760 ∴ nat. number = 2481·28. (The product.)

(2.) Log. 1·6213246 ∴ nat. number = 41·81427. (The quotient.)

(3.) Dif. of long. 3° 55′.

(4.) Sun's decl. 10° 16′ 48″ S.; zenith dist. 55° 7′ 50″; lat. 44° 51′ 2″ N.

(5.) True courses—S. ½ E. 14 miles; S. 2½ E. 24 miles; S. 1¼ W. 20 miles; S. 3 W. 26 miles; S. 7½ W. 27 miles; N. 6½

E. 26 miles; N. $3\frac{3}{4}$ E. 21 miles; N. by W. 20 miles—course S. $\frac{1}{4}$ E.; dist. 36 miles; lat. in 2° 1′ S.; long. in 5° 44′ 24″ E.

(6.) Mer. dif. lat. 1155·2; dif. long. 944·25—course S. 39° 16′ W.; dist. 1054·8 miles.

(7.) App. Gr. time, Oct. 24, 15 h. 24 m. 18 s.; sun's decl. 12° 0′ 0″·7 S.; true amplitude, W. 19° 59′ 10″ S.; variation, 0° 17′ 55″ W.

(8.) Mean Gr. time, Oct. 24, 1 h. 1 m. 46 s.; sun's decl. 11° 47′ 47″; mean time at ship, Oct. 23, 20 h. 16 m. 42 s.; long. 71° 16′ W.

(9.) Mean time at Gr., Oct. 23, 16 h. 23 m.; sun's decl. 11° 40′ 13″; true azimuth, N. 126° 59′ 6″ W.; variation, 25° 44′ W.

(10.) App. time of obs. before noon, 11 m. 6 s.; app. time of obs. at Gr., Oct. 12, 4 h. 49 m. 19 s.; sun's decl. 7° 29′ 32″ S.; arc 1 = 82° 29′ 57″; arc 2 = 12° 41′ 6″; lat. 20° 11′ 9″ S.

(11.) N. 59° 12′ 30″ W.; N. 82° 56′ 30″ E.; S. 53° 38′ 30″ E.; N. 77° 46′ W.; N. 58° 41′ 30″ E.; S. 78° 10′ W.

(12.) True dec. 16° 14′ 17″·4 N.; true alt. 61° 9′ 15″; lat. 12° 36′ 28″ S.

EXAMINATION PAPER, No. IV.

For Second Mate.

(1.) Multiply 7298 by 34·75, by common logarithms.

(2.) Divide 43875 by 195, by common logarithms.

(3.) In lat. 46° the departure made was 207 miles; required the difference of longitude by Parallel Sailing.

(4.) On October 10, 1866—long. 53° 35′ W.; the observed meridian altitude of the sun's lower limb was 74° 12′ 30″, the sun being south of the observer; index error, 1′ 50″ to subtract; height of the eye, 18 feet; required the latitude.

(5.) In the following day's work, correct the courses for leeway and variation; find the course and distance made good, and the latitude and longitude in, by inspection:

DAY'S WORK.

H.	K.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
1	5	4	SW. by W.	SSE.	½	Noon. I take my departure from a point on St Paul's Island, lat. 20° 59′ 50″ S., long. 55° 13′ 54″ E.; bearing by compass N. by W. distant 18 miles.
2	3	8				
3	4	8				
4	5	0				
5	5	5	N. by E. ¾ E.	SE.	0	
6	6	5				
7	7	0				
8	7	0				
9	7	4	SSW.	SE. by E.	1½	
10	6	6				
11	5	8				
12	6	2	..	..	..	Variation 1¼ points W.
1	6	0	NE. by E.	SE.	1½	
2	6	8				
3	6	8				
4	7	4				
5	6	4	SW. by S.	SE.	1	
6	5	6				
7	4	8	..	..	..	For the last 14 hours a current sets SE. by E., by compass, 1½ miles an hour.
8	4	2				
9	3	8	NE. ½ E.	SE. by S.	½	
10	4	2				
11	5	0				
12	5	0				

(6.) Required the course and distance, by Mercator's Sailing, from M. to N.:

Lat. of M. 51° 14′ N.	Long. of M. 12° 36′ W.
Lat. of N. 30° 56′ N.	Long. of N. 50° 3′ W.

Additional for Only Mate.

(7.) On October 27, 1866, at 5 h. 38 m. 35 s. P.M. apparent time at ship, in lat. 22° 0′ 27″ N., long. 120° 50′ 3″ W., the sun's setting magnetic amplitude was W. 28° 3′ S.; required the variation.

(8.) On October 27—A.M. at ship; lat. 38° 12′ S.; observed altitude of sun's lower limb, 35° 19′ 30″; index error, 1′ 53″ to subtract; height of eye, 21 feet; time by chronometer, October 27, 3 h. 12 m. 40 s., which had been found slow 3 m. 24 s. on mean Gr. time, September 12, and losing 1·8 s. daily; required the longitude.

Additional for Chief Mate.

(9.) On October 27, 1866, civil time, at 6 h. 25 m. 1 s. A.M. mean time at ship; lat. 53° 1′ S., long. 110° 22′ 15″ W.; observed altitude of sun's lower limb, 16° 10′ 50″, bearing by azimuth compass, E. ½ S.; index error, 2′ 25″ to add; height of eye, 24 feet; required the variation.

(10.) On October 27, 1866—P.M. at ship; lat. by account 36° 22′ S., long. 128° 35′ W.; the observed altitude of the sun's lower limb was 63° 48′, north of the observer, and near the meridian; index error, 1′ 32″ to subtract; height of eye, 21 feet; time by watch, October 27, 0 h. 54 m. 50 s., which had been found 4 m. 23 s. fast on apparent time at ship, but the ship had made 24′ dif. of long. to W. since the error for time had been found; required the true latitude.

Additional for Master Ordinary.

(11.) On April 1, 1866, the observed meridian altitude of β Orionis (Rigel) was 35° 43′ 20″ south of the observer; index error + 56″; height of the eye, 25 feet; required the latitude.

(12.) Correct the following courses for the local deviation of the compass (by the Table, page 402): SE. by E.; NW. by W. ½ W.; N. by E. ½ E.; W. by S.; E. by S.; SSE. ½ E.

Answers.

(1.) Log. 5·4041587 ∴ nat. number = 253605·5. (The product.)

(2.) Log. 2·3521825 ∴ nat. number = 225. (The quotient.)

(3.) Dif. of long. 4° 58′.

(4.) Sun's dec. 6° 43′ 4″ S.; zenith dist. 15° 37′ 26″; lat. 8° 54′ 22″ N.

(5.) True courses—S. 2¼ E. 18 miles; S. 4¼ W. 19 miles; N. ½ E. 26 miles; S. 2¼ W. 26 miles; N. 2¾ E. 27 miles; S. 3 W. 21 miles; N. 2¾ E. 18 miles; S. 6¼ E. 21 miles—course S. 52° E.; dist. 20·7 miles; lat. in 21° 12′ 32″ S., long. 55° 31′ 24″ E.

(6.) Mer. dif. lat. 1637·7; dif. long. 2247—course S. 53° 55′ W.; dist. 2068·5 miles.

(7.) App. Gr. time, Oct. 27, 13 h. 41 m. 55 s.; sun's dec. 13° 0′ 0″·1 S.; true amplitude, W. 14° 2′ 13″ S.; variation, 14° 0′ 29″ E.

(8.) Mean Gr. time, Oct. 27, 3 h. 17 m. 25 s.; sun's dec. 12° 51′ 28″; mean time at ship, Oct. 26, 20 h. 5 m. 10 s.; long. 108° 3′ 45″ W.

(9.) Mean time at Gr., Oct. 27, 0 h. 11 m. 30 s.; sun's dec. 12° 48′ 51″ S.; true azimuth, S. 90° 19′ 2″ E.; variation, 5° 56′ 32″ W.

(10.) App. time of obs. after noon, 49 m. 31 s.; app. time of obs. at Gr., Oct. 27, 9 h. 23 m. 51 s.; sun's dec. 12° 56′ 25″; arc 1 = 76° 45′ 42″; arc 2 = 23° 14′ 35″; lat. 36° 28′ 53″ S.

(11.) True dec. 8° 21′ 34″ S.; true altitude, 35° 38′ 6″; lat. 46° 0′ 20″ N.

(12.) Corrected courses—S. 48° 55′ E.; N. 71° 39′ 30″ W.; N. 19° 25′ 30″ E.; S. 68° 11′ W.; S. 68° 51′ E.; S. 24° 43′ 30″ E.

EXAMINATION PAPER, No. V.

For Second Mate.

(1.) Multiply 36500 by 2080, by common logarithms.

(2.) Divide 3876000 by 1200, by common logarithms.

(3.) On October 26, 1866, in longitude 115° 36′ W., the observed altitude of the sun's lower limb was 78° 12′ 35″, the sun being north of the observer; index error, 1′ 18″ to subtract; height of the eye, 19 feet; required the latitude.

(4.) In lat. 56° N., the departure made good was 161 miles; required the difference of longitude by Parallel Sailing.

(5.) By calculation on Mercator's principle, required the course and distance from H to K:

Lat. of H, 48° 34′ S.	Long. of H, 12° 13′ W.
Lat. of K, 27° 14′ S.	Long. of K, 13° 50′ E.

(6.) In the following day's work, correct the courses for

variation and leeway; find the course and distance made good, and the latitude and longitude arrived at, by inspection:

DAY'S WORK.

H.	K.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
1	7	0	S. by W.	ESE.	0	Noon. The departure is taken from Oumanack, lat. 52° 51′ N., long. 168° 50′ W., bearing by compass N. by E. distant 17 miles.
2	7	4				
3	7	6				
4	8	0	SW. by W.	SE. by E.	¼	
5	7	6				
6	7	4				
7	7	0	W. by S.	SE.	0	
8	7	0				
9	6	8				
10	6	8				
11	6	4				
12	7	0	SW. by S.	SSE.	1¼	Variation 1¼ east.
1	6	6				
2	6	4				
3	6	0				
4	6	0	E. by N.	S. by W.	¼	
5	7	0				
6	7	8				
7	7	2				
8	7	0	ENE.	South.	0	
9	7	0				
10	7	4	NE. by N.	SE. by S.	1	A current set the ship SSE. by compass 21 miles during the 24 hours.
11	7	6				
12	8	0				

Additional for Only Mate.

(7.) On October 30, 1866, the sun's setting amplitude was SW. by W. at 5 h. 11 m. 39 s. apparent time at ship, in lat. 40° N., long. 129° 42′ 30″ W.; required the variation.

(8.) On October 30, 1866—P.M. at ship; lat. 32° 43′ N., the observed altitude of the sun's lower limb was 28° 34′ 40″; index error, 2′ 10″ to subtract; height of the eye, 18 feet; time by chronometer, October 30, 11 h. 56 m. 43 s., which had been found 2 m. 28 s. slow on mean time at Greenwich, on October 1, at noon, and losing 2·4 s. daily; required the longitude.

Additional for Chief Mate.

(9.) On Oct. 30, 1866, civil time, at 7 h. 2 m. 59 s. A.M.

apparent time at ship; lat. 37° 59′ S.; long. 123° 28′ 30″ W.; the observed altitude of the sun's lower limb was 20° 36′ 40″, bearing by compass ESE.; index error, 1′ 50″ to add; height of the eye, 22 feet; required the variation.

(10.) On Oct. 30, 1866, civil time—A.M. at ship; lat. by account 40° 3′ S.; long. 150° 12′ E.; observed altitude of the sun's lower limb, near the meridian, 63° 11′ 30″, bearing N.; index error, 1′ 52″ to add; height of eye, 20 feet; time by watch, Oct. 20, 2 h. 30 m. 16 s., which had been found 2 h. 40 m. 12 s. fast for apparent time at ship, but the ship had made 21′ dif. long. to the east since the error for time had been determined; required the true latitude.

Additional for Master Ordinary.

(11.) Correct the following courses for local deviation (by Table, page 402): N. by W.; NW. by W.; SE. ½ E.; north; south; east; west.

(12.) On Oct. 31, 1866—long. 179° 30′ W.; observed the meridian altitude of α Aquilæ (Altair), 58° 52′ 40″; star north of the observer; index error, 2′ 8″ to subtract; height of the eye, 18 feet; required the latitude.

Answers.

(1.) Log. 7·8803562 ∴ nat. number = 75920000. (The product.)

(2.) Log. 3·5092026 ∴ nat. number = 3230. (The quotient.)

(3.) Sun's dec. 12° 34′ 38″ S.; zenith dist. 11° 36′ 58″; lat. 24° 11′ 36″ S.

(4.) Difference of longitude, 4° 48′.

(5.) Mer. dif. lat. 1543·4; dif. long. 1563; course N. 45° 21′ 41″ E.; dist. 1823·1 miles.

(6.) True courses—S. 2¾ W. 17 miles; S. 2¾ W. 22 miles; S. 7 W. 23 miles; N. 7¼ W. 34 miles; S. 6 W. 26 miles; S. 7¾ E. 28 miles; N. 7¾ E. 14 miles; N. 3¾ E. 23 miles; S. ¼

E. 21 miles; dif. lat. 47′·5 S.; dept. 41′·6 W.; course S. 49° W.; dist. 63 miles; lat. in 52° 3′ 30″ N.; long. in 169° 58′ 27″ W.

(7.) App. time at Gr., October 30, 13 h. 50 m. 32 s.; sun's dec. 13° 59′ 47″ S.; true amp. W. 18° 24′ 17″ S.; variation, 15° 20′ 43″ E.

(8.) Mean time at Gr., Oct. 30, 12 h. 0 m. 21 s.; sun's dec. 13° 58′ 30″ S.; mean time at ship, Oct. 30, 2 h. 30 m. 1 s.; longitude in 142° 35′ W.

(9.) App. time at Gr., Oct. 30, 3 h. 16 m. 53 s.; sun's dec. 13° 51′ 10″ S.; true azimuth, S. 88° 22′ 20″ E.; variation, 20° 52′ 20″ W.

(10.) App. time of obs. before noon, 8 m. 32 s.; app. Gr. time of obs., Oct. 29, 13 h. 57 m. 4 s.; sun's dec. 13° 40′ 12″ S.; arc 1 = 76° 17′ 26″; arc 2 = 26° 15′ 42″; lat. 39° 58′ 16″ S.

(11.) Corrected courses—N. 14° 25′ W.; N. 65° 33′ W.; S. 42° 33′ 30″ E.; N. 22′ W.; S. 16′ W.; S. 79° 16′ E.; S. 78° 10′ W.

(12.) True dec. 8° 31′ 8″ N.; true altitude, 58° 45′ 52″ N.; lat. 22° 43′ S.

EXAMINATION PAPER, No. VI.

For Second Mate.

(1.) Multiply 758900 by ·0135, by common logarithms.

(2.) Divide 1000 by ·125, by common logarithms.

(3.) In latitude 58°, the departure made was 214 miles; required the difference of longitude by Parallel Sailing.

(4.) On Oct. 12, 1866, in long. 63° 19′ W.; the observed altitude of the sun's lower limb was 44° 15′ 50″, the zenith being north of the sun; index error, 2′ 19″ to add; height of the eye, 24 feet; required the latitude.

(5.) In the following day's work, correct the courses for variation and leeway; find the course and distance made good, and the latitude and longitude, by inspection:

DAY'S WORK.

H.	K.	10ths.	Courses.	Winds.	Lee-way.	Remarks.
1	7	4	SSE. ½ E.	W. by S.	¼	Noon. I took my departure from light-house, Isle of May, lat. 56° 11′ 22″ N., long. 2° 32′ 47″ W., bearing by compass N. by W. distant 18 miles.
2	7	6				
3	8	0				
4	8	0				
5	7	8	SSE.	WSW.	0	
6	7	6				
7	7	4				
8	7	2				
9	6	8	SSE. ½ S.	West.	0	
10	7	2				
11	7	4				
12	7	6				
1	8	0	S. by E. ½ E.	WSW.	1½	
2	8	0	..	..	..	Variation, 2½ west.
3	8	4				
4	7	6				
5	7	8				
6	8	2				
7	8	0	SE. by S.	SW.	¼	A current set the ship, during the last 14 hours, 16 miles SE. by S. by compass.
8	7	6				
9	7	6				
10	7	6				
11	7	0				
12	7	2				

(6.) Required the course and distance, by Mercator's Sailing, from Cape Clear to Cape Finisterre:

Cape Clear, lat. 51° 24′ 55″ N., long. 9° 29′ 0″ W.
Cape Finisterre, lat. 42° 56′ 10″ N., long. 9° 16′ 15″ W.

Additional for Only Mate.

(7.) On Oct. 28, 1866, civil time—in lat. 52° 34′ N., long. 90° 35′ 15″ E., at 7 h. 11 m. 9 s. A.M. apparent time at ship, the sun's rising amplitude by compass was SSE.; required the variation.

(8.) On Oct. 28, 1866—P.M. at ship, in lat. 51° 12′ S., the observed altitude of the sun's lower limb was 20° 18′ 35″; index error, 1′ 17″ to subtract; height of eye, 21 feet; time by chronometer, Oct. 28, 1 h. 11 m. 16 s., which had been found 3 m. 12 s. slow on mean time at Greenwich, Oct. 5, at noon, and gaining 1·3 s. daily; required the longitude.

Additional for Chief Mate.

(9.) On Oct. 30, 1866, at 4 h. 36 m. 52 s. P.M. apparent time at ship; lat. 53° 17′ S., long. 31° 16′ 45″ E.; observed altitude of the sun's lower limb, 23° 16′ 35″, bearing by azimuth compass S. 79° 12′ W.; index error, 1′ 37″ to subtract; height of the eye, 24 feet; required the variation.

(10.) On Oct. 30, 1866—A.M. at ship; lat. by account 10° 50′ 30″ N., long. 47° 15′ W.; the observed altitude of the sun's lower limb was 65° 13′ 12″, bearing south; index error, 1′ 50″ to subtract; height of the eye, 20 feet; time by chronometer, Oct. 30, 3 h. 14 m. 16 s., which had been found fast on apparent time at ship 3 h. 21 m. 12 s., but the ship had made 14′ dif. long. to the *east* since the error for time had been determined; required the true latitude.

Additional for Master Ordinary.

(11.) Correct the following courses for local attraction (by Table, page 402): NNE.; NE. ½ E.; SSW.; SSE. ½ E.; WSW.; S. ½ E.

(12.) On December 31, 1866—in longitude 124° 12′ W., the observed meridian altitude α Leonis (Regulus) was 39° 18′ 20″ north of the observer; index error, 1′ 15″ to add; height of the eye, 24 feet; required the latitude.

Answers.

(1.) Log. 4·0105184 ∴ nat. number = 10245·15. (The product.)

(2.) Log. 3·9030900 ∴ nat. number = 8000. (The quotient.)

(3.) Difference of longitude = 6° 44′.

(4.) Sun's dec. 7° 28′ 59″ S.; zenith dist. 45° 31′ 25″ N.; lat. 38° 2′ 26″ N.

(5.) True courses—S. 3¼ E. 18 miles; S. 5 E. 31 miles; S. 4¼ E. 30 miles; S. 3¾ E. 29 miles; S. 5 E. 48 miles; S. 5¾ E. 45 miles; S. 5¼ E. 16 miles; dif. lat. 127·4 S.; dept. 172·5

E.; course S. $4\frac{3}{4}$ E.; dist. 214 miles; lat. in 54° 3′ 58″ N.; long. in 2° 28′ 55″ E.

(6.) Mer. dif. lat. 750·7; dif. long. 12′·75; course S. 0° 58′ E.; dist. 508·82 miles.

(7.) Apparent time at Gr., Oct 28, 1 h. 13 m. 30 s.; sun's dec. 13° 9′ 42″·2 S.; true amp. E. 22° S.; variation, 30′ west.

(8.) Mean time at Gr., Oct. 28, 1 h. 13 m. 58 s.; sun's dec. 13° 9′ 55″·1 S.; mean time at ship, Oct. 28, 4 h. 38 m. 29 s.; long. in 51° 7′ 45″ east.

(9.) App. time at Gr., Oct. 30, 2 h. 31 m. 45 s.; sun's dec. 13° 50′ 34″ S.; true azimuth, S. 98° 21′ 28″ W.; variation, 19° 9′ 28″ E.

(10.) App. time of obs., 6 m. before noon; app. time of observation at Gr., Oct. 30, 3 h. 3 m.; sun's dec. 13° 50′ 59″; arc 1 = 103° 51′ 16″; arc 2 = 24° 34′ 40″; lat. 10° 43′ 24″ N.

(11.) Corrected courses—N. 25° 50′ E.; N. 58° 41′ 30″ E.; S. 19° 14′ W.; S. 23° 55′ 30″ E.; S. 58° 12′ W.; S. 4° 38′ 30″ E.

(12.) True dec. 12° 36′ 58″ N.; true alt. 39° 13′ 40″; lat. 38° 9′ 22″ S.

ASTRONOMICAL TABLES.

DESCRIPTION OF ASTRONOMICAL TABLES.

(PAGES 422—425.)

The pages 1 and 2 from the *Nautical Almanac* for the month of October 1866 are here inserted because a great many of the Examples and Exercises contained in the work are given for that month. Page 1 contains the apparent right ascension and apparent declination of the sun, for the instant of apparent noon at Greenwich for each day of that month, with their hourly rate of change at the same instant.

These hourly changes are used for finding the app. R.A., and app. dec. at any other instant of Greenwich app. time, as follows: Take the difference between the dif. for one hour at the preceding and succeeding noons of the time for which the R.A. or dec. are sought, and state the following proportion: 24 hours is to *half* the time past the preceding app. noon, as the difference is to the correction to be applied to the dif. for 1 hour at the previous noon, to obtain the rate of change at the middle time. This correction is to be applied by addition to the dif. for 1 hour at the previous noon, when it is the less of the two, but by subtraction when it is the greater.

Thus, let it be required to find the sun's declination on October 15, 9 h. apparent time at Greenwich, we have $55''\cdot58 - 55''\cdot27 = \cdot31$ for the difference of the hourly change, and the half of 9 h. $= 4\frac{1}{2}$; hence,

$$24 \text{ h.} : 4\tfrac{1}{2} \text{ h.} :: \cdot31 : \cdot06 = \text{correction.}$$

This being subtracted from $55''\cdot58$ gives $55''\cdot52$, which, being multiplied by 9

the number of hours for which the correction is required, gives 499″·68 = 6′ 19″·68. This correction being added, since the declination is increasing, to that given at the previous noon, namely—

Declination at app. noon	=	S. 8° 32′ 14″·4
Correction 55·52 × 9	=	0° 6′ 19″·68
∴ Dec. Oct. 15, 9 h. app. Gr. T.	=	8° 38′ 34″·1 S

NOTE.—For nearly all the problems in Navigation, the correction will be obtained with sufficient accuracy by multiplying half the sum of the difs. for 1 hour at the previous and succeeding noons, by the number of hours and decimals of an hour for which the change of declination is required; which, in this case, would give the correction $\frac{55''\cdot58 + 55''\cdot27}{2}$ = 55″·42, which, being multiplied by 9, gives the correction to be applied to the declination 6′ 18″·78, being less than the former by ·9 of a second.

The column marked *Sidereal Time of the Sun's Semidiameter passing the Meridian* is useful for reducing a transit observation of either limb of the sun, when one only has been observed, to the transit of the centre. It is of no use at sea, since a transit cannot be observed aboard ship on account of the ship's motion.

The *Equation of Time* is the difference between apparent and mean time, and therefore serves for the conversion of either time into the other. The numbers here given, shew, for Greenwich apparent noon, the distance of the mean sun from the meridian, or the portion of time to be *added to or subtracted from* (according to the precept at the head of the column) Greenwich apparent noon, to obtain the corresponding mean time at the same meridian, or the time which ought to be shewn by the mean-time clock. It differs from the equation of time on page ii, because the equation itself varies in the interval between apparent and mean noon.

Where time is deduced from observations of the sun, the *immediate* result is *apparent* time; to convert it into mean time, the equation of time is necessary, and it is to be applied to apparent time, according to the precept at the head of the column. When it has to be corrected for a given instant of time, apply the dif. for 1 hour, exactly as directed above for declination.

PAGE 2.

The quantities here given under the name of the *Sun's Apparent Right Ascension* and *Declination* at mean noon, denote the *apparent* position of the true sun with reference to the equator, and the true equinox, at the instant the Greenwich mean-time clock indicates 0 h. 0 m. 0 s., or when the hour angle of the true sun is equal to the equation of time.

The Semidiameter of the Sun.—The numbers in this column express the

angle at the centre of the earth subtended by the sun's semidiameter, and are required for reducing observations of the limb to the centre, as in the instance of measuring the altitude of the sun's upper or lower limb, or the distance of the moon from the sun.

Equation of Time.—The numbers in this column are the values of the equation at the instant of mean noon, and therefore serve more particularly to convert *mean* into *apparent* time; for which purpose we have only to apply the equation according to the precept at the head of the column, and the *mean* time will be converted into the corresponding *apparent* time.

Sidereal Time at Mean Noon is the angular distance of the first point of Aries or the true vernal equinox from the meridian, at the instant of mean noon; it is therefore the right ascension of the mean sun, or the time which ought to be shewn by a sidereal clock at Greenwich, when the mean-time clock indicates 0h. 0m. 0s. The sidereal time here given is that in common use among astronomers, and expresses the actual hour-angle from the meridian, westward, of the true equinoctial point at the moment of observation. For further illustrations of the nature and uses of the quantities on these pages, see the Explanations in the *Nautical Almanac.*

TABLES OF THE FIXED STARS.

In these tables are given the mean right ascensions and declinations of the principal fixed stars for January 0 + 0·081 d., 1866, together with their annual variations.

North declination is distinguished by N., and south declination by S. The sign + prefixed to an annual variation of right ascension, indicates that the variation is to be *added to*, and the sign —, that it is to be *subtracted from*, the right ascension: also, for stars having *north* declination, + signifies *add*, and — *subtract;* but for stars of south declination, + denotes that the variation is to be *subtracted from*, and — that it is to be *added to*, the declination.

OCTOBER 1866 (PAGE 1, N. A.).

AT APPARENT NOON.

Day of the Week.	Day of the Month.	THE SUN'S *Apparent* Right Ascension.	Dif. for 1 hour.	*Apparent* Declination.	Dif. for 1 hour.	Sidereal Time of the Semidiameter passing the Meridian,*	Equation of Time, *to be subtracted from Apparent Time.*	Dif. for 1 hour.
		h. m. s.	s.	° ′ ″	″	m. s.	m. s.	s.
Mon....	1	12 29 36·39	9·059	S. 3 11 57.4	58·28	1 4·35	10 19·50	0·796
Tues....	2	12 33 13·95	9·072	3 35 15·0	58·19	1 4·40	10 38·44	0·782
Wed....	3	12 36 51·84	9·086	3 58 30·3	58·08	1 4·45	10 57·05	0·768
Thur....	4	12 40 30·08	9·101	4 21 42·8	57·96	1 4·50	11 15·32	0·754
Frid.....	5	12 44 8·67	9·116	4 44 52 3	57·82	1 4·55	11 33·23	0·739
Sat.....	6	12 47 47·65	9·133	5 7 58·3	57·67	1 4·60	11 50·76	0·722
Sun.....	7	12 51 27·03	9·150	5 31 0·5	57·50	1 4·66	12 7·88	0·705
Mon.....	8	12 55 6·82	9·167	5 53 58·4	57·32	1 4·72	12 24·59	0·687
Tues....	9	12 58 47·05	9·185	6 16 51·8	57·12	1 4·79	12 40·87	0·669
Wed.....	10	13 2 27·73	9·205	6 39 40·3	56·91	1 4·86	12 56·70	0·650
Thur....	11	13 6 8·87	9·225	7 2 23·4	56·68	1 4·93	13 12·07	0·630
Frid.....	12	13 9 50·49	9·245	7 25 0·7	56·43	1 5·00	13 26·96	0·610
Sat.....	13	13 13 32·61	9·266	7 47 32·0	56·16	1 5·08	13 41·36	0·589
Sun.....	14	13 17 15·23	9·287	8 9 56·6	55·88	1 5·16	13 55·25	0·567
Mon.....	15	13 20 58·39	9·309	8 32 14·4	55·58	1 5·24	14 8·60	0·545
Tues....	16	13 24 42·10	9·333	8 54 25·0	55·27	1 5·32	14 21·41	0·522
Wed.....	17	13 28 26·37	9·357	9 16 27·8	54·94	1 5·41	14 33·66	0·498
Thur....	18	13 32 11·21	9·381	9 38 22·5	54·60	1 5·50	14 45·33	0·474
Frid.....	19	13 35 56·65	9·406	10 0 8·7	54·24	1 5·59	14 56·42	0·449
Sat.....	20	13 39 42·70	9·432	10 21 46·0	53·86	1 5·68	15 6·89	0·423
Sun.....	21	13 43 29·38	9·459	10 43 14·1	53·47	1 5·78	15 16·74	0·397
Mon.....	22	13 47 16·71	9·486	11 4 32·6	53·06	1 5·87	15 25·94	0·370
Tues....	23	13 51 4·71	9·514	11 25 41·1	52·64	1 5·97	15 34·47	0·341
Wed.....	24	13 54 53·40	9·544	11 46 39·2	52·20	1 6·08	15 42·31	0·312
Thur....	25	13 58 42·80	9·574	12 7 26·6	51·75	1 6·18	15 49·44	0·282
Frid.....	26	14 2 32·94	9·605	12 28 2·9	51·28	1 6·28	15 55·84	0·251
Sat.....	27	14 6 23·83	9·636	12 48 27·7	50·78	1 6·39	16 1·49	0·220
Sun.....	28	14 10 15·47	9·668	13 8 40·6	50·28	1 6·50	16 6·38	0·188
Mon.....	29	14 14 7·90	9·701	13 28 41·3	49·76	1 6·61	16 10·50	0·155
Tues....	30	14 18 1·13	9·734	13 48 29·2	49·22	1 6·72	16 13·82	0·122
Wed.....	31	14 21 55·15	9·768	14 8 4·0	48·67	1 6·83	16 16·35	0·089
Thur....	32	14 25 49·97	9·801	S. 14 27 25·3	48·10	1 6·94	16 18·07	0·055

* *Mean* Time of the Semidiameter passing may be found by subtracting 0·18 s. from the *Sidereal* Time.

OCTOBER 1866 (PAGE 2, N. A.).

AT MEAN NOON.

Day of the Week.	Day of the Month.	The Sun's *Apparent* Right Ascension.	The Sun's *Apparent* Declination.	The Sun's Semi-diameter.*	Equation of Time, *to be added to Mean Time.*	Sidereal Time.
		h. m. s.	° ′ ″	′ ″	m. s.	h. m. s.
Monday.....	1	12 29 37·95	S. 3 12 7·4	16 1·4	10 19·64	12 39 57·59
Tuesday.....	2	12 33 15·56	3 35 25·3	16 1·6	10 38·58	12 43 54·14
Wednesday..	3	12 36 53·50	3 58 40·9	16 1·9	10 57·19	12 47 50·69
Thursday....	4	12 40 31·79	4 21 53·7	16 2·2	11 15·45	12 51 47·24
Friday.....	5	12 44 10·43	4 45 3·4	16 2·4	11 33·37	12 55 43·80
Saturday.....	6	12 47 49·45	5 8 9·7	16 2·7	11 50·90	12 59 40·35
Sunday.....	7	12 51 28·88	5 31 12·1	16 3·0	12 8·02	13 3 36·90
Monday.....	8	12 55 8·72	5 54 10·3	16 3·3	12 24·73	13 7 33·45
Tuesday.....	9	12 58 48·99	6 17 3·9	16 3·5	12 41·01	13 11 30·00
Wednesday..	10	13 2 29·71	6 39 52·6	16 3·8	12 56·84	13 15 26·55
Thursday....	11	13 6 10·90	7 2 35·9	16 4·1	13 12·21	13 19 23·11
Friday.....	12	13 9 52·56	7 25 13·4	16 4·4	13 27·10	13 23 19·66
Saturday.....	13	13 13 34·72	7 47 44·8	16 4·6	13 41·49	13 27 16·21
Sunday.....	14	13 17 17·39	8 10 9·6	16 4·9	13 55·38	13 31 12·77
Monday.....	15	13 21 0·59	8 32 27·5	16 5·2	14 8·73	13 35 9·32
Tuesday.....	16	13 24 44·34	8 54 38·2	16 5·5	14 21·53	13 39 5·87
Wednesday..	17	13 28 28·64	9 16 41·1	16 5·7	14 33·78	13 43 2·42
Thursday....	18	13 32 13·52	9 38 35·9	16 6·0	14 45·45	13 46 58·97
Friday.....	19	13 35 58·99	10 0 22·2	16 6·3	14 56·53	13 50 55·52
Saturday.....	20	13 39 45·08	10 21 59·6	16 6·6	15 7·00	13 54 52·08
Sunday.....	21	13 43 31·79	10 43 27·8	16 6·8	15 16·84	13 58 48·63
Monday.....	22	13 47 19·15	11 4 46·3	16 7·1	15 26·03	14 2 45·18
Tuesday.....	23	13 51 7·18	11 25 54·8	16 7·4	15 34·56	14 6 41·74
Wednesday..	24	13 54 55·90	11 46 52·9	16 7·7	15 42·39	14 10 38·29
Thursday....	25	13 58 45·33	12 7 40·3	16 7·9	15 49·51	14 14 34·84
Friday.....	26	14 2 35·49	12 28 16·6	16 8·2	15 55·91	14 18 31·40
Saturday.....	27	14 6 26·40	12 48 41·3	16 8·4	16 1·55	14 22 27·95
Sunday.....	28	14 10 18·07	13 8 54·1	16 8·7	16 6·43	14 26 24·50
Monday.....	29	14 14 10·52	13 28 54·7	16 8·9	16 10·54	14 30 21·06
Tuesday.....	30	14 18 3·76	13 48 42·5	16 9·2	16 13·85	14 34 17·61
Wednesday..	31	14 21 57·79	14 8 17·2	16 9·4	16 16·37	14 38 14·16
Thursday....	32	14 25 52·63	S. 14 27 38·4	16 9·7	16 18·09	14 42 10·72

* The Semidiameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

FIXED STARS, 1866 (FROM N. A.).

MEAN PLACES FOR JANUARY 0 + ·081 d.

Star's Name.	Mag.	Right Ascension.	Annual Variation.	Declination.	Annual Variation.
		h. m. s.	s.	° ′ ″	″
α Andromedæ.	2	0 1 27·894	+ 3·0857	N. 28 21 2·02	+19·900
γ Pegasi (*Algenib*)	3.2	0 6 20·225	3·0807	N. 14 26 18·29	20·028
β Hydri	3	0 18 39·799	3·2834	S. 78 0 35·40	20·249
α Cassiopeæ	var.	0 32 55·249	3·3601	N. 55 48 7·08	19·808
β Ceti	2	0 36 51·603	3·0124	S. 18 43 22·18	19·817
α Ursæ Minoris (*Polaris*)	2	1 9 57·614	19·6357	N. 88 35 42·29	19·127
α Eridani (*Achernar*)	1	1 32 43·085	2·2354	S. 57 55 5·20	18·424
ν Piscium	5.4	1 34 27·546	3·1135	N. 4 48 30·16	18·332
β Arietis	3.2	1 47 14·476	3·2955	N. 20 9 6·08	17·787
α Arietis	2	1 59 37·450	3·3656	N. 22 49 37·93	17·234
γ Ceti	3.4	2 36 21·506	3·1009	N. 2 40 8·38	15·382
α Ceti	2.3	2 55 16·531	3·1269	N. 3 33 42·59	14·357
α Persei	2	3 14 46·188	4·2462	N. 49 22 52·16	13·187
η Tauri	3	3 39 31·335	3·5517	N. 23 41 17·37	11·476
γ^1 Eridani	3	3 51 46·617	2·7943	S. 13 53 30·97	10·526
α Tauri (*Aldebaran*)	1	4 28 14·042	3·4349	N. 16 14 13·61	7·640
ε Leporis	4.3	4 59 47·267	2·5357	S. 22 33 11·94	5·137
α Aurigæ (*Capella*)	1	5 6 47·653	4·4211	N. 45 51 27·92	4·180
β Orionis (*Rigel*)	1	5 8 5·892	2·8797	S. 8 21 33·02	4·481
β Tauri	2	5 17 49·338	3·7870	N. 28 29 26·57	3·469
δ Orionis	2	5 25 9·719	3·0639	S. 0 24 4·59	2·997
α Leporis	3	5 26 49·272	2·6459	S. 17 55 13·92	2·895
ε Orionis	2	5 29 24·850	3·0413	S. 1 17 25·21	2·651
α Columbæ	2	5 34 48·010	2·1777	S. 34 8 48·91	2·199
α Orionis	var.	5 47 55·042	3·2460	N. 7 22 44·46	+ 1·051
α Argûs (*Canopus*)	1	6 20 58·762	1·3302	S. 52 37 24·71	− 1·834
γ Geminorum	2.3	6 29 58·236	3·4662	N. 16 30 38·52	2·562
α Canis Majoris (*Sirius*)	1	6 39 14·702	2·6452	S. 16 32 5·94	4·643
ε Canis Majoris	2.1	6 53 21·595	2·3579	S. 28 47 31·51	4·640
α^2 Geminorum (*Castor*)	2.1	7 26 2·784	3·8423	N. 32 10 44·72	7·431
α Canis Minoris (*Procyon*)	1	7 32 17·109	3·1448	N. 5 33 58·03	8·892
β Geminorum (*Pollux*)	1.2	7 37 6·739	3·6818	N. 28 20 48·81	8·305
ε Hydræ	3.4	8 39 40·697	3·1840	N. 6 54 30·33	12·909
ι Ursæ Majoris	3	8 50 1·063	4·1419	N. 48 33 54·93	13·826
ι Argûs	2	9 13 30·141	1·6017	S. 58 42 47·34	14·922
α Hydræ	2	9 21 0·095	2·9486	S. 8 4 45·97	15·389
θ Ursæ Majoris	3	9 23 52·608	4·0541	N. 52 17 9·44	16·139
ε Leonis	3	9 38 14·403	3·4195	N. 24 23 22·60	16·360
α Leonis (*Regulus*)	1.2	10 1 13·967	3·2026	N. 12 37 14·97	17·414
γ^1 Leonis	2	10 12 34·855	3·3169	N. 20 31 5·12	18·039
η Argûs	2	10 39 52·125	2·3085	S. 58 58 47·73	18·750
α Ursæ Majoris	2	10 55 26·102	3·7685	N. 62 28 24·51	19·353
δ Leonis	2.3	11 6 58·702	3·2030	N. 21 15 26·14	19·666
δ Hydræ et Crateris	3·4	11 12 38·524	2·9947	S. 14 3 14·32	19·451
β Leonis	2	11 42 13·322	3·0651	N. 15 19 15·78	20·097
γ Ursæ Majoris	2.3	11 46 46·152	3·1912	N. 54 26 22·91	20·026
ε Corvi	3	12 3 14·199	3·0751	S. 21 52 27·46	20·046
η Virginis	3.4	12 13 2·985	3·0649	N. 0 4 41·87	20·056
α^1 Crucis	1	12 19 9·648	+ 3·2624	S. 62 21 18·44	−19·934

FIXED STARS, 1866 (FROM N.A.).

Continued.

MEAN PLACES FOR JANUARY 0 + ·081 d.

Star's Name.	Mag.	Right Ascension.	Annual Variation.	Declination.	Annual Variation.
		h. m. s.	s.	° ′ ″	″
β Corvi	2.3	12 27 20·983	+ 3·1312	S. 22 39 19·87	−19·980
γ^1 Virginis	3.2	12 34 52·265	3·0373	S. 0 42 52·35	19·869
α Virginis (*Spica*)	1	13 18 8·136	3·1502	S. 10 27 39·71	18·939
ζ Virginis	3·4	13 27 52·024	3·0520	N. 0 5 25·77	18·534
η Ursæ Majoris	2	13 42 15·438	2·3732	N. 49 58 58·69	18·114
η Bootis	3	13 48 18·239	2·8582	N. 19 4 13·96	18·210
β Centauri	1	13 54 23·618	4·1585	S. 59 43 28·91	17·676
α Bootis (*Arcturus*)	1	14 9 32·962	2·7337	N. 19 52 53·05	18·913
ρ Bootis	4·3	14 26 3·301	2·5869	N. 30 57 39·96	15·981
α^2 Centauri	1	14 30 32·187	4·0332	S. 60 16 39·01	15·038
ι Bootis	2.3	14 39 8·026	2·6194	N. 27 38 25·97	15·406
α^2 Libræ	2.3	14 43 28·113	♈ 3·3052	S. 15 28 58·73	15·224
β Ursæ Minoris	2	14 51 7·863	− 0·2530	N. 74 42 10·26	14·755
β Libræ	2	15 9 47·884	+ 3·2184	S. 8 53 10·88	13·575
α Coronæ Borealis	2	15 29 0·840	2·5377	N. 27 10 3·01	12·348
α Serpentis	2.3	15 37 40·063	2·9489	N. 6 50 57·68	11·612
β^1 Scorpii	2	15 57 38·893	3·4769	S. 19 26 9·40	10·223
δ Ophiuchi	3	16 7 19·442	3·1359	S. 3 20 48·72	9·588
α Scorpii (*Antares*)	1.2	16 21 11·668	3·6660	S. 26 7 54·02	8·407
η Draconis	3.2	16 22 11·555	0·8228	N. 61 49 5·82	8·221
α Trianguli Australis	2	16 34 30·437	6·2783	S. 68 46 34·59	7·385
ζ Herculis	3.2	16 36 14·112	2·2623	N. 31 50 50·97	6·721
κ Ophiuchi	3·4	16 51 19·602	2·8339	N. 9 35 8·65	5·896
α Herculis	var.	17 8 32·208	2·7320	N. 14 32 43·71	4·420
θ Ophiuchi	3·4	17 13 46·852	3·6762	S. 24 51 43·38	3·999
β Draconis	3.2	17 27 24·296	1·3505	N. 52 24 6·00	2·839
α Ophiuchi	2	17 28 42·826	2·7807	N. 12 39 36·15	2·937
μ Herculis	3·4	17 41 12·849	2·3424	N. 27 48 3·46	2·376
γ Draconis	2·3	17 53 29·708	1·3923	N. 51 30 20·52	− 0·608
α Lyræ (*Vega*)	1	18 32 24·033	2·0304	N. 38 39 38·77	+ 3·115
β Lyræ	var.	18 45 7·865	2·2120	N. 33 12 31·51	3·896
ζ Aquilæ	3	18 59 14·902	2·7521	N. 13 40 0·24	5·056
δ Aquilæ	3·4	19 18 44·422	3·0240	N. 2 51 0·51	6·858
γ Aquilæ	3	19 39 53·251	2·8515	N. 10 17 20·31	8·472
α Aquilæ (*Altair*)	1.2	19 44 14·635	2·9274	N. 8 31 0·26	9·194
α^2 Capricorni	3.4	20 10 36·964	3·3327	S. 12 57 28·40	10·823
α Pavonis	2	20 15 1·999	4·7978	S. 57 9 38·10	11·117
α Cygni	2.1	20 36 51·788	2·0429	N. 44 48 10·24	12·681
α Cephei	3.2	21 15 22·764	1·4359	N. 62 1 5·95	15·107
β Aquarii	3	21 24 30·092	3·1628	S. 6 9 32·37	15·620
β Cephei	3	21 26 55·183	0·8011	N. 69 58 21·43	15·704
ι Pegasi	2.3	21 37 36·255	2·9480	N. 9 15 43·42	16·310
α Aquarii	3	21 58 53·945	3·0825	S. 0 58 11·05	17·308
α Gruis	2	21 59 46·483	3·8162	S. 47 36 28·72	17·179
ζ Pegasi	3.4	22 34 46·648	2·9870	N. 10 7 58·09	18·688
α Pis. Aus. (*Fomalhaut*)	1.2	22 50 14·325	3·3296	S. 30 19 54·21	18·960
α Pegasi (*Markab*)	2	22 58 5·204	2·9830	N. 14 29 5·81	19·307
γ Cephei	3·4	23 33 52·260	+ 2·3989	N. 76 53 4·55	+20·077

COMMERCIAL CHAPTER.

FLOATING property and population, and the connected interests, have special laws, courts, regulations, and officials for their protection and guidance. The department of the British government which undertakes the general superintendence of matters relating to merchant ships and seamen, is known as the *Board of Trade*, described in the Merchant Shipping Act as 'the Lords of the Committee of Privy Council appointed for the consideration of matters relating to trade and foreign plantations.' The special law courts are, the *High Court of Admiralty* in England, the highest tribunal for the hearing of maritime cases, civil and criminal; *Vice-Admiralty Courts*, held in various parts of the British Empire; and *Naval Courts*, consisting of from three to five members—namely, an officer in Her Majesty's royal navy not below the rank of lieutenant, a British consul, one or more masters of British vessels, or two British merchants, summoned when and where necessary, for the trial of offences against the Merchant Shipping Act. British *consuls*, or substitutes, are also appointed at most foreign ports for the protection of the commerce and subjects of the empire.

At the chief home-ports, the Board of Trade has offices, called mercantile marine or *shipping-offices*, for engaging and discharging crews, giving orders to enable seamen to send

money to their friends, receiving their deposits for the seamen's savings-banks, and other business, each with a superintendent, or, as he is generally called, a *shipping-master*. There are also the various *custom-houses* for the collection of the duties on goods exported or imported, the detection of smuggling, and the recording of commercial statistics; with bonded warehouses, revenue-cruisers, searchers, and superintending *collectors*, who act besides for shipping-masters where there is no shipping-office established. Various districts have *receivers of wreck*, usually the same person as the collector of customs, commissioned to take charge of all wreck and wrecked property coming ashore, to assist in saving life and property, to take command of all persons assisting, and assign them their duties, to demand the free use of boats, wagons, horses, and such like in the neighbourhood, if such help is needed, and to inquire into the circumstances and cause of the wreck. There are also *notaries-public*—persons authorised to draw up official statements made by a ship-master regarding damage by heavy weather, failure of merchants to furnish cargoes, and such like, called noting and extending *protests;* where there is no notary, a consul or a justice-of-the-peace acts instead.

To be entitled to the name, privileges, and advantages of a *British ship*, a ship must be owned by one or more natural-born British subjects who have not lost their rights as such, or by persons naturalised and resident, or bodies corporate, subject to the laws, and having their chief place of business in Her Majesty's dominions; and every vessel, except very small craft, must be registered at a port, for that reason called her *port of registry*, and her name, description, tonnage, official number, signal-flags, and owner's and master's names entered on a document called the *ship's register*.

The commander of a British ship is called the *master*, popularly the captain, who in general must hold a certificate from the Board of Trade, to entitle him to act in that capacity. Usually, also, the first and second mates must hold certificates. The rest of the mariners consist of able seamen, ordinary sea-

men, and apprentices or boys. The master, officers, and crew may belong to any nation. In certain places, as entrances to harbours, arms of the sea, and channels, where the navigation of a ship is attended with more than ordinary difficulty from local causes, it is compulsory to take on board a person called a *pilot*, who is licensed to take charge of ships in such places.

Vessels which trade on the coasts of the United Kingdom are termed *coasters;* those on the same coasts, or trading to places on the coast of Europe between Brest and the river Elbe, are said to be engaged in the *home-trade*, and vessels trading to ports outside those limits are said to be engaged in the *foreign trade.*

The law holds a ship-owner, and therefore also the ship-master, to be a common carrier, and all common law binding on carriers in general is binding on the ship-owner; but as he has many dangers and obstacles to contend with, as wreck, capture, piracy, mutiny, &c. which carriers by railway, canal, or road generally escape, his agreements are made to suit the emergencies and dangers of the undertaking.

The use of merchant ships being to carry goods and passengers from one port to another, there are usually three persons, or bodies of persons, concerned. First, there is the *ship-owner*, who provides the vessel; second, the *merchant*, who supplies or receives the cargo, and who pays the ship-owner a sum, called freight, for its carriage; and, third, the *insurer* or underwriter, who insures the vessel and freight for the ship-owner, and the cargo for the merchant, if they so please. The ship-master is the accredited agent of the ship-owner, and, indeed, in cases of difficulty, where no instructions can be got, is sometimes compelled to act as agent for both cargo-owner and insurer. His powers are extensive, but must be cautiously exercised; and in making contracts for the employment of the ship, receiving freights, and drawing bills on his owners, or in his more critical duties, as pledging the ship, freight, or cargo, selling part of the cargo, repairing the vessel, or selling her remains, he is expected to have everything minutely detailed,

verified, and justified documentarily. The merchant or cargo-owner is called the exporter, shipper, freighter, or consignor when he supplies the cargo, and the importer or consignee when he receives it. The shipper may have, and often has, an accredited agent to look after his interests at the port where the cargo is to be delivered. The insurer, usually called the underwriter, may be a private individual, an association of ship-owners for mutual insurance, or a company, with or without charters; in fact, any one may be an insurer. Insurances are generally effected by persons called *insurance-brokers*, who act as middlemen between the insurer and the insured. The surveying, classification, and registering of vessels for insurance purposes for all parties, individuals, and clubs, are managed by a society whose affairs are superintended by a committee of ship-owners, merchants, and underwriters, consisting of thirty-five members—twenty-four elected in London, and eleven at the out-ports, meeting at a set of rooms, called *Lloyd's*, in London. Their classes of ships are A1, A in red, Æ, E, and I. The agents appointed at most ports for the protection of the interests of insurers, whether individuals or associations, are usually called *Lloyd's agents*.

That there may be no misunderstanding as to the contracts made by the ship-owner and merchant, or between them and the underwriter, several documents are drawn up for each engagement of a ship; and as all disputes are settled by the terms of these documents, read in the light of Acts of Parliament, previous decisions, and common justice, it is necessary to have them worded clearly and fully, and to understand, by close study of their terms, the duty and liability of each party concerned. The principal documents are three—the *charter-party*, or the stamped agreement made between the shipper and the ship-owner; the *bill of lading*, or the stamped receipt signed by the ship-master when he receives the goods on board, binding him to carry the cargo to the place the merchant has agreed for; and the *policy of insurance*, or the stamped contract of insurance of ship, freight, or cargo. It is requisite for the ship-master to

obtain and keep all other documents or forms necessary for the lawful sailing of the ship. Among them are the *official* and *ship's log*, in which must be entered all births, deaths, and marriages on board, the character of each of the crew, desertions, transfers of seamen to the royal navy, sales of deceased seamen's effects, offences either punished or to be prosecuted for, all damage to ship and cargo or casualties however slight, strandings, collisions, protests, surveys of ship or cargo, and similar matters, all properly certified and witnessed, a full and faithful record of such matters being the best means of insuring confidence and dispatch when settlements and adjustments have to be made; the *ship's articles*, in which are recorded the names and signatures of the crew, their wages, and conditions regarding their food, medicines, &c.; the *victualling-bill*, a warrant obtained by the ship-master at the custom-house to ship stores required for the use of the crew and the passengers, and containing a list of such stores; a *bill of health;* and any other special warrants required in times of sickness or war, or by special treaties. Ships also carry a full statement of the cargo, when and where shipped, and by and for whom, and for what place, in a *cargo-book* or *manifest*.

Suppose now that, in the ordinary flow of commerce, it is required to send a cargo from this country to some port outside the coasting limits. An agreement is at once made between the person or persons having cargo to send and a ship-owner. This agreement is the charter-party, and the ship is said to be chartered. The ship-owner binds himself to supply a good and tight ship properly and fully found in every particular to enable her to proceed on her voyage expeditiously and safely, which includes a competent master and officers, and a sufficiently numerous and skilful crew, abundant store of food, water, and medicines, and all the necessary documents, as well as a strong hull, good masts, stout sails and rigging, and sufficient ballast. If any damage happens through failure in these respects, the ship-owner must bear his own loss, and is liable besides to an action for damages at the suit of the shipper. The shipper binds

himself to have ready and ship a certain amount of cargo; and the bargain about freight and various other matters is often entered in this agreement. If one party fails to be ready at the appointed time and place, the other may seek another ship or cargo, and put in a claim for loss sustained.

Certain custom-house formalities must now be observed. If the goods are bonded or warehoused, the exporter presents a list of them in a *bond-note* at the custom-house, and finds security that the goods shall be shipped, exported, and landed at the place for which they are destined, or otherwise satisfactorily accounted for to the Commissioners of Customs. If the goods are free, the bond-note is dispensed with. In all cases, particulars of the cargo to be shipped are entered on papers called *shipping-bills*, and presented at the Customs, the master also signifying his intention to load or enter outwards. The goods thus entered in the shipping bill or bills may then be shipped under the eye of the Customs' searcher, after which the items in the various shipping-bills are collected in one document called a *content*. Just before sailing, the ship-master presents this content to the collector, together with his victualling-bill for the ship's stores, and the receipts for light-dues, &c. These papers form a *clearance* outwards, which fact is attested at the custom-house by a small card, called a *cocket*, being stamped and sealed to the victualling-bill; and this stamped document the master carries to sea, as his authority from the Customs for sailing. The masters of coasting-vessels require simply to present their cargo-books at the Customs, and receive a permit to sail, called a *transire*. No vessel can legally proceed to sea without a clearance or transire.

When the shipping-bills are filled up, the goods are sent by the shipper to the quay alongside the ship, or sent off in his boats to the place where the ship is lying ready, and the ship-master becomes responsible for the goods the moment they are thus brought to his ship. Usually some person, as the mate, receives the goods, gives an ordinary receipt for them, and enters them in the cargo-book. The bills of lading, which are the

special receipts, are usually signed by the master after the goods named in them are actually on board, and he should be sure to recall the first receipts before doing so. If necessary, there are separate bills of lading for each separate part of the cargo. The master should be very careful what he is signing on these bills, as he thereby binds himself to deliver the goods to the consignee named, on payment of freight and other charges, in like good condition as he receives them, except so far as he is protected by the saving-clause, that he shall not be liable for damage arising from the act of God (winds, storms, and natural causes), the king's enemies (pirates and other foes), fire, and all and every other dangers and accidents of the seas, rivers, and navigation of whatever nature and kind (foundering, stranding, &c.), and any other exceptions to suit the particular voyage which may be agreed upon; that is, as a common carrier he binds himself to carry and deliver the goods unless prevented by irreparable disaster, and that alone. He should therefore be careful not to sign for goods he does not get, nor to sign that they are in good condition when he only has the shipper's word for it. Mistakes and want of caution in signing are a fruitful cause of dispute. The bill of lading often serves the purpose of a charter-party also.

The merchant is only allowed a certain time, called *lay-days*, for stowing the cargo, and subjects himself to a daily payment, called *demurrage*, if he detains the ship over time—the master, in such a case, noting a protest at the end of the lay-days before claiming the demurrage.

The master is responsible for the judicious stowing of the cargo, except where the shipper appoints a person called a *stevedore* to ship and stow it in the hold.

The ship being now ready for sea, and the shipping-office and Customs regulations all complied with, dues paid, crew engaged and shipped, the ship's register, articles, and log-book, victualling-bill and clearance, and all other requisite papers, copies of the charter-parties, bills of lading, and insurance policies on board, and a pilot, if one be necessary, the ship sets sail.

The master must now strictly follow the route agreed on, or

the shortest route, to the port or ports at which he is to call, with as much speed as may be, and he must on no account deviate from that course without justifiable necessity. *Deviation*, indulged in without sufficient reason, clears the underwriter from all responsibility for loss happening after the time of deviation, and the ship-owner is liable to the shipper for the value of the goods if they should afterwards be lost. Suppose, however, that the voyage has been a prosperous one in every respect, and that the port of destination is safely reached. The master must hoist his signal for a pilot, and not attempt to enter harbour himself till he has waited a reasonable time without being able to obtain assistance, and any regulations in force at the port must be acted up to. On being safely moored, the master must immediately report his ship, crew, and cargo to the chief officer of Customs, as well as to the British consul, from whom he has to obtain the requisite clearance, and any other document by which landing of the cargo is permitted; the ship's articles must also be delivered to either of these authorities, who keeps them while the ship remains there. It is likewise the practice, especially if bad weather has been experienced, to note a protest against winds and weather before a notary or the consul; but in doing so, both in this and any other case of protest, no statement should be made but such as can be clearly proved by the ship's log.

And now comes a most important question. To whom shall the ship-master deliver the cargo? While the ship has been pursuing her voyage, the shipper has posted to his agent at the port of destination, or to the consignee there, invoices of the goods, and a copy of the bills of lading indorsed to the party who is to receive the goods. The master must therefore deliver the goods to the party who presents to him the *indorsed* bill of lading. If the consignee should have become bankrupt or nearly so, however, and the goods are not really his own, shipped to his own order and paid for, but are actually the property of the shipper, the shipper may instruct the ship-master to detain the goods, and the master must so detain them. This is called *stoppage in transitu*. The party receiving the goods must, on

their delivery, pay the freight due in terms of the agreement for the carriage of the goods, and any other charges or claims which are exigible, otherwise the ship-owner has a lien on the goods until a settlement is made. If the consignee is not liable, the shipper is; but the bill of lading ought to be clear in its terms regarding the payment of the freight. The consignee only has the lay-days in which to discharge the cargo, and must pay demurrage, if he detains the vessel, for overtime.

Thus, in a voyage without accident or delay, the ship-owner earns his freight, the shipper has his goods landed according to his bargain, and the underwriter clears his premium, and thus all interests are benefited.

In getting another charter and cargo for the return-voyage home or elsewhere, all local requirements must be satisfied, and the usual documents and forms obtained; but any special information will be obtained from the merchant or consul, or instructions may probably be received by post or telegraph from home.

In arriving home with a cargo from abroad, the Customs officers must have free access to every part of the ship. The master must report his ship within twenty-four hours at the custom-house, shew the bills of lading there, if they are asked for, and answer all the collector's questions, the importer seeing to the entry of the cargo and the payment of duties; and he must deliver his log-book and ship's articles at the shipping-office, and see his crew properly discharged there, and their wages paid so far as due. All smuggling is punished by forfeiture and fines or imprisonment.

It remains to explain the courses to be adopted in cases of difficulty at sea, or at an intermediate port. And here it will be seen that the great increase of regularly running steam mail-packets, and the extension of land and ocean telegraphic wires, will enable the ship-master, in many cases of difficulty on shore, to communicate with the ship-owner, cargo-owner, and underwriter; and this is the proper course to take when it is practicable. Where this is not possible, he should get the very best advice he can from the consul, Lloyd's agent, or other persons

of trust, and act according to the best of his judgment—never, on any consideration, however great the difficulty, giving up his authority and command to any person, whether Lloyd's agent or another, and constantly keeping in mind that his great duty is to deliver the cargo, with as little delay as possible, to the proper party at the port named in the charter-party.

If, in the course of the voyage, bad weather or the dangers of the sea damage the ship or spars so that she cannot proceed safely without repairs, and she consequently puts into some intermediate port, none but the most necessary repairs are permitted, and the ship must proceed with as little delay as possible to resume the course from which she has thus lawfully deviated. Wanting medical assistance, aiding ships in distress, avoiding capture, and the forcible carrying of the vessel out of her course by mutineers or pirates, are also regarded as justifiable reasons for deviation.

If, on careful survey at an intermediate port, it is found that the ship is too seriously injured to pursue her voyage, the master should try to forward the cargo by another ship, or in some other way. Often, however, in such cases, an amicable arrangement is made with the charterer or his representative, by which a part of the freight, in proportion to the part of the voyage accomplished, called a *pro rata* freight, is paid, and the cargo is given up.

In case of wreck, the master and crew are bound, without any claim for salvage, to use the utmost exertions and endeavours to save vessel and cargo; indeed, wages are forfeited if the opposite can be proved. The master should accept such offered assistance from salvors as he may deem proper, but he can repel by force any one intruding into his vessel without his leave.

If the vessel breaks up, he should secure as much cargo as possible, and the ship's papers, particularly the log-book. He should stay by the wreck while any chance of saving anything remains, and should communicate with the ship-owner, shipper or consignee, and underwriter. He should enter a full and minute account in the log-book, and note a protest before a

notary; and copies of protests, besides all surveys of ship or cargo, should be sent in this and other cases to the ship-owner and shipper.

If it is impossible by any exertions or experiment to restore the wreck to a sea-worthy state, the master may sell it, but he should consult the best authorities first, and should also, if it be possible in any way, obtain permission from the owner. Cargo saved should be forwarded, returned to the port of lading, or warehoused till instructions arrive from the shipper. If the goods are of an extremely perishable nature, and it is impossible to communicate with the shipper, they may be sold to avoid utter destruction.

Full and complete documentary evidence of procedure for the justification of the master's conduct in such matters, and authentic copies of protests, surveys, and salvage agreements, awards, and vouchers, should be preserved; and in case of repairs to the vessel necessitating the discharge of the cargo, the various items of expense for unloading, warehousing, and reshipping, should be kept distinct from each other.

In cases of collision between vessels,* the owner of the vessel which is to blame is liable; and where both vessels are to blame, the total loss is added up and divided equally between them.

In settling claims for salvage, the ship-master usually executes a salvage-bond, binding himself to act up to the award of the court or the arbitrator, and giving security for performance.

Vessels cannot be cleared for a blockaded port, but if the blockade happens after a legal agreement is made, the ship-owner is not discharged. The rule seems to be, to wait for the removal of the blockade, so as to deliver the cargo and earn the freight, or to carry it to the nearest port, where it can be profitably sold.

To raise money on a voyage, the master may draw bills on his owner, or raise money on his own credit. When all other

* See rules for avoiding collisions, pages 392–399.

means fail, he may hypothecate or pledge the ship's bottom, or the ship and freight, or the ship, freight, and part of the cargo. It should be done by advertisement, the lowest offer of maritime interest, which is very high, being accepted, and the lender must satisfy himself that there is a necessity for the expenditure to meet which the bond, usually called a *bottomry bond*, is executed. Where several bonds are raised, the last is payable first, and the first last: if the vessel arrives safely, the lender is paid his principal and interest; if not, he loses his money. This method of raising money cannot be too cautiously resorted to, and communication should be made with the owner when possible. The master has not the right to sell cargo at an intermediate port to raise money, unless the sanction of all parties concerned can be obtained, except in cases of dire necessity, and then the sale must be proved to have been a necessitous one. No court can order such a sale.

To meet troubles arising between the ship-master and his crew, the Merchant Shipping Act contains a number of statutory punishments to be inflicted after prosecution before the proper court, for drunkenness, desertion, disobedience, mutiny, &c.; and a full detail of the offence is to be entered in the official log-book to enable the master to prosecute. This does not interfere with the master's right to use lawful force to repress mutiny or violence, or to inflict moderate punishment after consulting with his officers. For undeserved or excessive punishment, the seaman can sue the master for damages in the ordinary courts, and the master is bound to allow a seaman to go ashore at any port, on his application, to make a complaint to a justice-of-the-peace, consul, or naval officer in command of one of Her Majesty's ships. The ship-master can only be deprived of his command by a vice-admiralty or naval court on being proved grossly unfit for his duty.

The general duties and responsibilities of the various parties in such commercial transactions are thus seen. The plan they adopt to meet the expense of losses by the perils of the sea is the ordinary one of insurance. The underwriter will insure the

ship-owner against the entire loss of the ship, and against the loss of spars, sails, cables, &c. carried away in heavy weather, in getting off a lee-shore, or escaping from an enemy. Ordinary wear and tear of the ship, and damage by rats and other vermin, are not insured against, and in charging the underwriter for expense of repairs for which he may be liable, it is the custom to deduct one-third of the expense of both material and labour. The owner may also insure against loss of his freight.

The underwriter will insure the cargo-owner against total loss of his goods, and for partial loss under certain conditions and exceptions stated in a foot-note in the policy. The term *standing* used in this foot-note signifies, in law, 'taking the ground under any extraordinary circumstances of time or place by reason of some unusual or accidental occurrence.'

The loss short of total which each party concerned sustains and bears as his own is generally called *particular average.*

It may often happen, however, in a perfectly good and properly managed ship, with the cargo judiciously stowed, that it is necessary deliberately to *cut* away masts, cables, or rigging, to *throw* overboard cargo, or even to *run* the vessel aground, so as to save the ship and cargo from greater or total loss. By a very old law, any such sacrifice being deliberately and intentionally made in sound judgment for the good of all parties, the losses on ship, freight or cargo thereby occasioned, are made good by what is called a *general average*—that is, a general levy on all interests, ship, freight, and cargo, in proportion to their clear values after the charges and expenses of the voyage are deducted. It is the custom in such cases to get the shares of the levy adjusted at the port of destination, and the master should be careful, before delivering the cargo, to get the consignees of the cargo to sign an *average bond*, binding them to pay their share, and naming a person who is to make the necessary legal adjustment; and the fullest and minutest details of expenses and agreements of all kinds must be brought home by the master.

Particular average and general average expenses, as well as

total *abandonment*, are made good by the underwriter, with such exceptions as are named in the policies. In the case of total abandonment, the underwriter has the right to everything that remains or can be rescued.

The policies sometimes do not name the values of the interests insured, but leave them to be declared and ascertained afterwards should a loss happen, and are thus *open;* where the value is named, the policy is termed a *valued* one.

A policy of insurance is void, and the party bears his own loss, if there has been any material misrepresentation or concealment, intentional or accidental, in obtaining it, or if the conditions or risks on which it is granted are in any way broken, departed from, or added to.

GLOSSARY OF NAUTICAL TERMS.

[*The following is a tolerably comprehensive, though by no means an exhaustive List.*]

Aback, the situation of the sails, when their surfaces are pressed aft against the mast by the force of the wind, tending to force the vessel astern.

Abaft, or **Aft**, the sternmost part of the ship, or towards the stern. *The mast rakes aft*—that is, leans towards the stern.

Abaft the beam, denotes the relative situation of any object to the ship, when the object is placed in any part of that arch of the horizon which is contained between a line at right angles with the keel and that point of the compass to which the ship's stern is directed.

Aboard, the inside of a ship.

About, the situation of a ship as soon as she has tacked or changed her course.

About ship! or **Ready about!** the order to the ship's or boat's crew to prepare for tacking.

A-cock-bill, the position of the yards topped up at an angle with the deck; also, the position of an anchor hanging up and down the ship's side.

Adrift, the state of a ship broken from her moorings, and driving about without control.

Afloat, buoyed up by the water from the ground.

Afore, all that part of a ship which is forward or towards the stem.

After, a phrase applied to any object in the hinder part of the ship, as the after-hold, the after-sails, &c.

Aground, the situation of a ship when her bottom or any part of her rests on the ground.

Ahead. Any thing which is situated on that point of the compass to which a ship's bow is directed, is said to be ahead of her.

A-lee, the position of the helm when it is put in the direction opposite to that from which the wind blows, forcing the ship's head up to the wind in moving ahead.

All hands! the whole ship's company.

All in the wind, the state of a ship's sails, when they are parallel to the direction of the wind, so as to shake or shiver.

Aloft, up in the tops, at the mast-heads, or anywhere about the higher rigging.

Along-shore, along the coast; a course which is in sight of the shore, and nearly parallel to it.

Along-side, side by side, or joined to a ship, wharf, &c.

Amain, anything done suddenly or at once.

Amidships, the middle of a ship, either with reference to her length or breadth.

Anchor, the apparatus by which a ship is held. *The anchor is foul*—that is, the cable or other entangling object has got about the fluke or arms of the anchor. *The anchor is a-peak*—that is, directly under the hawse-hole of the ship. The anchors, ranked according to weight, are the *sheet anchor, best bower, small bower, stream*, and *kedge*. The parts of an anchor are the *ring* for fastening the cable to; the *stock*, either of wood or iron; the *shank*, or the middle piece, connected at the *crown* with the two *arms*; the *flukes* or *palms*, the triangular pieces at the ends of the arms; and the *bills*, the points at the extremity of the flukes.

An-end, the position of any mast, &c. when erected perpendicularly on the deck. The topmasts are said to be an-end, when they are hoisted up to their usual station.

A-peak. The cable having been drawn so tight as to bring the ship directly over her anchor, the anchor is said to be a-peak.

Arming, tallow put into the hollow at the bottom of the sounding-lead, to bring up a specimen of the bottom.

Ashore, on the shore, as opposed to aboard. It also means aground.

Astern, any distance behind a ship, as opposed to ahead.

At anchor, the situation of a ship held by her anchor or anchors.

Athwart, across the fore-and-aft line of a ship. *Athwart hawse*, across the direction of a vessel's head; or across her cable. *Athwartships*, across the ship from one side to the other.

Atrip, when applied to the anchor, means that the anchor is drawn out of the ground, and hangs in a perpendicular direction by the cable or buoy-rope.

Avast, a term used for stop; as, *avast heaving*, do not heave any more.

A-weather, towards the wind; the situation of the helm when the tiller is put in the same direction as that from which the wind blows —forcing the ship's head from the wind in moving ahead. Opposite to *a-lee*.

Aweigh, the same as atrip when applied to the anchor.

Awning, a shelter or screen of canvas spread over the decks of a ship, or over a boat, to keep off the rain or the heat of the sun.

Back and fill, to, to trim the sails in a contrary wind in order to go backwards and forwards across a river, letting the tide take the ship to windward.

Back the anchor, to, to carry out a small anchor ahead of the large one, in order to support it in bad ground, and to prevent it from loosening or coming home.

Back the sails, to, to arrange them in a situation that will occasion the ship to move astern.

Balance, to, to contract a spanker or fore-and-aft mainsail, making the sail nearly triangular.

Bale. *Bale the boat*—that is, heave the water out of her.

Ballast is either pigs of iron, lead, stones, or gravel, which last is called shingle ballast; and its use is to bring the ship down to her bearings in the water. *Trim the ballast* —that is, spread it about and lay it even.

Bar, a bank or shoal at the mouth of a harbour.

Bare poles. When a ship has no sail set, she is under bare poles.

Barge, a large double-banked boat.

Barnacle, a shell-fish often found on a vessel's bottom.

Barque, a three-masted vessel, with tops and yards to the foremast and mainmast, but neither top nor yards on the mizzen-mast.

Barrel bulk, the space occupied by certain goods in a ship's hold.

Batten, a thin piece of wood. *To batten down the hatches*, is to lay battens upon the tarpaulins, which are over the hatches in bad weather, and nail them down that they may not be washed off; also put on rigging to keep it from chafing.

Beacon, a post or buoy erected over a shoal or sand-bank, as a warning to seamen to keep at a distance; also, a signal placed on the shore.

Beam, the breadth of the vessel at its widest part.

Beam-ends. A vessel is on her beam-ends when turned over so much on one side, that her masts are inclined towards the horizontal.

Beams, strong pieces of timber stretched horizontally across a ship, to support the decks, and retain the sides at their proper distance.

'Bear a hand,' make haste to assist; despatch.

Bearing signifies the point of the compass on which any two or more places lie from each other, or how any place lies from the ship by the compass; or it may be said to bear on the beam, abaft the beam, on the bow, on the quarter, the head or stern, &c.

Bearings of a ship, that line which is formed by the water upon her sides when she is properly loaded.

Bear up, to, to alter the course, turning the ship's head more away from the wind. Equivalent to *keep her away*, and *bear away*.

Beating to windward, proceeding as nearly as possible in a direction towards the wind, by steering alternately close-hauled on the starboard and port tacks.

Becalm, to, to intercept the current of the wind, in its passage to a ship, by any contiguous object. A ship is becalmed when there is no wind to act upon her sails.

Becket, a circular handle of rope.

Before the beam, denotes an arch of the horizon comprehended between the line of the beam (which is at right angles to the keel) and that point of the compass to which the stem is directed.

Before the wind, having the wind exactly aft.

Belay, to, to make a rope fast by turns round a pin, called a *belaying pin*. Large ropes are belayed to strong pieces of wood called *kevels*.

Bells. The time at sea is announced by striking on a bell, beginning at noon with eight strokes, called eight bells. One bell is struck at 12½ h., two bells at 1 h., three bells at 1½ h., four bells at 2 h., and so on up to eight bells at 4 o'clock. One bell is then recommenced at 4½ h., and the number is again increased by one every half-hour up to eight bells; the eight bells are thus completed every four hours.

Bend, to, to apply to and fasten—as, *bend the sails*, apply them to the yards and fasten them; *unbend the sails*—that is, cast them off, and take them from the yards; *her sails are unbent*, she has none fixed; *bend the cable*, make it fast to the anchor. A *bend* is a knot by which one rope is made fast to another.

Bends, the thickest outside planking of a vessel, extending from a little below the water-line up to the bulwarks, being the strongest part of a vessel's side.

Bewspred. See **Heap.**

Berth, a place—as, *the ship's berth*, the place where she is moored; *an officer's berth*, his place in the ship to eat or sleep in; *berth the ship's company*—that is, allot them their places to mess in; *berth the hammocks*, point out where each man's hammock is to hang.

Between or 'tween decks, the space contained between any two decks of a ship.

Bight of a rope, the double part of a rope when it is folded. *Bight*, a narrow inlet of the sea.

Bilge, that nearly horizontal part of a ship's bottom on each side of the keel.

Bilge-water is that which, by reason of the construction of a ship's bottom, lies on her floor, and cannot go to the well of the pump.

Binnacle, the box that contains the compasses upon deck.

Bitts, perpendicular pieces of timber going through the deck, to secure cables, ropes, &c. to.

Block, a small wooden or metal apparatus composed of a shell, enclosing a travelling wheel called a *sheave*, for reeving ropes through. A combination of blocks and ropes is called a *tackle*.

Bluff. A bluff-bowed vessel is one which is full and square forward.

Board. To make *a board* is to make a stretch upon any tack when a ship is close-hauled. *The ship makes a stern-board*—that is, she loses way in working on a wind.

Board, to. To board a ship is to enter it.

Boatswain, the officer who has charge of all the cordage, rigging, anchors, &c.

Bobstay, the stay holding the bowsprit down.

Body-post, a piece of timber rising from the keel before the stern-post.

Bold shore, a steep coast, permitting the close approach of shipping.

Bollard heads, timber heads left in the ship's side, clear of the planking, to which anchor-stoppers, hawsers, &c. are secured.

Bolsters, pieces of soft wood covered with canvas, on which the eyes of the shrouds rest. They are placed on the trestle-trees to prevent the rigging from being cut by the outer edge of the trestle-tree.

Bolt-rope, the rope going round a sail to which the canvas is sewed. The side ropes are called *leach ropes*, that at the top the *head-rope*, and that at the bottom the *foot-rope*.

Bonnet of a sail is a piece of canvas laced to the sail in moderate weather to hold more wind. By taking the bonnet off, the sail becomes a *storm-sail*.

Boom, a spar used to extend the bottom of a fore-and-aft sail or of a studding-sail.

Boot topping, cleaning a ship's bottom, or that part which lies immediately under the surface of the water, and daubing it over with tallow, or with a mixture of tallow, sulphur, rosin, &c.

Both sheets aft, the situation of a ship sailing right before the wind.

Bow, or Bows, the rounded part of the foremost end of a vessel.

Bow grace, a frame of old rope or junk, laid out at the stem and sides of a ship, to prevent her from being injured by ice.

Bowlines, lines made fast to the sides of the sails to haul them forward when upon a wind, which being hauled taut, enables the ship to come nearer to the wind; *on a taut bowline*, close-hauled.

Bowse, to, to pull upon any body with a tackle.

Bowsprit, a large and strong spar which stands out from the bows of a ship.

Boxhauling, a particular method of wearing a ship by backing the head sails, when the swell of the sea renders tacking impracticable.

Boxing, an operation somewhat similar to boxhauling. It is performed by laying the head sails aback, to receive the greatest force of the wind in a line perpendicular to their surfaces, in order to turn off the ship's head if she has approached too near the wind in consequence of the wind drawing ahead, or from bad steering.

Braces, the ropes by which the yards are turned about to trim the sails to the wind.

Brace the yards, to, to move the yards, by means of the braces, to any direction required. *To brace about*, to brace the yards round for the contrary tack; *to brace sharp*, to brace the yards to a position in which they will make the smallest possible angle with the keel for the ship to have headway; *to brace up*, to lay the yard more fore and aft; *to brace in*, to lay it more nearly square with the fore-and-aft line; *to brace to*, to brace the head yards a little aback to assist the motion of the ship's head in tacking.

Brails, a name peculiar to certain ropes belonging to the mizzen, used to truss it up to the mast. But it is likewise applied to all the ropes which are employed in hauling up the bottoms or lower corners of the other great sails. *To brail up*, to haul up a sail by means of the brails, making it ready for furling. See **Buntlines**.

Break bulk, to, the act of beginning to unload a ship.

Break sheer, to. When a ship at anchor is forced, by the wind or current, from that position in which she keeps her anchor most free of herself and most firmly in the ground, so as to endanger the tripping of her anchor, she is said to break her sheer.

Breaming, burning off the accumulations on a ship's bottom.

Breast-fast, a rope employed to confine a ship sideways to a wharf or to some other ship.

Bridle, a rope with both ends made fast to the sides (leeches) of square-sails, to which the bowlines are attached.

Brig, a two-masted vessel, with yards and

tops on both masts. The masts are termed foremast and mainmast. Vessels differing from a brig in having no top to the mainmast, and carrying a fore-and-aft mainsail, instead of the square mainsail and trysail, are called *brigantines*.

Bring by the lee, to. See **Broach to.**

Bring to. Same as **Heave to.**

Broach to, to incline suddenly to windward of the ship's course, in consequence of a sea striking her stern, or through bad steering, &c. so as to present her side to the wind, and endanger her oversetting. The difference between *broaching to* and *bringing by the lee* may be thus defined: suppose a ship under great sail is steering south, having the wind at NNW., then west is the weather-side, and east the lee-side. If, by any accident, her head turns round to the westward, so that her sails are all taken aback on the weather-side, she is said to *broach to*. If, on the contrary, her head declines so far eastward as to lay her sails aback on that side which was the lee-side, it is called *bringing by the lee*.

Broadside, the whole side of a vessel.

Broken backed, the state of a ship which is so loosened in her frame as to droop at each end.

Bulk, the whole cargo when stowed in the hold. *Stowed in bulk*, is stowed loose, not in casks or bags (grain, for instance).

Bulkhead, a partition across a vessel dividing her into distinct compartments.

Bulwarks, the boarding round the sides of a vessel above the deck.

Bum-boats are those which attend alongside vessels with provisions, fruit, &c.

Bumpkins, or **Boomkins,** short booms or projecting pieces of timber on each bow and quarter.

Bunt, the middle part of a sail.

Bunting, or **Buntine,** thin woollen stuff for making flags, ships' colours, &c.

Buntlines, ropes fastened to the foot-rope of square sails to haul up the body of the sail to the middle of the yards for furling.

Buoy, a floating cask, piece of wood, or other body, moored upon shoals to shew where the danger is; also used at anchor, to shew where the anchor lies.

By the board, over the ship's side.

By the head, the state of a ship when she is loaded so as to draw more water forward than aft.

By the wind, sailing as close as possible to the direction of the wind, which is generally within six points of it.

Cable, a large strong rope or chain for holding a vessel at anchor. A ship's cable is usually 120 fathoms, or 720 feet, in length. Cables are now generally of chain, and are called *chain-cables;* a large one is 150 fathoms long. The measure called a cable's length is 100 fathoms.

Camel, a machine for lifting vessels over a shoal or bar. Sometimes used in launching to give buoyancy.

Canvas, the cloth of which sails are made. The coarsest and strongest is called No. 1; the lighter kinds are numbered from 2 to 8.

Cap, a strong thick block of wood, having two large holes through it, the one square, the other round; used to keep the upper mast in its proper position. There is also one at the end of the bowsprit. Now sometimes of iron.

Capsize, to, to overturn. *The boat is capsized* —that is, overset. *Capsize the coil of rope* —that is, turn it over.

Capstan, a solid barrel of wood turning round usually on a vertical axis, used, with the assistance of levers called capstan-bars, to weigh the anchor out of the ground, and also for setting up the shrouds and other work where a great purchase is required.

Careen, to, to incline a ship on one side, by shifting the cargo or stores on one side, or by the application of a strong purchase to her masts, so that her bottom on the other side may be cleansed by breaming. To lie over.

Carlings, short pieces of timber running fore and aft, connecting one beam with another to distribute the strain of the masts, capstan, &c. among the several beams so connected.

Carry away, to, to break—as, a ship has carried away her bowsprit—that is, it is broken off.

Carvel-built. A vessel is carvel built when the planks are laid in smoothly, and not lapped over as they are in clench or clincher built vessels.

Casting, trimming the sails in order to turn the ship's head round, so as to bring the wind on either side of the ship after it has blown some time right ahead. It is particularly applied to a ship about to weigh anchor.

Cat-heads, large timbers on ships' bows with sheaves in them, by which the anchor is hoisted after it has been hove up from the bottom by the cable. See **Cat the anchor.**

Cat's-paw is a light air of wind perceived at a distance in a calm, sweeping the surface of the sea very lightly.

Cat the anchor, to, is to hook the cat-block to the ring of the anchor, and haul it up close to the cat-head. The rope used in thus hauling is called the *cat-fall.*

Caulking is filling the seams between the planks of a ship with oakum.

Chain-plates, strong links or plates of iron fastened to the ship's sides under the chains, to which the lower dead-eyes are fastened.

Channels, platforms projecting outwards from the ship's side to which the shrouds are fastened, for the purpose of giving them a greater angle than they could have if fastened to the ship's side, and of course giving them a greater power to secure the mast. Sometimes called chains.

Chase, a vessel pursued by some other. *Chaser*, the vessel pursuing.

Cheeks, projections on each side of a mast, upon which the trestle-trees rest. The upper parts of the cheeks are termed *hounds*.

Cheerily, a phrase implying heartily, quickly, cheerfully.

Chinse, to, to fill the deck seams with oakum, and serve them over with melted resin.

Claw off, to, the act of turning to windward from a lee-shore to escape shipwreck, &c.

Clear is variously applied: The weather is said to be clear when it is fair and open; the sea-coast is clear when the navigation is not interrupted by rocks, &c. It is applied to cordage, cables, &c. when they are disentangled, so as to be ready for immediate service. In all these senses it is opposed to *foul*. *To clear the anchor* is to get the cable off the arms or flukes, and to disencumber it of ropes ready for dropping. *Clear hawse*, the cables are directed to their anchors without lying athwart the stem. *To clear the hawse* is to untwist the cables when they are entangled by a cross, an elbow, or a round turn. See **Elbow in the hawse**.

Clear out, to, the act of leaving one port for another.

Cleat, a piece of iron or wood used in various parts of a vessel for fastening or belaying ropes to.

Clenched, made fast, as the cable is to the ring of the anchor. *A chain-cable* is *shackled* to the ring of the anchor.

Clew-lines, are ropes which come down from the mast to the lower corners of the sails, and by which the corners or clews of the sails are hauled up. The clew-line of a fore-sail or mainsail is called a *clew garnet*.

Clew of a sail, the two lower corners of square sails, but the aftermost corner only of fore-and-aft sails, the other lower corner being called the tack. See **Sheet** and **Tack**.

Clew up, to, to haul up the clews of a sail to its yard by means of the clew-lines.

Close-hauled, the same as **By the wind**, on or close by wind.

Club haul, to, a method of tacking a ship when it is expected she will miss stays on a lee-shore.

Coamings, a raised boundary to the hatchways, to keep water from going down into the hold.

Coasting, the act of making progress along the sea-coast of any country.

Coil, to, to lay a rope up in a ring, one turn over another.

Come home, to. The anchor is said to come home when it loosens from the ground by the effort of the cable, and approaches the place where the ship floated.

Coming to, denotes the approach of a ship's head to the direction of the wind.

Companion, a covering over a staircase leading to a cabin.

Compressor, a large movable iron lever for stopping and securing the cable by preventing it running through the chain-pipe.

Con, to, to direct. *To con a ship* is to direct the man at the helm how to steer.

Counter, the part of a vessel's stern that rakes outwards.

Course, the point of the compass upon which the ship sails. *The ship is under her courses*—that is, has no sail set but the mainsail and foresail.

Coxswain, the person who steers and has charge of a boat.

Crank. *The ship is crank*—that is, she has not sufficient cargo or ballast to render her capable of bearing much sail. It may also be owing to faulty construction, or injudicious stowage of cargo.

Cringles, a number of short pieces of rope with both ends worked into the rope which is sewed on the sides of a sail.

Cross-jack (pronounced *cro-jack*), the lower yard on the mizzen-mast.

Cross-trees, two pieces of timber or iron lying athwart-ships on the top of the trestle-trees; they support the *top* on a lower-mast, and spread the topgallant rigging at a topmast-head.

Crowfeet, a number of small lines used to suspend awnings by.

Cuddy, a cabin in the fore-part of a boat.

Cut and run, to, to cut the cable and make sail instantly, without waiting to weigh anchor.

Cutter, a small boat. Also a one-masted vessel of a particular rig.

Cutwater, a sharp projection forward of the vessel's bows, being the foremost part of a vessel's prow.

Davits, wooden or iron outriggers to hoist a boat up to. *Fish-davit*, a spar with a sheave or roller at its end, used in fishing the anchor.

Deaden a ship's way, to, to impede her progress through the water.

Dead-eyes, blocks of wood with three holes through which the lanyards of the rigging are rove, without sheaves.

Dead-lights, a kind of shutter for cabin windows, used in bad weather only.

Dead-water, the eddy of water, which appears like a whirlpool, closing in with the ship's stern as she sails on.

Dead-wind, the wind right against the ship, or from the very point to which she wants to go.

Deck, the planked floor resting on the beams of a vessel.

Derrick, a single spar supported by stays and guys, used with a purchase attached for hoisting.

Dismasted, the state of a ship that has lost her masts.

Dog, a short iron bar with a fang or tooth at one end, and a ring at the other.

Dog-vane, a small vane with feathers or bun-

tine, placed on the ship's quarter, by which to see the direction of the wind.

Dog-watch, the watches from four to six, and from six to eight respectively, in the evening.

Doubling, the act of sailing round or passing beyond a cape or point of land.

Douse, to, to lower suddenly or slacken.

Down-haul, a rope attached to the top part of jibs, staysails, and studding-sails, for hauling them down.

Drag the anchor, to, to trail it along the bottom after it is loosened from the ground.

Draught, the depth of water required for a vessel to float in.

Draw, to, when a sail is inflated by the wind, so as to advance the vessel in her course, the sail is said to *draw*. *To draw a jib* is to shift it over the stay to leeward, when it is aback, so as to make it fill.

Drive, to, to drag the anchors in a gale of wind; also, to scud before a gale. See **Scud**.

Drop anchor, to, the same as *to anchor*. *To drop astern*, the retrograde motion of a ship.

Drum-head, the top of a capstan.

Duck, cloth, lighter and finer than canvas, used for small sails.

Dunnage, a quantity of loose wood, &c. laid at the bottom of a ship's hold to keep the goods from being damaged.

Earings, small ropes used to fasten the upper corners of sails to the yards.

Ease, to, Ease away, or **Ease off**, to slacken gradually; thus, *ease* the bowline, *ease* the sheet.

Ease the ship, to, to put the helm hard a-lee, when the ship is expected to plunge her head deep in the water when close-hauled; this brings the ship's head up to the wind.

Ebb-tide, the state of the tide when it is falling.

Edge away, to, to decline gradually from the shore or from the line of the course which the ship formerly held in order to go more large.

Edge in with, to, to advance gradually towards the shore, or any other object.

Elbow in the hawse is when a ship, being moored, has, upon the shifting of the tides, gone round *twice* the wrong way, so as to lay the cables one over the other: having gone *once* wrong, she makes a *cross* in the hawse, and *three* times wrong, she makes a *round turn*.

End for end, a term used when a rope runs all out of a block, and is unreeved: or in coming to an anchor, if the stoppers are not well put on and the cable runs all out, it is said to have gone out end for end. To reverse the ends of a rope.

End on, when a ship advances to a shore, rock, &c. without any apparent possibility of preventing her, she is said to go *end on* for the shore, &c.

Ensign, the national colours.

Even keel. When the keel is horizontal, a ship is said to be upon an *even keel*.

Eye, the loop or circular part of a stay or shroud where it goes over a mast.

Fag, the end of any rope which is become untwisted; to prevent which the ends of ropes are wound round with pieces of twine, called *whipping*.

Fair, a general term for the disposition of the wind when favourable to a ship's course.

Fair-way, the channel of a narrow bay, river, or harbour, in which ships usually keep in their passage up and down.

Fake, a circle of any rope or cable made in coiling it.

Fall, that part of a tackle to which the power is applied in hoisting.

Fall aboard of, to, to strike or encounter another ship when one or both are in motion.

Falling off denotes the motion of the ship's head from the direction of the wind. It is used in opposition to *coming to*.

False keel, an additional keel below the main keel. It protects the main keel if the ship should take the ground; and also, by offering resistance sidewards to the water, lessens the vessel's leeway.

Fathom, a measure of six feet.

Fender, a piece of wood or a soft substance of rope, tow, or cork, sometimes covered with canvas, used for hanging over the side of a vessel, to protect it from striking or chafing when alongside anything.

Fid, a square bar of wood or iron, placed through the hole in the lower end of a topmast, to support the weight of the mast; also a large piece of wood, tapered, used to extend the strands and layers of ropes in splicing, in opening eyes, &c.

Fife rail, a rail round a mast, used in belaying ropes.

Fill, to, to brace the sails so as to receive the wind in them, and advance the ship in her course, after they have been either shivering or braced aback.

Fish, a large piece of wood. *Fish the mast*, apply a large piece of wood to it to strengthen it.

Fish-hook, a large hook by which the fluke of the anchor is brought up to the bulwarks or rail of the vessel after having been hove up from the bottom and catted; the *tackle* which is used for this purpose is called the fish-tackle, and the *davit* is called the fish-davit. The operation is called *fishing the anchor*.

Flare, the term describing the vessel's sides when they go outwards from the perpendicular.

Flat aback, the situation of the sails when their surfaces are pressed aft against the mast by the force of the wind.

Flat in, to, to draw in the aftermost lower corner or clue of a sail towards the middle of the ship to give the sail a greater power to turn the vessel. *To flat in forward*, to

draw in the fore-sheet, jib-sheet, and fore-staysail-sheet towards the middle of the ship. See **Sheet.**

Flaw, a sudden breeze or gust of wind.

Fleet-ho! the order to shift the position of a block or fall so as to haul with more advantage.

Floating, the state of being buoyed up by the water from the ground.

Flood-tide, the state of the tide when it flows or rises.

Floor, the bottom of a vessel, on each side of the keelson.

Flowing sheets, the position of the sheets of the principal sails when they receive the wind more nearly perpendicularly than when close-hauled, but more obliquely than when the ship sails before the wind. A ship going two or three points large has *flowing sheets.*

Fly, the whole of a ship's flag, except the upper inner corner, where the *union* is marked.

Fog signals. See page 395.

Foot-rope, a rope stretching along the under side of a yard, for the seamen to stand on when reefing or furling the sails.

Force over, to, to force a ship violently over a shoal by a great quantity of sail.

Fore, the parts of a ship's frame, mast, sails, rigging, &c. that lie near the stem. *Fore and aft*, throughout the whole ship's length, lengthways of the ship, opposite to athwartships.

Fore-castle, that part of the upper deck before the fore-mast; also the forward part of the vessel under the deck.

Fore foot, a piece of timber at the forward end of the keel; the bottom of the stem rests upon it. The after end of the keel is called the *heel* of the vessel.

Fore-reach, to shoot ahead, especially when going about from one tack to the other.

Forward, towards the fore-part of a ship.

Fother, to, or **Fodder**, to draw a sail filled with oakum under a leak in a vessel's bottom to stop it.

Foul is used in opposition both to clear and fair. Thus, as opposed to clear, foul weather, foul bottom, foul ground, foul anchor, foul hawse; as opposed to fair, foul wind.

Foul anchor, the anchor with a turn of the cable round it.

Founder, to, to sink at sea by filling with water.

Frap, to, to pass ropes round a sail to prevent it from blowing about.

Freshen, to. When the wind increases it is said to freshen; also, *to freshen* is to relieve a rope by shifting it to prevent it wearing through by chafing; *to freshen the ballast* is to alter its position.

Full, the situation of the sails when they are kept distended by the wind.

Full and by, sailing close-hauled on a wind; also, the order given to keep the sails full and at the same time close to the wind.

Furl, to, to wrap or roll a sail close up to the yard, boom, or stay to which it belongs, and secure it.

Futtock shrouds, short shrouds secured to the lower mast just below the *top*. *Futtock timbers*, the timbers in a ship's frame above the floor timbers.

Gaff, the top spar of a ship's mainsail.

Gage, or **Gauge**, the depth of water a vessel draws; also her position in regard to another vessel, as weather-gage, lee-gage.

Gain the wind, to, to arrive on the weather-side or to windward of some ship or fleet in sight, when both are sailing as near the wind as possible.

Galley, the house on deck where the cooking is done.

Gammoning, a strong rope passed round the bowsprit and into the cutwater to steady the bowsprit; it is further secured by shrouds and bobstays.

Gangway, that part of a ship's side both within and without, by which persons enter and depart.

Garboard strakes, the ship's lowest planking outside, nearest to the keel, running fore and aft.

Gasket, the rope which is passed round the sail to bind it to the yard or boom, when it is furled, called bunt, quarter, or yard-arm gasket, according to position.

Gather, to. A ship is said to gather on another as she comes nearer to her.

Gimbleting, the action of turning the anchor round by the stock, so that the motion of the stock appears similar to that of the handle of a gimblet, when employed to turn the wire; to turn anything round on its end.

Girt, the ship is girt with her cables when she is too tightly moored.

Goose-neck, an iron ring fitted to the end of a yard or boom.

Goose-wings of a sail, the clews or lower corners of a ship's mainsail or foresail when the middle part is furled or tied up to the yard.

Grapnel, a small anchor with several claws.

Grappling-iron, a thing of the nature of an anchor, with four or six flukes to it.

Gripe, a projection forward at the lowest part of the stem to keep the vessel's head up to the wind. The ship gripes when she turns her head too much to the wind.

Grommet, a piece of rope laid in a circular form, and used for large boats' oars instead of rowlocks, and also for many other purposes.

Grounding, laying the ship ashore in order to repair her. It is also applied to running aground accidentally.

Ground tackle, everything belonging to a ship's anchors, and which are necessary for

anchoring or mooring, such as cables, hawsers, warps, springs, buoy-ropes, &c.

Ground tier, the tier of casks which is lowest in the hold.

Guesswarp, a rope used to secure or haul a boat ahead with. It must be carried out from the ship, and the end of it returned, the distance being guessed.

Guntackle-purchase, a rope working through two single blocks.

Gunwale, the upper edge of a ship's or boat's side.

Guy, a rope attached to anything to guide or steady it.

Gybing, the act of shifting any fore-and-aft sail from one side of the mast to the other.

Hail, to, to call.

Halyards, or **Halliards**, the ropes by which yards, gaffs, and sails are hoisted.

Hammock, a canvas bed, suspended at each end.

Handing. The same as furling.

Handsomely, carefully, slowly.

Handspike, a long wooden lever, used for heaving at the windlass.

Hard a-weather, put the helm quite up to windward.

Hatchway, an opening in a ship's deck forming a passage into the holds, or from one deck to another.

Haul, to, to pull.

Haul the wind, to, to direct the ship's course nearer to the point from which the wind blows.

Hawse-holes, the holes in the bows of the ship through which the cables pass. *To clear hawse* is to untwist the cables where a ship is moored and has got a foul hawse; *athwart hawse* is to be across or before another ship's head.

Hawser, a large rope.

Head, the ornamental work at the prow of a vessel: a carved figure is called a figure-head; carved work bending like the head of a violin, a *fiddle-head;* if bending the contrary way, a *billet-head*.

Head-fast, a rope employed to confine the head of a ship to a wharf or to some other ship. *Headmost*, the situation of any ship or ships which are the most advanced in a fleet. *Head sails*, all the sails which belong to the foremast and bowsprit. *Mast-head*, the upper end of a mast from where the rigging is placed to the top of the mast.

Head-sea. When the waves meet the head of a ship in her course, they are called a head-sea. It is likewise applied to a single wave coming in that direction.

Head to wind, the situation of a ship when her head is turned to the point from which the wind blows, as it must when tacking.

Head-way, the motion of advancing, used in opposition to stern-way.

Heave, to, to turn about a capstan, windlass, or other machine of the like kind by means of bars, handspikes, &c. *To heave ahead*, to advance the ship by heaving in the cable or other rope fastened at some distance before her.

Heave astern, to move a ship backwards by an operation similar to that of heaving ahead. *To heave down*, to careen. *To heave in the cable*, to draw the cable into the ship, by turning the capstan or windlass. *To heave in stays*, to bring a ship's head to the wind, by a management of the sails and rudder, in order to get on the other tack. *To heave out*, to unfurl or loose a sail; more particularly applied to the staysails; thus, loose the topsails and heave out the staysails. *To heave short*, to draw so much of the cable into the ship, that she will be almost perpendicular over her anchor. *To heave tight or taut*, to heave till the rope or cable becomes straightened. *To heave the capstan*, to turn it round. *To heave the lead*, to throw the lead overboard, in order to find the depth of water. *To heave the log*, to throw the log overboard, in order to ascertain the velocity of the ship. *Heave handsomely*, heave gently or leisurely. *Heave hearty*, heave strongly and quickly.

Heave of the sea, is the power that the swell of the sea has upon a ship in driving her out, or faster on, in her course.

Heel, the lower end, as of a mast, stern-post, &c.

Heel over, to, to lie on one side. *She heels to port*—that is, inclines or lies down upon her port or left side.

Helm, the apparatus by which the ship is steered, including both the wheel and the tiller, as one general term. *Helm's a-lee*—that is, the tiller is quite down to leeward.

High and dry, the situation of a ship when so far run aground as to be seen above water-mark.

Hitch, to make fast in a peculiar manner.

Hog, a flat rough broom used for scrubbing vessels' bottoms. The operation is called *hogging*.

Hogged, the state of a vessel when she has been strained so as to droop at each end.

Hold, is the space between the deck and the bottom of a ship, where the cargo, stores, &c. lie. *To stow the hold* is to place the cargo, &c. in it.

Home implies the proper situation of any object—as, to *haul home* the topsail sheets is to extend the bottom of the topsail to the lower yard, by means of the sheets. In stowing the hold, a cask or other object is said *to be home*, when it lies close to some other object. An anchor *comes home*, when it is loosened from the ground, and hove in towards the vessel.

Hounds. See **Cheeks**.

Housing a mast, lowering the mast as much as possible, without taking the rigging off the mast-head. The *housing* of a mast is that

part between the heel of the mast and the deck, or all the part inside the ship.

Hove-to, keeping the ship stationary by making one sail act against another, as in waiting for a boat, &c.

Hulk, a ship without masts or rigging; also a vessel used to get masts into or out of ships by means of sheers, whence called a *sheer-hulk*.

Hull of a ship, the body of it.

Hull down is when a ship is so far off, that only her masts are visible.

In stays. See **Heave in stays.**

Jack stays, ropes, or strips of wood or iron, stretched close along a yard, to which the head of a sail is attached.

Jacob's ladder, a ladder with rope sides and wooden steps.

Jaws, the inner ends of booms or gaffs, hollowed in, something like the shape of a jaw; also called a *throat*.

Jeers, tackles for hoisting the lower yards.

Jigger, a small tackle for hoisting or hauling.

Jolly-boat, a small boat.

Junk, old rope condemned and cut up.

Jury-mast, a temporary or occasional mast erected in a ship in the place of one which has been carried away by accident, &c. *A jury-rudder*, a temporary rudder.

Keckling, a covering of old ropes, to prevent the surface of cables from chafing.

Kedge, a small anchor with an iron stock, used for warping.

Keel, the principal piece of timber in a ship, running fore and aft at the lowest part of the ship.

Keelson, a piece of timber placed over the keel on the middle of the floor timbers, and serving to unite the former to the latter.

Keep away, to, to alter the course, turning the ship's head more away from the wind for a little time, to avoid some ship, danger, &c. *Keep away* is likewise said to the steersman, who is apt to go to windward of the ship's course. *To keep full*, to keep the sails distended by the wind. *To keep hold of the land*, to steer near to or in sight of the land. *To keep off*, to sail off or keep at a distance from the shore. *To keep the land aboard*, the same as to *keep hold of the land*. *To keep the luff*, to continue close to the wind. *To keep the wind*, the same as to *keep the luff*.

Kentledge, pigs of iron for ballast, laid on each side of the keelson.

Kevel-head, or **Kevel**, a strong piece of wood bolted to some part of the bulwarks for belaying ropes to.

Kink, a twist or turn in a cable or rope.

Knees, crooked timbers uniting the beams to the ship's side.

Knight-heads, strong timbers on each side of the stem, continued high enough to form a support for the bowsprit.

Knot, a division of the log-line, answering in the calculation of the ship's velocity to one nautical mile. A complication of the strands of a rope.

Labour, to, to roll or pitch heavily in a sea.

Laid up, the situation of a ship when moored in a harbour for want of employment.

Land-fall, the first land discovered after a sea voyage. A *good land-fall* implies the land expected or desired; a *bad land-fall* the reverse.

Land-locked, the situation of a piece of water so surrounded with land as to exclude the prospect of the sea, unless over some intervening land.

Lanyards, ropes rove through dead-eyes for setting up rigging; also, a rope made fast to anything to secure it.

Large, a vessel is said to be going large when she has the wind free, that is, is not close-hauled.

Lash, to bind.

Launch, a large boat. The long-boat is sometimes so called.

Lay, to come or go, as lay aloft, lay aft.

Leading-wind, a fair wind.

Lead-line, a graduated line with a heavy cylindrical piece of lead attached for ascertaining the depth of water. See **Arming.** The *deep-sea lead* is for measuring great depths. Various patents are in use.

Leak, a chink or hole in the sides or bottom of a ship, through which the water enters into the hull.

Lee, or **Leeward**, that part opposite the direction from which the wind blows; the opposite to *windward*. *Lee-gage*, a ship or fleet to leeward of another is said to have the lee-gage. *Lee lurches*, the sudden and violent rolls which a ship often takes to leeward, in a high sea, particularly when a large wave strikes her on the weather-side. *Lee-quarter*, that quarter of a ship which is on the lee-side. *Lee-shore*, that shore upon which the wind blows. *Lee-side*, that half of a ship lengthwise, which lies between a midships line drawn fore and aft and the side which is farthest from the wind. *Lee-tide*, a tide that sets to leeward. *Under the lee* of anything, is having that between the ship and the wind.

Lee-board, a board attached to the lee-side of a flat-bottomed vessel, when sailing close-hauled, to prevent it drifting to leeward.

Leeches, or **Leaches**, the borders or edges of a sail at the sides. The foremost edge of a staysail or jib is called the *luff*.

Leeway, the lateral movement of a ship to leeward of her course; or the angle which the line of her way makes with a line in the direction of her keel.

Lie along, to, to be pressed down sideways by a weight of sail in a fresh wind.

Lie to, to, to retard a ship in her course by reducing sail or arranging the sails in such a manner as to counteract each other with nearly an equal effort, and render the ship almost immovable with respect to her progressive motion or head-way.

Life lines, ropes along yards or elsewhere for the seamen to hold on by.

Lifts, the ropes which come to the ends of the yards from the mast-heads, by which the yard-arm is supported, and by which they are suspended when lowered down.

Lighter, a large boat, used in loading and unloading ships.

Limbers, gutters formed in the lower part of a ship's floor-timbers, very near the keel; forming a channel for water, and communicating with the pump-well throughout the whole length of the floor; covered with limber-boards.

List, inclination to one side. *The ship has a list to port*—that is, she heels to port.

Log-line, the graduated line by which, with the aid of a sand-glass called the log-glass, the ship's rate of going is ascertained.

Log-slate, a slate on which the ordinary work of the ship is marked. It is copied into the *log-book* every 24 hours.

Long-boat, the largest boat carried by merchant-ships.

Long sea, a uniform motion of long waves.

Look-out, a watchful attention to some important object or event that is expected to arise. Thus persons on board of a ship are occasionally stationed to *look out* for signals, other ships, land, &c.

Loose, to, to unfurl or cast loose any sail.

Lubber's hole, a hole in the *top*, next the mast.

Lubber's point, a vertical mark in the bowl holding the steering compass, to shew the direction in which the ship's head lies.

Luff, to put the helm towards the lee-side of the ship in order to sail nearer to the wind; also, the forward edge or leech of fore-and-aft sails.

Luff-tackle, a purchase comprising a single and double block.

Lugger, a small vessel carrying sails bent to a yard hanging obliquely to the mast. Such sails are called *lug-sails*, a name also given to a small square sail carried by boats.

Lying to under bare poles, the situation of a ship when all her sails are furled and her helm is lashed a-lee; by which she lies nearly with her side to the wind and sea, her head being somewhat inclined to the direction of the wind.

Made, composed of different pieces.

Make a board, to, to run a certain distance upon one tack, in beating to windward. *To make sail*, to increase the quantity of sail already set, either by unreefing or by setting others. *To make stern-way*, to move with the stern foremost. *To make the land*, to discover it from afar. *To make water*, to leak.

Man, to, to place men in any position, as on the yards, in a boat, &c.

Man-ropes, ropes used in going up and down a vessel's side.

Marline, a fine kind of spun-yarn used for twisting or winding round a rope. The operation is called *marling*.

Marline-spike, an iron pin, sharp at one end and having a hole at the other.

Mast, the upright timber on which the yards and sails are set.

Masts. See explanation, page 457.

Mend service, put on more service.

Messenger, a rope used for heaving in a cable by the capstan.

Midships. See **Amidships**.

Miss stays, to, a ship is said to *miss stays* when her head will not fly up into the direction of the wind, in order to get her on the other tack.

Moon-sail, a small sail sometimes carried in light winds above the sky-sail.

Moor, to, is to secure a ship, both ahead and astern, with two or more anchors. *Mooring*, securing a ship in a particular station by chains or hawsers, which are either fastened to an adjacent shore or to anchors at the bottom. *Moorings*, anchors or chains laid down ready for a ship to be secured to them.

Mouse, a kind of ball or knob.

Narrows, a small passage between two lands.

Neap tides, the tides at the first and last quarters of the moon; they are not so high, low, or rapid as spring tides. A ship is said *to be beneaped* when she has not water enough to take her off the ground, or over the bar, &c. at neap tides.

Nippers, a number of yarns, with small rope twisted round, used to fasten the messenger to the cable in heaving up the anchor.

Nock, the forward upper corner of a sail set on a boom; also called the *throat* of the sail.

No higher, not to bring the ship's head nearer to the wind. An order used when the ship is close-hauled.

Nothing off, to bring the ship's head nearer to the wind. An order used when the ship is being close-hauled.

Nun buoy, a buoy tapering at each end.

Oakum, old yarns untwisted and pulled open; used for caulking, &c.

Off and on. When a ship is beating to windward, so that by one board she approaches towards the shore, and by the other stands out to sea, she is said to stand *off and on* shore.

Offing, to seaward from the land. *A ship in the offing* is to seaward, at a distance from the land.

On board, within the ship.

On the beam, any distance from the ship's s

Glossary

Lie to, to, to retard a ship in her course, or
ducing sail or arranging the sails
a manner as to counteract each other
nearly an equal effort, and making the
almost immovable with respect to the
gressive motion of head-way.

Life lines, ropes along yards
the seamen to foot ropes

Lifts, the ropes which
yards from the mast-heads
yard-arm is supported
are suspended when

Lighter, a large boat
loading ships

Limbers,
a ship
forming a
casing
which
limber-holes

List,
list

Log-line

Log-slate

Long-boat

line with her beams—that is, at right angles to the keel.

On the bow, an arch of the horizon, comprehending about four points of the compass on each side of that point to which the ship's head is directed. Thus, the ship in sight bears three points on the starboard bow—that is, three points towards the right hand, from right ahead.

On the quarter, an arch of the horizon, comprehending about four points of the compass on each side of that point to which the ship's stern is directed.

Open, the situation of a place exposed to the wind and sea. It is also said of any distant object or port to which the sight or passage is not interrupted.

Open hawse, when the cables of a ship at her moorings lead straight to their respective anchors, without crossing, she is said to ride with an *open hawse*.

Outhauler, a rope used for hauling out the tack of a jib, or the clew of a boom-sail.

Out-of-trim, the state of a ship when she is not properly balanced for the purposes of navigation.

Outrigger, a spar for spreading an extra sail beyond the vessel's side.

Overboard, out of the ship; as, he fell overboard.

Overhaul, to clear away and disentangle any rope, or to examine anything; also, to overtake.

Over-rake. When a ship at anchor is exposed to a head-sea, the waves of which break in upon her, the waves are said to *over-rake* her.

Painter, a rope secured in the bows of a boat, and used for making her fast: a *shank-painter*, a rope attached to the shank of an anchor.

Palm, a piece of leather fitting the hand, with a thimble, used in sail-making; also, the fluke of an anchor.

Parbuckle, to, to raise or lower a spar or cask by single ropes passed round it.

Parcel a rope, to, is to put a quantity of tarred canvas, called parcelling, upon it. It is then said to be tarred and parcelled. *To Parcel a seam* is to lay a narrow piece of canvas over it after it is caulked.

Parliament heel, the situation of a ship when she is careened, so that the upper part of her bottom on the other side can be cleaned. See **Boot topping**.

Parrel, the rope or iron collar which confines a yard at its centre to the mast.

Parting, being driven from the anchors, by the breaking of the cable.

Partners, strengthening timbers fitted into the deck round masts, capstans, bitts, &c.

Patent reefing topsails. The sail is reefed by being rolled up on the yard or spar, which revolves for that purpose, and is set in action from the deck.

Pawl, or **Paul**, a short bar of wood or iron fixed close to the capstan or windlass of a ship, to prevent those machines from rolling back, or giving way, when they are charged with any great effort.

Pay, to, to daub or cover the surface of any body with pitch, tar, marine glue, &c. in order to protect it from injury by the weather.

Pay away, to, or **Pay out**, to slacken a cable or other rope, so as to let it run out.

Pay off, to, to move a ship's head from the wind.

Pazaree, a rope secured to the corner (clew) of a foresail, used for trimming the sail when before the wind.

Peak. See **A-peak**.

Peak, the upper and outer corner of a spanker or trysail; the outer end of a gaff.

Pennant, or **Pendant**, a long streamer of bunting carried at the mast-head; also, a pointed flag; also, the square flag carried by a commodore, Royal Navy; also, a long strap attached to the end of a yard, &c. with a block or a hook at one end, through which to reeve a brace or hook a tackle.

Pinnace, a boat between a launch and a cutter in size.

Pintles, the metal bolts used for hanging a rudder in its place.

Pitching, the movement of a ship, by which she alternately plunges her head and stern into the hollow of the sea.

Pole, the highest (part) mast of a ship, as, the sky-sail pole; usually painted.

Poop, a raised deck in the after-part of a vessel, built over the ordinary deck. When no poop-deck or forecastle-deck is raised, the ordinary deck is said to be *flush*.

Pooping, the shock of a high and heavy sea upon the stern or quarter of a ship, when she scuds before the wind in a tempest.

Port (formerly larboard), the name given to the left-hand side of the ship, when looking forward—as, the ship heels to port, top the yards to port, &c.; also, a harbour or haven.

Ports, holes in the ship's sides.

Port tack. A vessel is on the port tack when she carries her boom over the starboard quarter.

Port the helm! the order to put the helm over to the port side, so as to force the ship's head to starboard in moving ahead.

Press of sail, all the sail that a ship can set or carry.

Preventer, an additional rope employed at times to support any other, when the latter suffers an unusual strain, particularly when blowing fresh, or in a gale of wind.

Pudding, a pad made of ropes, and put round the mast under the lower yards; supported by a rope or strap called a *dolphin*.

Pump-well, an enclosure round the pumps and mainmast.

Purchase, any sort of mechanical power employed in raising or removing heavy weights.

Quarter, that part of a vessel's sides extending from the main-chains to the stern.

Quarter-deck, the portion of the upper deck abaft the mainmast.

Quarter-wind is the wind which blows from that part of the horizon situated on the quarter of the ship. See **On the quarter**.

Raft, a float composed of timber, spars, empty casks, or anything buoyant which is available; used in shipwreck, &c.

Rake, the leaning of a mast from the perpendicular.

Range of cable, a sufficient length of cable drawn upon deck, and arranged on the windlass before the anchor is cast loose, to admit of its sinking to the bottom without any check.

Ratlines, the small horizontal ropes fastened to the shrouds, by which the men go aloft.

Reach, the distance between any two points on the banks of a river, wherein the current flows in an uninterrupted course.

Ready about! a command to the crew, implying that all the hands are to be attentive and at their stations for tacking.

Reef, the part of a sail between its head and the first row of short ropes (called *points*), or between two such rows. This quantity of sail is reduced in taking in one reef, two such quantities for two reefs, and so on. A square sail reefs from its top, and a fore-and-aft sail from its foot. Also, rocks lying near the surface of the water.

Reef points, a number of ropes made fast to the sails for the purpose of reefing.

Reeve, to, to pass the end of a rope through any hole, as the channel of a block, the cavity of a thimble, &c.

Relieving tackle, a tackle hooked to the tiller in a gale of wind, in case of accident to the wheel or tiller ropes.

Rendering, the giving way or yielding freely to the efforts of some mechanical power. It is used in opposition to jambing or sticking.

Ribs of a ship, a figurative expression for the timbers which form the sides of a ship.

Ride at anchor, to, is when a ship is held by one or two anchors ahead, and is not driven by wind or tide. To *ride athwart* is to ride with the ship's side to the tide.

Riding bitts, timber heads (bitts) fixed amidships in the fore-part of the deck, to which, with the assistance of stout ropes with a knot or hook, called *stoppers*, and the compressor, the cable is secured when riding at anchor.

Rigging, a general name given to all the ropes employed to support the masts, and to arrange, extend, or reduce the sails; also, the common term for the shrouds with their ratlines, as the main rigging.

Rigging out a boom, running out a pole at the end of a yard to extend the foot of a sail.

Righting, restoring a ship to an upright position, either after she has been careened, or after she has been pressed down on her side by the wind.

Right the helm, to, is to bring it amidships, after it has been pushed either to starboard or port.

Ring-tail, a light studding-sail, shaped like a jib, set abaft the spanker in light winds.

Roach, the forward edge (leech) of a fore-and-aft sail; also, the curved shape of the bottom of a square sail.

Road, or **Roadstead**, a place near the land where ships may anchor, but which is not sheltered.

Robands, or **Ropebands**, pieces of rope having an eye worked at one end; used in pairs to tie the upper edges of the sails to their respective yards or gaffs.

Rolling, the motion by which a ship oscillates from side to side.

Rolling-tackle, extra tackles for steadying the yards in a heavy sea.

Rope is composed of threads of hemp or other stuff. These threads are called yarns; a number of yarns twisted together form a strand; and three or more strands twisted together form a rope. The various kinds of ropes used in ships, in order of strength, are hawsers (used in towing and for heavy strains), warps, ropes, lines, and small stuff. Cables are now usually made of chain, for the sake of extra strength.

Rough-tree, an unfinished spar.

Rounding, old ropes fastened on the cable, or a spar, to keep it from chafing.

Rounding-in, pulling upon any rope which passes through one or more blocks in a direction nearly horizontal; as, *round in* the weather-braces.

Rounding-up. Similar to *rounding-in*, except that it is applied to ropes and blocks which act in a perpendicular direction.

Round-turn, the situation of the two cables of a ship at anchor, after they have been three times crossed by the swinging of the ship.

Rowsing, pulling up a cable or rope without the assistance of tackles.

Rudder, the apparatus by which the ship is steered, hanging on the stern-post by pintles and braces.

Rule of the road. Rules for avoiding collisions (see page 395).

Rullock, or **Rowlocks**, the niche in a boat's gunwale, in which the oars rest in pulling.

Run, the aftermost part of a ship's bottom, where it rises and narrows towards the stern-post. Run is also the distance sailed by a ship; and is likewise used by sailors to imply the agreement to work a single passage from one place to another.

Run out a warp, to, to carry the end of a rope out from a ship, in a boat, and fasten it to some distant object, so that the ship may be moved by pulling on it.

Lag to leeward, to, to make considerable leeway.

Lagged, the state of a vessel when she has been strained, so as to droop in the centre.

Lailing trim, in, is said of a ship when in the best state for sailing.

Sails. See page 458.

Schooner, a vessel with two masts, called foremast and mainmast, and no tops. A three-masted schooner has three masts, and no tops.

Scotchman, a large batten widened at the end, or a piece of hide, or rope, used to prevent chafing.

Scud, to, to go right before a gale of wind; going in this direction without any sail set, is called *spooning*. *Scud*, low-flying clouds.

Scull, to, to propel a boat by means of a single oar at the stern. A *scull*, a small oar scooped out in the blade.

Scuppers, holes cut through the ship's sides for letting any water run off the decks overboard.

Scuttle-butt, a cask kept on deck to hold water for daily use.

Scuttles, round holes in the ship's side for ventilation.

Scuttling, boring or cutting holes through the bottom or sides of a ship, either to sink her, or to unlade her expeditiously when stranded.

Sea. A large wave is so called. A *long sea* means a uniform and steady motion of long and extensive waves; a *short sea*, on the contrary, is made when they run irregularly, are broken, and interrupted. *Cross-sea*, *much sea*, *head-sea*, are also used.

Seams, spaces between the planks of a vessel's deck or sides.

Sea-room, a sufficiently clear distance from the coast or any dangerous rocks, &c.

Seize, to, to bind or make fast by turns of small stuff, called *seizing*.

Selvagee, a strong neat strap made by twisting a small line (marline) round several rope yarns.

Send or **sand, to**, the ship's head or stern falls suddenly and violently into the trough of the sea.

Serve, to, to wind spun-yarn about a rope to prevent it from chafing or fretting. The *service* is the stuff so wound about the rope.

Serving-board, a small board or mallet for winding and tightening small stuff, as spun-yarn, round a rope. See **Serve.**

Set sail, to, to unfurl and expand the sails to the wind, in order to give motion to the ship.

Set up rigging, to, to tighten the shrouds, backstays, &c. by means of tackles on the lanyards.

Shackle, an iron link, with a movable bolt, for connecting chains together.

Shape a course, to, to direct or appoint the track of a ship, in order to prosecute a voyage.

Sharp up, the position of the yards when braced as nearly fore-and-aft as possible.

Sheathing, a covering, usually of copper sheets, nailed to the bottoms of wooden vessels.

Sheave, the roller or wheel in a block on which the rope runs.

Sheave-hole, a hole in a spar or block for a rope to work through.

Sheer. The sheer of the ship is the curve under the gunwale between the head and the stern upon her side. Also, a vessel's position when riding by a single anchor. *The ship sheers about*—that is, she goes to one side or the other of her anchor.

Sheer-hulk, an old vessel fitted with sheers.

Sheer off, to, to remove to a greater distance.

Sheers, two or more spars raised up and lashed together near their upper ends, for the purpose of getting a mast out or in.

Sheet, a rope used in setting or unfurling a sail, to keep the corner or clew of the sail down to its place. See **Clew.** In square sails, the sheets run through the extremities of the yard. In sails extended on booms, the sheet hauls the boom over from one side of the vessel to the other. The sheet keeps down the after-clew of the jib, the inner-clew of a studding-sail, and the lee-clew of the courses. See **Tack.**

Sheet home, to, to haul the sheets of a sail quite out to the block at the end of the yard-arm or boom.

Shift the helm, to, to alter its position from right to left, or from left to right.

Ship, a vessel with three masts, all of which have tops and yards.

Ship, to, to take any person, goods, or thing on board. It also implies to fix anything in its proper place—as, *to ship the oars*, to fix them in their rowlocks.

Ship-shape, in a seaman-like manner.

Shivering, the state of a sail when fluttering, through the wind striking its edge or leach.

Shoal, shallow.

Shoe, a block of wood curved and hollowed for the point of the fluke of the anchor to rest on, to prevent the anchor from tearing the planks on the ship's bow; also, a piece of wood for the heels of sheers to rest on.

Shore, a general name for the sea-coast of any country; also, a prop or stanchion.

Shorten sail, to, used in opposition to *make sail.*

Shrouds, a range of large ropes extending from the mast-heads downwards to the right and left, to support the masts and enable them to carry sail.

Sinnett, a braid formed by plaiting rope-yarns.

Sister-block, a sort of double block with two sheaves above one another.

Skin, the sides of a ship's hold.

Slack, the part of a rope that hangs loose. *Slack in stays*, said of a vessel which works slowly in tacking.

Slack-water, the interval between the ebb and

flow of the tide, when no motion is perceptible in the water.

Slings, part of the gear for securing a yard to the mast.

Slip-rope, a rope with its bight round anything, and both ends on deck, so that it may be easily slipped.

Slip the cable, to, to let it run quite out, when there is not time to weigh the anchor.

Sloop, a small vessel with one mast. A vessel of any rig in charge of a *commander*, Royal Navy.

Slue, to, to turn anything about its axis without removing it; to turn anything round or over.

Small stuff, spun-yarn, marline, and the smallest kinds of rope.

Snatch-block, a single block with an opening in its side or bottom to receive the bight of a rope.

So! an order to stop hauling when the object is in its intended position.

Sound, to, to get the depth of water.

Spar, the general term for all masts, yards, booms, gaffs, and such like (see page 456).

Spill, to, to shake the wind out of the belly of a sail, when it is drawn up in the brails, in order to furl or reef it.

Spilling lines are ropes rove in bad weather to keep the sails from being blown away, when they are clewed up.

Splice, to make two ends of ropes fast together by untwisting them, and then interweaving the strands of one piece with the strands of the other.

Split, the state of a sail rent by the violence of the wind.

Spoon-drift, water swept from the tops of the waves in a tempest, and flying like a vapour before the wind.

Spray, the sprinkling of a sea driven occasionally from the top of a wave, and not continual as in a spoon-drift.

Spring. A spring is a rope led from aft and made fast to the cable (if at anchor), or to an object a short distance off, in order to turn the ship's head round.

Spring a mast, yard, &c. to, to strain a mast, yard, &c. in blowing weather, so that some of its fibres are broken, and it is rendered unsafe for use. *To spring a leak:* when a leak first commences, a ship is said to *spring a leak*. *To spring the luff:* a ship is said to *spring her luff*, when she yields to the effort of the helm, by sailing nearer to the wind than before.

Spring-stays are rather smaller than the stays, and are placed above them to assist them.

Spring-tides are the tides at new and full moon, which flow highest and ebb lowest.

Spun-yarn, a cord formed by twisting together two or three threads of hemp or other stuff of which a rope is made.

Squall, a sudden violent blast of wind.

Square. This term is applied to yards that are very long, as *taunt* is to high masts.

Square the yards, to, to brace the yards so as to hang horizontally and at right angles with the keel.

Stanchions, fixed upright posts of wood, iron, &c. used to support anything.

Stand on, to, to continue advancing. *To stand in*, to advance towards the shore. *To stand off*, to recede from the shore. *Stand by*, be prepared.

Starboard, the right-hand side of the ship when looking forward. *Starboard tack:* a ship is said to be on the *starboard tack* when sailing with the wind blowing upon her starboard side.

Starboard the helm! an order to push the helm to the starboard side so as to force the ship's head to port when moving ahead.

Stay a ship, to, to arrange the sails and move the rudder so as to bring the ship's head to the direction of the wind, in order to get her on the other tack. *To stay a mast* is to incline it forward or aft, or on one side, by means of the stays.

Stays, large ropes coming from the mast-heads down to some other mast or to some part of the vessel to support the masts.

Steady! keep the ship's head in the same direction.

Steerage, that part between decks which is forward of the cabin.

Steerage-way, such amount of progressive motion of a ship as will give effect to the shifting of the helm.

Steering, the art of directing the ship's way by the movement of the helm.

Steeve, to, to turn up. *The bowsprit steeves too much*—that is, it is too upright.

Stem, a piece of timber into which the two sides of a ship are united at the fore-end, rising from the fore-part of the keel to form the bows.

Stem the tide, to. When a ship is sailing against the tide at such a rate as enables her to overcome its power, she is said to *stem the tide*.

Step, a block of wood secured to the vessel's keel, in which the bottom (heel) of a mast is placed.

Stern, the aftermost part of a vessel.

Stern-board, trimming the sails so as to force the ship astern.

Sternfast, a rope confining a ship by her stern to any other ship or wharf.

Sternmost, the furthest astern; opposed to *headmost*.

Stern-post, a piece of timber rising from the after part of the keel to form the stern.

Sternway, the motion of a ship going stern foremost.

Stiff, the condition of a ship when she will carry a great quantity of sail without hazard of oversetting. It is used in opposition to *crank*.

Stoppers, large ropes with a knot at one end and sometimes a hook at the other; used for various purposes; thus, being fastened

to the cable in different places abaft the bitts, they are an additional security to the ship at anchor.

Stow, to, to arrange and dispose a ship's cargo.

Strand, one of the twists or divisions of which a rope is composed. It also implies the sea-beach.

Stranded, this term, applied to a hawser or rope, signifies that one of its strands is broken; applied to a vessel, it means that she has run aground.

Stream the buoy, to, to let it fall from the ship's side into the water, previously to casting anchor.

Stretch out! the order to men in a boat to pull strongly.

Strike, to, to lower or let down anything, as sending down a mast, sails, colours.

Surf, the swell of the sea that breaks upon shore or upon any rock.

Surging, slipping the hawser up the barrel of the capstan; or slacking the cable suddenly.

Swab, a mop made of old rope for cleaning and drying the decks.

Sway away, hoist.

Sweep, a long oar used for propelling vessels.

Sweeping, the act of dragging the bight or loose part of a rope along the surface of the ground, in a harbour or road, in order to drag up something lost.

Swell, the fluctuating motion of the sea either during or after a storm.

Swifter, the forward shroud to a lower mast.

Swifting in, steadying the shrouds in their places, before putting the ratlines on them.

Swinging, the act of turning a ship round her anchor at a change of wind or tide.

Tack, the rope by which that corner or clew (of the sails called courses), which is the *weather clew* for the time being, is hauled forward and down towards the deck. The tack of a fore-and-aft sail is the rope that keeps down the lower forward corner; and of a studding-sail, the lower outer corner. The part of the sail to which the tack is attached is also called the tack. The *starboard tacks are aboard* when the vessel is sailing on the starboard tack, the *port tacks* when she is on the port tack.

Tack, to, to turn a ship about from one tack to another, passing her head to the wind. The opposite of *wearing*.

Tackle, a mechanical power obtained by a rope rove through one or more blocks. The purchases so formed are called by various names, according to their uses or arrangement of blocks.

Taffrail, or **Taffarel**, the uppermost part of a ship's stern.

Tail, a rope attached to a block to make it fast to rigging or spars.

Taking in, the act of furling the sails. Used in opposition to *setting*.

Tarpaulin, canvas covered with tar or some other composition, so as to make it waterproof; used for covering hatches, boats, &c.

Taunt, high or tall. Particularly applied to masts of extraordinary length.

Taut, tight.

Tell-tale, an instrument in the front of the poop-deck, to shew the position of the tiller; a compass hanging from the beams of the cabin.

Tending, the turning or swinging of a ship round her anchor in a tide-way at the beginning of ebb and flood, attending to the helm, and if necessary setting some sail, so as to keep turns out of the cables.

Thus! an order to the helmsman to keep the ship in her present situation, when sailing with a scant wind.

Thwart. See **Athwart.**

Thwarts, the seats across a small boat.

Thwart-ships. See **Athwart-ships.**

Tide, to, to work in or out of a river, harbour, or channel in a calm or head-wind, by favour of the tide; anchoring whenever it becomes adverse.

Tide it up, to go with the tide against the wind.

Tide-way, that part of the river in which the tide ebbs and flows strongly. *Tide-rode*: a vessel at anchor, which swings by the force of the tide, is said to be tide-rode: opposite to *wind-rode*.

Tier, a row; as a tier of casks, a tier of ships, &c. *Tier of a cable*, a range of the fakes or windings of a cable which are laid within one another, in a horizontal position. *Cable-tier*, the place where cables are stowed.

Tiller, a large piece of wood or metal put into the head of the rudder, by means of which the rudder is moved.

Timber, a general term for all large pieces of wood used in ship-building. The keel, stem, stern-post, and timbers form a vessel's outer frame.

Top, a platform placed at the head of a lower mast to spread the topmast rigging, and for the convenience of the seamen going aloft. It rests on the lower cross-trees and trestle-trees.

Topping, pulling one of the ends of a yard or boom higher than the other by hoisting on the lift. The rope for so doing is called the topping lift.

Tow, to, to draw a ship in the water by a rope fixed to a boat or other ship which is rowing, sailing, or steaming on.

Tow-line, a rope used in towing.

Transoms, certain beams or timbers extended across the stern-post of a ship to form and strengthen her after-part.

Traverse, to, to go backwards and forwards, or from side to side. *The compass traverses* when it swings freely on its centre.

Treenails, long wooden pins employed to connect the planks of the ship's side and

bottom with the corresponding timbers and beams.

Trestle-trees, two strong pieces of timber, placed horizontally fore and aft on opposite sides of a mast-head. They underlie and support the cross-trees and top, and the fid of the mast above. See **Cheeks**.

Trice, to, Trice up, to haul up and fasten.

Trim, the state or disposition by which a ship is best calculated for the purposes of navigation. *To trim the hold*, to arrange the cargo properly. *To trim the sails*, to dispose the sails in the best arrangement for the course which a ship is steering. *In ballast trim*, only ballast on board.

Trip the anchor, to, to loosen the anchor from the ground, either by design or accident.

Trough of the sea, the hollow between two waves.

Truck, a round piece of wood put upon the top of the highest masts, with sheaves on each side for the halyards of the flags to reeve through.

Truss, part of the gear for securing a yard to the mast.

Trysail, a fore-and-aft sail set with a gaff, and hoisting on a small mast behind the lower mast, called a trysail-mast. The name is usually restricted to the sail so carried at the mainmast of a full-rigged brig; those carried at the foremast and mainmast of ships and barques being called *spencers*, and that at the mizzen-mast a *spanker*.

Tumbling home, the state of a ship's sides when they fall in above the bends; the opposite to *flaring out*.

Tye, a rope connected at one end with a yard, and at the other with a tackle for hoisting.

Unbend, to, to take the sails off their yards and stays; to cast loose the anchor from the cable; to untie two ropes. Generally, to cast off or untie.

Unbitt, to, to remove the turns of a cable from the bitts.

Under foot is said of an anchor that is directly under the ship.

Under sail, or **Under-way**. When a ship is sailing, she is said to be *under-way*.

Under the lee of the shore, is to be close under the shore which lies to windward of the ship.

Unfurl, to, to cast loose the gasket of the sail.

Union, the upper inner corner of the ensign or national colours. *Union down*, the ensign hoisted with the *union* at the lower corner as a signal of distress. See **Fly**.

Unmoor, to, to reduce a ship to the state of riding at single anchor, after she has been moored. See **Moor**.

Unreeve, to, to draw a rope out of a block, thimble, &c.

Unrig, to, to deprive the ship of her rigging.

Unship, to, to discharge anything from a vessel, or to remove anything out of its place.

Uvrou, the piece of wood by which the legs of the crow-foot are extended.

Vane, a small kind of flag and pointer at a mast-head, traversing on a spindle, to shew the direction of the wind.

Vang, a rope for steadying a gaff, leading from the outer end of the gaff (the peak) to the ship's sides.

Veer, to, to let or pay out—as, veer away the cable; also, to shift—as, the wind veers—that is, it shifts, changes.

Veer and haul, to, alternately drawing a rope in and slackening it, as in warping, until the vessel gets headway.

Very well thus, keep the ship's head steady in the same direction; an order used when the ship is sailing close-hauled.

Viol, or **Voyal**, a block through which the messenger passes in weighing the anchor. A large messenger is called a *viol*.

Waist, that part of the upper deck between the quarter-deck and the fore-castle.

Wake, the path or track impressed on the water by the ship passing through it, leaving a smoothness in the sea behind it. A ship is said to come into the **wake** of another when she follows her in the same track.

Wales are strong timbers that go the whole length of a ship a little above her water-line.

Warp, a small hawser employed occasionally to remove a ship from one place to another.

Warp, to, to haul the ship by means of a warp made fast to some fixed object. When there is no fixed object, and the warp is made fast to a kedge instead, the operation is called *kedging*.

Watch, the two portions of the crew appointed to do the ordinary duties of the ship alternately with each other. The length of time each watch is on duty is also called a watch. There are seven watches in the day—viz., from noon to 4 h. P.M., 4 h. to 6 h., 6 h. to 8 h. (the last two called dog-watches), 8 h. to midnight, midnight to 4 h. A.M., 4 h. to 8 h., and 8 h. to noon.

Water-borne, the state of a ship when there is barely a sufficient depth of water to float her off the ground.

Water-line, the line made by the water's edge when a ship has her full proportion of cargo, stores, &c. on board.

Water-logged, the state of a ship which has become heavy and inactive on the sea, from a great quantity of water having leaked into her.

Water-tight, the state of a ship when not leaky.

Wear the ship, to, to change a ship's course from one tack to the other, by turning her stern to the wind.

Weather, the direction from which the wind

blows. To weather anything is to pass to windward of it.

Weather-beaten, shattered by a storm. *Weather-bitt*, an additional turn of the cable about the end of the windlass. *Weather-board*, the side of a ship that is to windward. *Weather-gage*, when a ship or fleet is to windward of another, she is said to have the *weather-gage* of her. *Weather-quarter*, that quarter of the ship which is on the windward side. *Weather-side*, the side upon which the wind blows. A *weatherly* ship, one that makes little leeway. *Weather-helm*, used to denote that the ship tends to bring her head up to the wind, requiring the helm to be put a little in that direction, or, as it is called, *up*.

Weigh anchor, to, to heave up an anchor from the bottom to the ship's bows.

Wheel, a wheel to the axis of which the tiller ropes are attached to give greater power in steering a ship. Various patents are used. In one, the wheel works a powerful screw, the tiller ropes being dispensed with.

Whip-purchase, a purchase formed by a rope rove through a single block. To *whip*, to hoist by a whip; also, to secure the end of a rope from unravelling by winding twine round it.

Winch, a mechanical power composed of a horizontal spindle with a wheel or crank at the end, working either by hand or by steam.

Wind-bound, kept in port by a contrary wind.

Windlass, the winch. Several patent kinds are in use.

Wind-rode. When a ship is at anchor, and the wind being against the tide, is so strong as to overcome its power and keep the ship to leeward of her anchor, she is said to be *wind-rode*. Opposite to *tide-rode*.

Wind's eye, the point from which the wind blows.

Windward or Weather tide, a tide that sets to windward.

Windward, to, towards that part from which the wind blows.

Wings, the parts of the hold which are next the vessel's sides.

Weald, to bind round with ropes.

Work a ship, to, to direct the movements of a ship by trimming the sails and managing the rudder.

Work to windward, to, to make a progress against the direction of the wind.

Worming, to wind small stuff spirally between the strands of a rope to give it a neat appearance.

Yacht, a vessel of pleasure of any size or rig.

Yard-arm, the ends of a yard.

Yards, the spars tapering slightly at each end, and hung by the centre to a mast, to spread square sails upon.

Yawing, the motion of a ship when she goes off her course to the right or left.

Yoke, a top piece fitting on a boat's rudder, with lines attached for steering; it is chiefly used for small boats instead of a tiller.

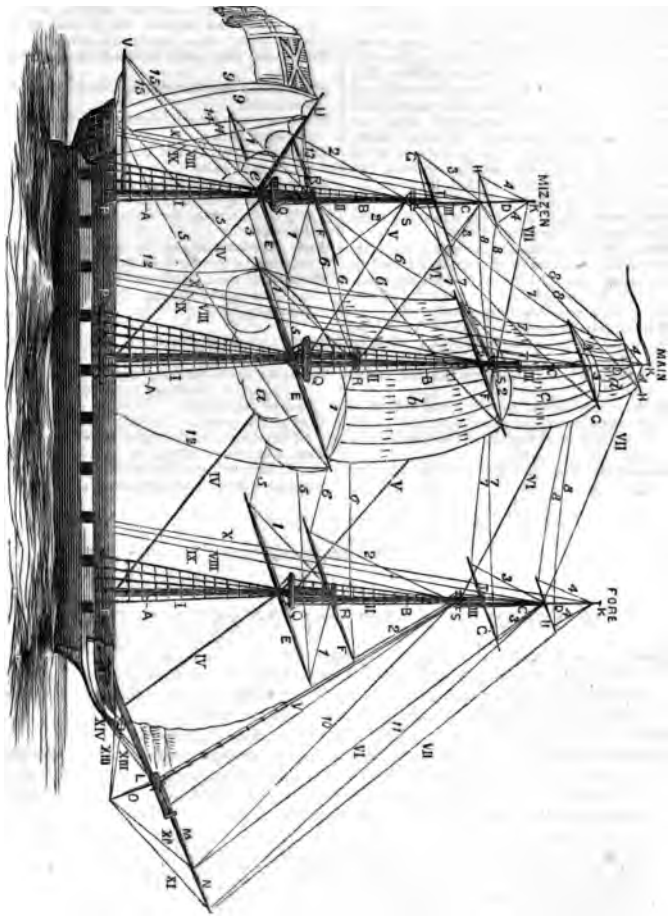
DIAGRAM OF A SHIP'S SPARS AND RIGGING.

Spars, &c.—A, mast; B, topmast; C, topgallant-mast; D, royal-mast; E, yard; F, topsail-yard; G, topgallantsail-yard; H, royal-yard; K, truck; L, bowsprit; M, jib-boom; N, flying jib-boom; O, martingale; P, chains; Q, top; R, cap; S, cross-trees; T, topmast cap; U, gaff; V, boom, or spanker-boom.

Sails.—*a*, main-sail; *b*, top-sail; *c*, topgallant-sail; *d*, royal; *e*, spanker.

Standing Rigging.—I. Shrouds; II. Topmast shrouds, crossed by ratlines; III. Topgallant shrouds; IV. Stay; V. Topmast stay; VI. Topgallant-mast stay; VII. Royal stay; VIII. Topmast backstay; IX. Topgallant-mast backstay; X. Royal backstay; XI. Flying jib-boom martingale stays; XII. Jib-boom martingale stays; XIII. Martingale backstays; XIV. Bobstays.

Running Rigging.—1, lifts; 2, topsail lifts; 3, topgallant-sail lifts; 4, royal lifts; 5, braces; 6, topsail braces; 7, topgallant braces; 8, royal braces; 9, peak signal halyards; 10, jib-stay; 11, flying jib-stay; 12, preventer-braces or jumpers; 13, peak halyards; 14, vangs; 15, topping lifts.


MIZZEN
MAIN
FORE

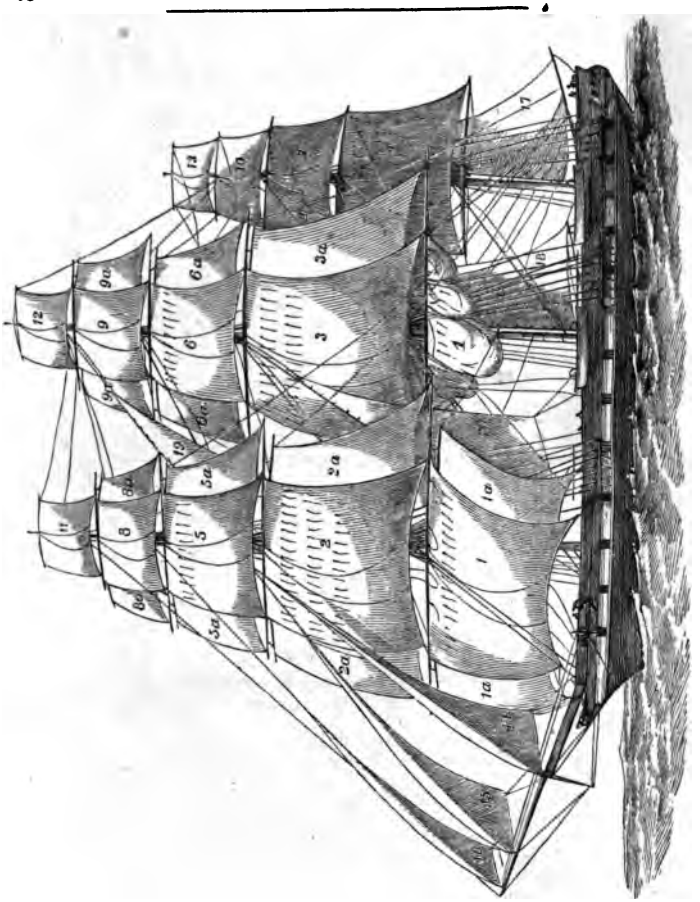


DIAGRAM OF A SHIP'S SAILS.

1, courses; 1*a*, studding-sails; 2, fore-topsail; 2*a*, studding-sails; 3, main-topsail; 3*a*, studding-sails; 4, mizzen-topsail; 5, fore-topgallant-sail; 5*a*, studding-sails; 6, main-topgallant-sail; 6*a*, studding-sails; 7, mizzen-topgallant-sail; 8, fore-royal; 8*a*, studding-sails; 9, main-royal; 9*a*, studding-sails; 10, mizzen-royal; 11, fore-skysail; 12, main-skysail; 13, mizzen-skysail; 14, fore-topmast-staysail; 15, jib; 16, flying jib; 17, spanker; 18, spenser; 19, main-royal-staysail; 20, main-topgallant-staysail; 21, mizzen-royal-staysail.

EDUCATIONAL COURSE Continued.

[illegible] and Mathematics.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] Keeping.

[illegible] Theoretical and Practical.

[illegible]

Key to Algebra.
[illegible]

[illegible] and Problems in Algebra.
[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Key to Practical Mathematics.
[illegible] Pages, Price 3s. 6d.

Mathematical Tables.
336 Pages, Price 3s.

Geometrical Chart, with Coloured Diagrams. *In a Sheet, Price 2s. 6d.*

The Sciences.

[illegible] of Knowledge.
[illegible], *Price 6d., stiff cloth.*

Introduction to the Sciences.
[illegible] *Wood-cuts, Price 1s.*

[illegible] *revised and improved.*

Motion and Motion.
[illegible], 35 *Wood-cuts, Price 10d.*

[illegible]anics, Theoretical & Practical.
[illegible], 110 *Wood-cuts, Price 1s.*

[illegible]statics and Pneumatics.
[illegible], 44 *Wood-cuts, Price 10d.*

[illegible]tics.
[illegible] *Wood-cuts, Price 1s.*

[illegible]
[illegible] *Wood-cuts, Price 1s.*

[illegible]
[illegible] *Wood-cuts, Price 1s.*

Electricity. By Dr Ferguson.
284 *Pages, 148 Wood-cuts, Price 3s. 6d.*

Meteorology.
82 *Pages, 11 Wood-cuts, Price 1s.*

Natural Philosophy. Vol. I.
340 *Pages, 207 Wood-cuts, Price 3s.*

Natural Philosophy. Vol. II.
404 *Pages, 181 Wood cuts, Price 3s.*

Chambers's Scientific Charts.
Containing upwards of [illegible] TORIAL ILLUSTRATIONS. [illegible] *each, or mounted on cloth* [illegible]

Geology.
228 *Pages, 53 Wood-cuts, Price 1s.*

Zoology. Edited by R. [illegible]
262 *Pages, 243 Wood-cuts, Price 1s.*

EDUCATIONAL COURSE—Continued.

Inorganic Chemistry, with Old and New Nomenclature. *New Edition.* 396 *Pages*, 69 *Wood-cuts*, *Price* 3*s.* 6*d.*

☞ Prepared by the late PROFESSOR GEORGE WILSON, and carefully revised by DR STEVENSON MACADAM.

Practical Chemistry, with Old and New Nomenclature. A Companion to the Laboratory and Testing-Room. By DR MACADAM. *Price* 2*s.* 6*d.*

Organic Chemistry, with New Notation. (*In preparation, Nov.* 1866.)

Navigation. By JAMES PRYDE, F.E.I.S.

Animal Physiology. [*In preparation.*

108 *Pages*, 39 *Wood-cuts*, *Price* 1*s.* 6*d.*

Vegetable Physiology.

128 *Pages*, 80 *Wood-cuts*, *Price* 1*s.* 6*d.*

Political Economy.

In simple language. 160 *Pages*, *Price* 1*s.* 6*d.*

Miscellaneous Questions with Answers. By W. CHAMBERS, F.R.S.E., F.G.S. *Price* 2*s.* 6*d.*

Historical and Miscellaneous Questions, with Answers. *Price* 4*s.* 6*d.*

Being the Historical and Miscellaneous Questions bound together.

Questions on British History. *Price* 1*s.*

Historical Questions. *Price* 2*s.* 6*d.*

Latin Series.

Edited by DR SCHMITZ, late Rector of the High School, Edinburgh. Illustrated with copious English Notes and Prefaces.

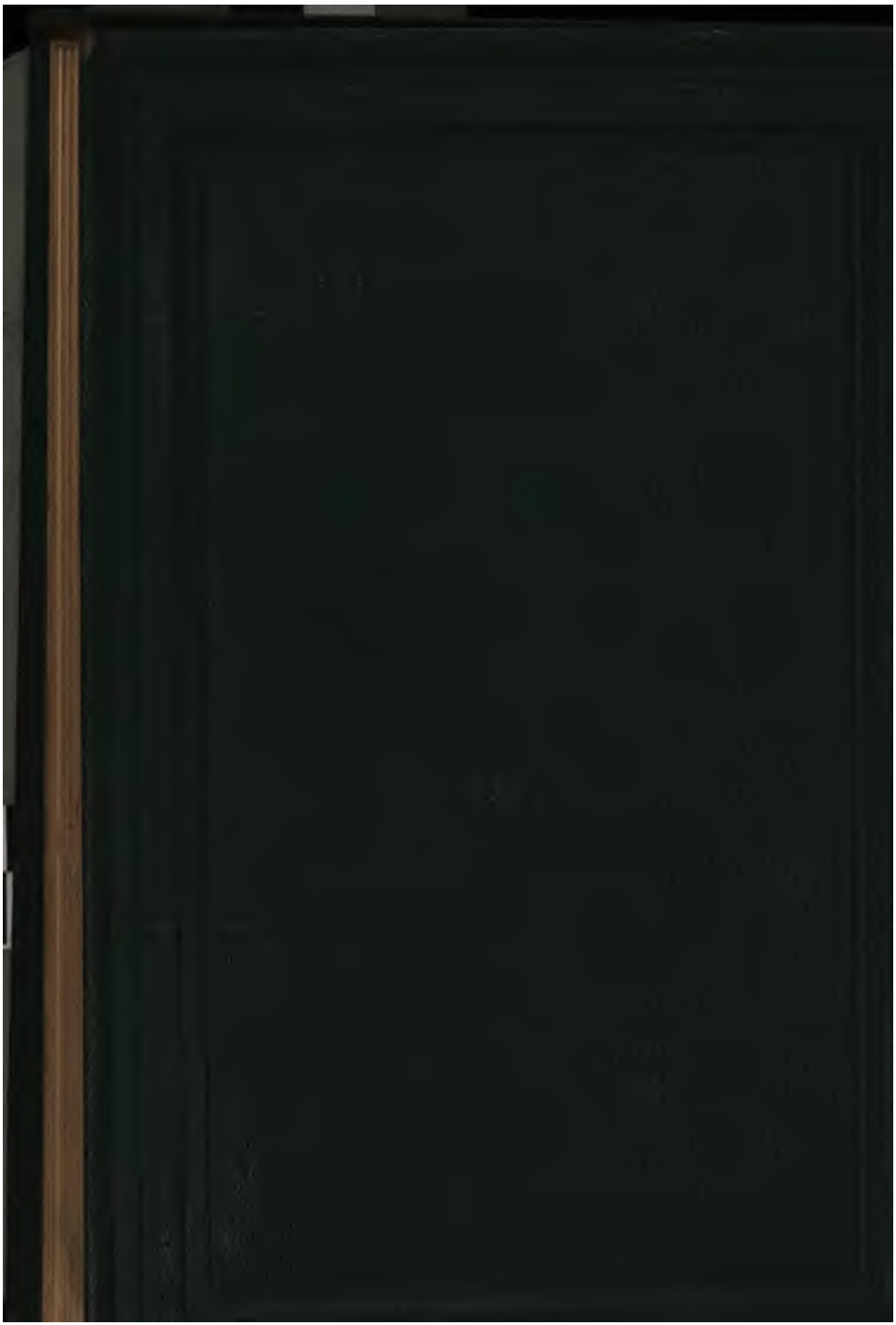
	Pages.	s.	d.
Ruddiman's Latin Rudiments,	76	0	10
Latin Grammar, Elementary,	212	2	0
——————, Advanced,	444	4	0
Latin Exercises, Elementary,	148	1	6
——————, Advanced,	272	2	6
Key to Advanced Latin Exercises,	142	2	0
Phædrus's Fables,	126	1	6
Nepos,	222	2	0
Cæsar,	256	2	6
Sallust,	176	1	6
Quintus Curtius,	352	3	0
Ovid,	272	3	0
Horace,	320	3	0
Virgil—Vol. I.; Bucolics and Æneid, Books I. to VI.	304	3	0
Virgil—Vol. II.; Æneid, Books VII. to XII., and Georgics; with INDEX NOMINUM ET RERUM,	296	3	0
Livy,	364	3	0
Cicero,	320	3	0
Latin Dictionary—			
Latin and English, bound,	784	6	0
Latin-English Part, "	418	3	6
English-Latin Part, "	366	3	6

German Series.

Edited by DR AUE, late German Master in the High School, Edinburgh.

	Pages.	s.	d.
First German Reading-Book,	153	1	6
Second German Reading-Book,	222	2	6
German Grammar, Elementary,	144	1	6
German Grammar, Advanced,	346	3	0
Phrase-Book, English-German,	156	1	6
German Synonyms, Dictionary of,	192	1	6
German Dictionary—			
German and English, bound,	954	7	6
German-English Part, "	448	4	0
English-German Part, "	506	4	0

Minor Educational Course.

No. 1. Introduction to Reading, 1½*d.*
2. Reading Lessons, 1½*d.*
3. Arithmetical Primer, 1½*d.*
——————— Key, 1*d.*
4. Grammatical Primer, 1½*d.*
5. Outlines of Geography, 1½*d.*
6. History, 1½*d.*

The above Treatises (the Key excepted) in One Volume, Price One Shilling.